

An Estimable Equilibrium Model of Labor Market Search and Schooling Choice*

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1 Introduction

A large number of papers, both theoretical and applied, have examined labor market phenomena within the search and matching framework, with some embedded in a simple general equilibrium setting.¹ Virtually all of the empirical work performed using this framework has assumed that individual heterogeneity, determined by the time of entry into the labor market, is exogenously determined. Perhaps the most important, observable correlate of success in the labor market is schooling attainment. In this paper we extend the standard search and matching framework to allow for endogenous schooling decisions.²

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¹A large number of macroeconomic labor applications are cited in Pissarides (2000) and the recent survey by Shimer (2005). In terms of econometric implementations of the model, examples are Flinn and Heckman (1982), Eckstein and Wolpin (2005), Postel-Vinay and Robin (2002), Dey and Flinn (2005), Cahuc et al (2006), and Flinn (2006).

²There are a number of ambitious empirical papers which estimate life cycle individual decision rule models of schooling choice and labor market behavior, such as Keane and Wolpin (1997) and Sullivan (2010). This approach has been extended to allow for the endogenous determination of rental rates for various types of human capital, most notably by Lee (2005) and by Lee and Wolpin (2006). However, these

We develop a simple model of schooling investment decisions, where higher levels of schooling investments are (generally) associated with better labor market environments. Individuals are differentiated in terms of initial ability, a , and the heterogeneity in this characteristic, along with the structure of the labor market, is what generates equilibrium schooling distributions. As is standard, we utilize axiomatic Nash bargaining to determine the division of the surplus between workers and firms. For simplicity, and due to the nature of the data we utilize, we assume that employed individuals do not receive alternative offers of employment, i.e., there is no on-the-job search.³

There is a longstanding literature examining the essence of the hold-up problem and the role contracts play to reduce, or altogether avoid, hold-up (see Malcomson 1997; Acemoglu 1996 and 1997; and Card et al. 2009 for reviews). At the core of the problem is the notion that investments must be made before agents meet and, thus, greater market frictions generally lead to more serious hold-up problems. Acemoglu and Shimer (1999) examine the potential for hold-up problems in frictional markets and the ways in which markets can internalize the resulting externalities. Their focus is on identifying ways in which hold-up and inefficiencies can be mitigated in labor markets characterized by ex-ante worker and firm investments and search frictions and find that this can be achieved in wage-posting models with directed search.⁴

In a recent paper, Card et al. (2009) use matched employer-employee records from an Italian administrative data set to estimate within-job models of rent-sharing and hold-up. They find strong evidence of rent sharing and that workers receive lower wages at firms with higher capital per worker, which is consistent with the absence of hold up. Unlike this work, our model assumes only one-sided (worker) investment because we do not have access to firm investment data. We do not consider this to be an important limitation, however, as most empirical studies of hold-up problems have focused on the issue on the demand side, and hold-up effects on workers' investments have been examined to a lesser degree. Furthermore, our theoretical model can easily be extended to examine how premarket investment for firms is appropriated by their bargaining power when bargaining over division of the surplus from the match.

The generalized Nash bargaining power parameter associated with the worker has a direct impact on the extent of the hold-up problem the worker faces vis-a-vis pre-market schooling investment decisions. While there are a number of estimates of the bargaining power parameter within models of Nash bargaining and matching, the estimates tend to vary significantly with the assumptions made regarding the presence of on-the-job (OTJ)

frameworks do not allow investigation of surplus division issues and the hold-up problem since they are based on a competitive labor market assumption.

³Adding on-the-job search alters the details of what constitutes "bargaining power" in the market, but not the fact that a lack of "generalized" bargaining power, which may include the possibility of renegotiation of contracts as in Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al (2006), is positively affects the individual's incentive to invest in human capital.

⁴It is well-known that wage-posting models have their requirements of commitment to mitigate the incentives of firms to renegotiate contracts with individual workers.

search, and given OTJ search, the nature of the renegotiation process, as well as the data set used in estimation. In their search, matching, and Nash bargaining frameworks, Dey and Flinn (2005), Cahuc et al. (2006), and Flinn and Mabili (2009) found that allowing for OTJ search substantially reduced the estimate of the worker's bargaining power parameter in comparison with the case in which OTJ search was not introduced (e.g., Flinn 2006). To some degree, this is a result of allowing for Bertrand competition. When competition between firms is introduced, substantial wage gains over an employment spell can be generated simply from this phenomenon, even when the individual possesses little or no bargaining power in terms of the bargaining power parameter. Indeed, the (approximately) limiting case of this is that considered by Postel-Vinay and Robin (2002), in which workers possessed no bargaining power whatsoever. While the hold-up problem would seem to be particularly severe in this case, to the extent that individuals would have no incentive to invest in human capital, this is not the case when Bertrand competition between competing potential employers occurs, which is when the individual can recoup some of the returns to her pre-market investment. Incentives to invest in their model are directly related to the contact rates with other potential employers in the course of an employment spell, most importantly, as well as the other rates of event occurrence (i.e., the offer arrival rate in the unemployed state and the rate of exogenous separation).

As our model structure makes clear, simply estimating separate behavioral models of the labor market for different schooling classes is at a minimum inefficient, and, more seriously, may lead to misinterpretations of labor market structure. For this reason, whenever possible, potentially endogenous individual characteristics acquired before or after entry into the labor market should be incorporated into the structure of the search, matching, and bargaining model. In order to do so in a tractable manner requires stringent assumptions regarding the productivity process, bargaining, etc., as is evident in what follows. Using our simple, and quite tractable model, we are able to make some preliminary judgements regarding the impact of hold-up on schooling investment. We find that it is quite sensitive.

The plan of the paper is as follows. In Section 2, we develop a bargaining model in a partial equilibrium framework, with education decisions made prior to entering the labor market. Section 3 extends the basic model to allow schooling submarkets to be characterized by different vectors of primitive parameters, such as contact and dissolution rates. Section 4, not available in this draft, extends the model to allow for the endogenous determination of contact rates through vacancy creation decisions of firms and school-level specific unemployment rates. In Section 5 we describe the sample used to estimate the model and discuss identification of model parameters and the estimator used. Section 6 contains the empirical analysis, as well as an illustration of the strong impact of bargaining power on schooling investment. Section 7 will cont the model is estimated. We present the estimation results in Section 5. In Section 6, these estimates are used to perform welfare experiments that involve the selection of an optimal minimum wage. A conclusion is offered in Section 7 (not available in this draft given its preliminary and incomplete nature).

2 Model with Homogeneous Schooling Markets

The output at a match is given by

$$y = ah \theta,$$

where θ is i.i.d. with c.d.f. G , h is the individual's human capital level, and a is individual ability, which is a permanent draw from the distribution F with corresponding density f , which has support (\underline{a}, \bar{a}) , with $0 \leq \underline{a} < \bar{a} < \infty$.

We restrict our attention to the case of $S = 2$, where $s = 1$ corresponds to high school and partial college, roughly, and $s = 2$ to college completion.⁵ Going to college increases the individual's productivity, so that

$$h_1 = 1 < h_2,$$

where the first equality is essentially an inconsequential normalization.

The problem can be made completely straightforward if we make the following set of assumptions.

1. All parameters describing the labor market are independent of schooling status with the exception of h . (This can easily be weakened, which is done in the next section.)
2. The flow value of unemployment to a type a individual with schooling level s is given by

$$b(a, s) = b_0 ah.$$

This last assumption is similar to that made in Postel-Vinay and Robin (2002) and in Bartolucci (2009).

Since we use data from the Current Population Survey, and thus the information only consists of a point sample of the labor market process, we assume no on-the-job (OTJ) search. We begin by considering the case in which, across schooling "submarkets," all job search environments are identical (i.e., they have identical parameters $\alpha_1 = \alpha_2$, $\eta_1 = \eta_2$, etc.). In this case, the value of search to an individual of type (a, h) can be summarized solely in terms of the product $\nu \equiv ah$, and the value of unemployed search to such an individual is given by $V(\nu)$. In terms of the Nash bargaining problem, the worker-firm pair solves

$$\max(V(w, \nu) - V(\nu)) V(w, \theta)^{1-\beta},$$

⁵This classification was determined to some extent empirically. Our original classification scheme grouped together all those sample members who had completed some level of schooling beyond high school. We found that those who had attended college but not completed it were far more similar, in terms of labor market outcomes, to those with only a high school education than to those who had completed four years of college. As a result, we grouped together all those who had not completed at least a four-year college degree. Even with this classification, over 1/3 of our sample of 30-34 year old males fell into schooling class $s = 2$.

where

$$\begin{aligned} V(w, \nu) &= \frac{w + \eta V(\nu)}{\rho + \eta} \\ V(w, \theta, \nu) &= \frac{\theta\nu - w}{\rho + \eta}. \end{aligned}$$

Note that we have assumed that the firm's outside option under Nash bargaining is equal to 0, which is consistent with the common free entry condition that drives the value of an unfilled vacancy to 0. The solution to the Nash bargaining problem yields

$$w(\theta, \nu) = \alpha\theta\nu + (1 - \alpha)\rho V(\nu),$$

and since

$$\rho V(\nu) \equiv y^*(\nu) = \nu\theta^*(\nu),$$

we have

$$w = \nu(\alpha\theta + (1 - \alpha)\theta^*(\nu)). \quad (1)$$

In terms of the value of unemployed search given ν , we have

$$\begin{aligned} \rho V(\nu) &= b_0\nu + \lambda \int_{*(\)} (V(\nu, \theta) - V(\nu)) dG(\theta) \\ \Rightarrow \nu\theta^*(\nu) &= b_0\nu + \frac{\lambda\alpha\nu}{\rho + \eta} \int_{*(\)} (\theta - \theta^*(\nu)) dG(\theta). \end{aligned} \quad (2)$$

Since this last equation is independent of ν , we have

$$\theta^*(\nu) = \theta^* \text{ for all } \nu,$$

which means that the reservation output value for an individual of ability a with schooling level s is simply

$$y^*(a, s) = ah \theta^*. \quad (3)$$

This result makes the consideration of the schooling choice problem considerably more straightforward. When an individual of type a has schooling level s and enters the labor market, the expected value of the labor market career is given by $V(ah)$. Then for a type a individual, the value of schooling level s at the time of entry into the labor market is

$$V(ah) = \rho^{-1} ah \theta^*.$$

There is no monetary cost associated with completing schooling level 1, and the present value of the monetary cost associated with completing schooling level 2 is given by $c_2 > 0$ at time τ_1 , when schooling level 1 is completed. The first time that the individual can decide to exit school is at the completion of compulsory schooling, which is $s = 1$. The additional

time it takes to complete schooling level 2 is given by τ_2 . We assume for simplicity that the cost c_2 includes all monetary and psychic costs incurred during the completion of schooling level 2. Then an individual of type a will choose schooling level 2 if and only if

$$\begin{aligned} \exp(-\rho\tau_2)\rho^{-1}ah_2\theta^* - \rho^{-1}a\theta^* &\geq c_2 \\ \Rightarrow a\{\exp(-\rho\tau_2)h_2 - 1\} &\geq \frac{\rho c_2}{\theta^*}. \end{aligned}$$

The left hand side is linear in a , and strictly increasing when

$$\exp(-\rho\tau_2)h_2 > 1. \tag{4}$$

Now assume that

$$\underline{a}\{\exp(-\rho\tau_2)h_2 - 1\} < \frac{\rho c_2}{\theta^*} \tag{5}$$

$$\bar{a}\{\exp(-\rho\tau_2)h_2 - 1\} > \frac{\rho c_2}{\theta^*}, \tag{6}$$

We have the following result.

Proposition 1 *Under conditions (4), (5), and (6), there exists a unique value*

$$a^* = \frac{\rho c_2}{\theta^* \{\exp(-\rho\tau_2)h_2 - 1\}},$$

such that an individual of type a chooses $s = 2$ if and only if

$$a^* \leq a \leq \bar{a}.$$

Proof. The fact that the payoff to college is linearly increasing in a and that for the least able person choice $s = 1$ dominates and for the most able person choice $s = 2$ dominates ensures that there exists a marginal ability type, a^* , who is indifferent between the two choices and that the sets of individuals choosing each schooling level are connected. ■

There are two comments we wish to make regarding this result. First, (4) is required for any agent to acquire schooling beyond the mandatory level. If this condition is not satisfied, no individual completes college. Since we do see individuals completing college in the data, a substantial number of them, this condition is required for the model to have any prima facie validity. Second, given the satisfaction of (4), the more able individuals go to college. In this sense, the net payoff to college attendance is supermodular in the two arguments (a, s) , where a is continuous and s is discrete (in fact, binary in the case we are currently considering).

2.1 Comparative Statics Results

Given the simplicity of the decision rule, comparative statics results are easily derived. For the most part, they are intuitively reasonable, which is a strength of this modeling setup.

The focus of the paper is schooling decisions. In our two schooling class model, we can summarize the schooling distribution in terms of the probability that a population member graduates from college, the likelihood of which is

$$P_2 \equiv P(s = 2) = \tilde{F}(a^*),$$

where \tilde{F} denotes the survivor function associated with the random variable a . The results are:

1. $\partial P_2 / \partial c_2 < 0$. The proportion of the population attending college is decreasing in the direct costs of college attendance.
2. $\partial P_2 / \partial \tau_2 < 0$. The proportion of the population attending college is decreasing in the time it takes to complete college, which is simply another form of (opportunity) cost associated with continuing education.
3. $\partial P_2 / \partial h_2 > 0$. This is perhaps the most intuitive result. The greater the impact on labor market productivity, the more individuals complete college.
4. $\partial P_2 / \partial \theta^* > 0$. Now θ^* is not a primitive parameter of course, but most primitive parameters characterizing the labor market only affect the schooling decision through θ^* , which is a determinant of the value of search for all agents (recall that the critical output level for job acceptance is $ah \theta^*$). Through this value, we can determine the impact of the most of the various labor market parameters on the schooling decision.
 - (a) $\partial P_2 / \partial \lambda > 0$. Increases in the arrival rate of offers increase θ^* , and hence increase the value of having a higher productivity distribution.
 - (b) $\partial P_2 / \partial \eta < 0$. Increases in the (exogenous) separation rate decrease θ^* and hence the value of becoming more productive when matched with an employer.
 - (c) $\partial P_2 / \partial b_0 > 0$. Increases in the “baseline” flow value of occupying the unemployment state increase the value of that state and the value of going to college.
5. $\partial P_2 / \partial \rho < 0$. To be consistent with the definitions of the utility flow associated with employment, which is equal to the wage, the cost measure c_2 is defined as

$$\begin{aligned} c_2 &= \int_0^{\infty} e^{-\rho t} \tilde{c}_2 dt \\ &= \frac{\tilde{c}_2}{\rho} (1 - \exp(-\rho \tau_2)). \end{aligned}$$

Then the critical schooling ability level is given by

$$a^* = \frac{\tilde{c}_2(1 - \exp(-\rho\tau_2))}{\theta^* \{\exp(-\rho\tau_2)h_2 - 1\}}.$$

It follows that

$$\begin{aligned} \operatorname{sgn} \left(\frac{\partial a^*}{\partial \rho} \right) &= \operatorname{sgn} \{ \tau_2 \exp(-\rho\tau_2) \tilde{c}_2 \theta^* [\exp(-\rho\tau_2)h_2 - 1] \\ &\quad - \tilde{c}_2(1 - \exp(-\rho\tau_2))(\exp(-\rho\tau_2)h_2 - 1) \frac{\partial \theta^*}{\partial \rho} \\ &\quad + \tilde{c}_2(1 - \exp(-\rho\tau_2)) \tau_2 \exp(-\rho\tau_2) h_2 \theta^* \}. \end{aligned}$$

The terms on the first and third line of the right hand side are unambiguously positive. Since θ^* is decreasing in ρ , we have our result.

The main comparative statics result, which is the focus of the paper, concerns the effect of bargaining power α on schooling. While the result is obvious at this point, we state it more formally than the other results.

Proposition 2 *Increases in bargaining power on the workers' side of the market result in increases in schooling level. or*

$$\frac{\partial P_2}{\partial \alpha} > 0.$$

2.2 Empirical Implications

Here we consider the model's implications for the labor market outcomes of individuals in the two schooling classes. In particular, unemployment rates and wage distributions for the two schooling classes.

2.2.1 Unemployment Experiences

Under our modeling assumptions, the steady state unemployment rate for an individual of type ν ($= ah$) is independent of ν . This is due to the fact that the likelihood that any job is acceptable to an individual of type ν is simply $\tilde{G}(\theta^*)$, which is obviously independent of ν . The proportion of time and individual of type ν spends in unemployment, or the steady state probability that they will occupy the unemployment state, is simply

$$P(U|\nu) = \frac{\eta}{\eta + \lambda \tilde{G}(\theta^*)} = P(U).$$

Thus, the assumption that the primitive parameters are identical across schooling groups produces the implication that there is no difference in unemployment experiences across schooling groups.

2.2.2 Wage Distributions

The distributions of wages are obvious modifications of those derived for the homogeneous schooling markets case. We assume that the support of the common matching distribution G is the nonnegative real line, and the G is everywhere differentiable on its support with corresponding density g . We have established that the schooling continuation set is defined by $[a^*, \bar{a}]$. Now, from (1) we know that

$$\theta = \frac{-(1-\alpha)\theta^*}{\alpha},$$

where $\nu = ah$ and the lower limit of the wage distribution for an individual of type ν is $\underline{w}(\nu) = \nu\theta^*$. Then the cumulative distribution function of wages for a type ν individual is

$$F(w|\nu) = \frac{G(\alpha^{-1}(-(1-\alpha)\theta^*)) - G(\theta^*)}{\tilde{G}(\theta^*)}, \quad w \geq \nu\theta^*,$$

and the corresponding conditional wage density is given by

$$f(w|\nu) = \frac{1}{\alpha\nu} \frac{g(\alpha^{-1}(-(1-\alpha)\theta^*))}{\tilde{G}(\theta^*)}, \quad w \geq \nu\theta^*.$$

Now we consider the wage densities by schooling class. For this purpose, we write

$$f_{|}(w|a, s) = \frac{1}{\alpha ah} \frac{g(\alpha^{-1}(\frac{w}{s} - (1-\alpha)\theta^*))}{\tilde{G}(\theta^*)}, \quad w \geq ah\theta^*.$$

Then the marginal density of wages in schooling class s is given by

$$f_{|}(w|s) = \frac{1}{\alpha h \tilde{G}(\theta^*)} \int a^{-1} g(\alpha^{-1}(\frac{w}{ah} - (1-\alpha)\theta^*)) dF(a|s), \quad w \geq h \underline{a}(s)\theta^*,$$

where $\underline{a}(s)$ denotes the lowest ability individual who makes schooling choice s . Given the simple form of the schooling continuation decision, the density of wages among those with a high school education is

$$f_{|}(w|s=1) = \frac{1}{\alpha \tilde{G}(\theta^*)} \int_{-}^{*} a^{-1} g(\alpha^{-1}(\frac{w}{a} - (1-\alpha)\theta^*)) \frac{dF(a)}{F(a^*)}, \quad w \geq \underline{a}\theta^*, \quad (7)$$

while the density of wages among the college-educated population is

$$f_{|}(w|s=2) = \frac{1}{\alpha h_2 \tilde{G}(\theta^*)} \int_{*}^{-} a^{-1} g(\alpha^{-1}(\frac{w}{ah_2} - (1-\alpha)\theta^*)) \frac{dF(a)}{\tilde{F}(a^*)}, \quad w \geq a^* h_2 \theta^*. \quad (8)$$

The conditional wage densities for the two schooling groups differ, then, not only because college education improves the productivity of any individual who acquires it, but

also through the systematic selection induced on the unobserved ability distribution F by the option of going to college. In terms of the conditional (on s) wage distributions, we note that the upper limit of the support of both distributions is ∞ . The distributions do differ in their lower supports, with this lower bound equal to $\underline{a}\theta^*$ for those with high school education and $a^*h_2\theta^*$ for those with college. Since $a^*h_2 > \underline{a}$, the lower support of the distribution of the college wage distribution lies strictly to the right of the high school wage distribution.

Proposition 3 *The wage distribution of the college educated first order stochastically dominates that of the high school educated.*

Proof. Since $h_2 > h_1 = 1$, for any a , $F(w|a, h_2)$ first order stochastically dominates $F(w|a, h_1)$. For any s , $F(w|a', s)$ first order stochastically dominates the distribution $F(w|a, s)$ whenever $a' > a$. Since $a' \geq a^* > a$ for all $a' \in [a^*, \bar{a}]$ and $a \in [\underline{a}, a^*)$, the mixture distributions are strictly ordered in the sense

$$F_{|1}(w|1) \geq F_{|2}(w|2) \text{ for all } w \geq \underline{a}\theta^*.$$

■

From this result, it immediately follows that the average wage is greater among the college educated. More importantly, the wages of the college-educated exceed those with a high school education at every quantile of the respective distributions.

Before proceeding to investigate some extensions of the basic model, we want to examine some descriptive evidence regarding the empirical implications of the model. The data used in all of the empirical analysis below will be described in more detail in the sequel. In terms of the general characteristics of the sample, it is drawn from monthly Current Population Survey samples from 2005, and consists of males living in CPS households who were between the ages of 30 to 34, inclusive when interviewed. The “high schooling” category, corresponding to $s = 2$, consists of individuals who have completed (at least) a four year college program. The “low schooling” category is all others. The hourly wage data are taken from sample members who were employed at the time of the interview, and are the actual hourly wages if the individual is paid on this basis or are imputed by dividing usual weekly earnings by usual weekly hours. We eliminated outliers by trimming the lowest and highest 2.5 percent of wage observations from both schooling subsamples.

Figure 1.a and 1.b contain the plots of the wage distributions by schooling group. The minimum wage observed (after trimming) for the low schooling group is 6.00 and for the high schooling group is 7.50. We see from these figures that the wages of the low schooling group members are highly concentrated in the range 6 to 20 dollars, while the high schooling group wages show considerably more dispersion. Figure 1.c displays the distribution of total wages. The percentage of the wage sample who have completed a college degree is 33.5, thus the total wage distribution is heavily shaped by the the low

schooling wage distribution. This is not the case at high wage levels, where virtually all observations are associated with high schooling level individuals.

Proposition 3 implies that the high schooling wage distribution first order stochastically dominates the low schooling wage distribution. Figure 1.d presents evidence regarding this implication. The figure plots

$$F(w_{(1)}|s = 1) - F(w_{(1)}|s = 2)$$

for an increasing sequence of wages, $w_{(1)} < \dots < w_{(n)}$. First order stochastic dominance implies that all values in this sequence should be nonnegative, and the figure strongly bears out this claim.

3 Separate Schooling Sub-Markets

We continue within the partial equilibrium setting of the previous section, but consider relaxing some of the more restrictive (from an empirical perspective) features of that model. In particular, we know from the large number of structural estimation exercises involving search models that the primitive parameters across sub-markets are often found to be markedly different (see, for example, Flinn (2002)). In particular, it is often noted that the unemployment rate differs across schooling groups, with those with lower completed schooling yields often having lengthier and more frequent unemployment spells. As we saw above, such a result is not consistent with the assumption that all primitive labor market parameters are the same across schooling classes.

The situation we consider is one in which each schooling class inhabits a sub-labor market, which has its own market-specific parameters (λ, η, α) . The parameter ρ , being a characteristic of individual agents (individuals and firms), is assumed to be homogeneous across labor markets, as is the baseline unemployment utility flow parameter b_0 . The match productivity distribution G is also identical across markets. In terms of the productivity of an individual, nothing has changed from the previous case, since $y(a, s, \theta) = ah\theta = \nu\theta$, so that the distribution of y is a function of the scalar ν and the common (to all matches) distribution G . However, it is no longer the case that the critical match value will be the same across schooling sub-markets. Because primitive parameters differ across markets, ν is no longer a sufficient statistic for the value of search of an individual; instead (ν, s) is. This is clear if we reconsider the functional equation determining the value of search in the homogeneous sub-markets case, which was given in (2), adapted to the heterogeneous case. Then we have

$$\nu\theta^*(\nu, s) = b_0\nu + \frac{\lambda\alpha\nu}{\rho + \eta} \int_{\theta^*(\nu, s)}^{\infty} (\theta - \theta^*(\nu, s))dG(\theta).$$

The solution $\theta^*(\nu, s)$ now clearly is independent of ν , but is not independent of s . Thus there is a common critical value $\theta^*(s)$ shared by all individuals with schooling choice s , which is independent of their ability a .

We now turn to the schooling choice decision in this case. The critical match value for an individual of type a in schooling market s is given by $ah \theta^*(s) = \nu \theta^*(s)$, so that the value of unemployed search in this submarket is given by $\rho^{-1} \nu \theta^*(s)$. Then we have that the net value of college education to an individual of type a is

$$\exp(-\rho\tau_2) \rho^{-1} ah_2$$

Comparative statics results are fundamentally different in this case in the sense that certain market-specific primitive parameters only impact the value of unemployed search within their particular submarket. By simple extension of the homogeneous results above, the results regarding $\partial P_2/\partial c_2 < 0$ and $\partial P_2/\partial \tau_2 < 0$ remain the same, since the cost structure of acquiring schooling is identical in the two cases. It is also clearly the case that $\partial P_2/\partial h_2 > 0$. The main departure from the previous case regards the presence of $\theta^*(1)$ and $\theta^*(2)$. We note that

1. $\partial P_2/\partial \theta^*(1) < 0$. As before, $\theta^*(1)$ is not a primitive parameter, but the primitive parameters specific to submarket 1 only affect the schooling decision through $\theta^*(1)$. Then
 - (a) $\partial P_2/\partial \lambda_1 < 0$. Increases in the arrival rate of offers in the low-schooling market increase $\theta^*(1)$, and increase the relative value of a low schooling level.
 - (b) $\partial P_2/\partial \eta_1 > 0$. Such an increase decreases the value of a low schooling level.
2. $\partial P_2/\partial \theta^*(2) > 0$.
 - (a) $\partial P_2/\partial \lambda_1 > 0$
 - (b) $\partial P_2/\partial \eta_1 < 0$
3. Perhaps most interesting is the impact of market-specific bargaining powers α on the schooling decision. When there is one bargaining power parameter that holds throughout all educational labor markets, the meaning of hold-up is relatively unambiguous. When there are market-specific bargaining power parameters, a relative notion of hold-up is more appropriate. Clearly we have

$$\frac{\partial P_2}{\partial \alpha_1} < 0.$$

$$\frac{\partial P_2}{\partial \alpha_2} > 0.$$

It is important to note that α_2 could be quite low, and yet a substantial proportion of agents may choose the high schooling level if α_1 is significantly lower yet.

3.2 Empirical Implications

There are a few obvious differences in the empirical implications of the homogeneous and heterogeneous labor market models.

3.2.1 Unemployment

The characteristic scalar ν is no longer sufficient for describing an individual's probability of

5 Econometric Issues

We will devote most of our attention to identification of the primitive parameters in the homogeneous markets case, that is, the model specification in which all parameters are the same across markets. We also confine our attention to the partial equilibrium version of the model in which the contact rate λ is treated as a primitive parameter.

5.1 Identification

The primitive parameters in the homogeneous markets case are $\rho, b, \lambda, \eta, F, G, h_2$, and c_2 . Much of the identification analysis can be conducted using results from Flinn and Heckman (1982) after noting which of the parameters determine labor market outcomes explicitly once we condition on the observed value of schooling, s .

As we showed above, conditional on s , randomness in labor market outcomes (across individuals and over time) is generated by the two independent random variables ν and θ . As we showed, under our model assumptions the critical match value θ^* is independent of s . We also showed that the model implies that all individuals with an ability level less than a^* chose $s = 1$, while all others choose $s = 2$ (college completion). Under the normalization $h_1 = 1$, the minimum wage that could be observed for the low-schooling group is

$$\underline{w}_1 = \theta^* \underline{a},$$

while the lowest wage the could be observed for the high-schooling group is

$$\underline{w}_2 = \theta^* a^* h_2.$$

Just as Flinn and Heckman (1982) showed that parametric assumptions were, in general, necessary to recover the parameters of the wage offer distribution in the partial-partial equilibrium search case, they will also be necessary here for similar reasons. Parametric assumptions on F also include the specification of the support of the distribution, of course. In this case, we assume that $\underline{a} = 1$, so that $\underline{w}_1 = \theta^*$. Then from Flinn and Heckman (1982), we know that

$$\hat{\theta}^* = \min\{w\}_{\in S_1},$$

where S_1 is the set of sample members in the low-schooling group, is a superconsistent estimator of θ^* when there is no measurement error in wages. We will discuss the no measurement error assumption below when we consider the implementation of the estimator.

The value of a^* , that characterizes the schooling decision rule, is a function of all of the parameters in the model, including the the flow cost of attending school, \tilde{c}_2 . It is clear that this “free” primitive parameter only enters the schooling decision directly, and thus for estimation purposes we can treat a^* as a parameter to be estimated. If all other parameters determining a^* are identified, then the estimated value of a^* can be inverted to yield an estimate of \tilde{c}_2 since a^* is monotone in \tilde{c}_2 .

As shown in Flinn (2006), for example, knowledge of the proportion unemployed and the average duration of unemployed search from a point sample of this model is sufficient to identify the rate parameters λ and η in conjunction with wage distribution information. The wage distribution information is required to compute an estimate of $\tilde{G}(\theta^*)$. Given a consistent estimate of this quantity, then

$$\hat{\lambda} = (\bar{t} \times \widehat{\tilde{G}(\theta^*)})^{-1}$$

is a consistent estimator for λ , since the sample mean of the duration of unemployed search is a consistent estimator of the analogous population moment. Similarly, a consistent estimator of η is given by

$$\hat{\eta} = (\bar{t} \times \tilde{p})^{-1},$$

where \tilde{p} is the sample proportion in the employment state.

Identification of the distributions of the components determining total match productivity is more challenging using only point sample wage data. As is evident from (7) and (8), the schooling-specific wage distributions are mixtures of truncated lognormal distributions, with the mixing distribution, F , representing the (truncated, under the model) distribution of abilities within schooling level s . As stated above, parametric assumptions are required for the identification of $G(\theta)$, and in the empirical work below we make the common assumption that the match values are lognormally distributed, so that

$$G(\theta; \mu, \sigma) = \Phi\left(\frac{\ln \theta - \mu}{\sigma}\right), \theta \in R_+,$$

where Φ is the standard normal c.d.f. Thus G is assumed to be completely characterized given knowledge of the two parameters μ and σ .

We have also chosen to make parametric assumptions regarding the ability distribution. In particular, we assume that the c.d.f. of a is given by a (truncated) one parameter (negative) exponential,

$$F(a) = 1 - \exp(-\kappa(a - 1)), a \geq 1, \kappa > 0.$$

Knowing how difficult precise estimation is even of parametric mixing distributions except in extremely large samples, we chose this admittedly restrictive one parameter distribution. If we had access to multiple spell information (per individual), then the restriction that a is constant across spells would undoubtedly make the recovery of F considerably easier, and consequently, we could be less restrictive regarding the specification of F .

Under our model specification and the nature of the decision rules used by agents, the distribution of wages by schooling level are functions of the parameters (both primitives and decision rules): $\mu, \sigma, \kappa, \alpha, h_2, \theta^*, a^*$. (Recall that we already have access to a superconsistent estimator for θ^* .) We use a method of moments estimator to recover these parameters using characteristics of the sample wage distribution for each of the two schooling levels

along with information on the sample proportion that has completed four years of college. We use the conditional wage distribution information by partitioning the sample space of wages into 9 intervals, and then use 8 of the intervals in the construction of the method of moments estimator. We choose 8 thresholds, $w^{(1)} < \dots < w^{(8)}$, and then use the sample proportions (by schooling class) in the intervals $(0, w^{(1)}]$, $(w^{(1)}, w^{(2)}]$, ..., $(w^{(7)}, w^{(8)}]$. We compute the corresponding model proportions given a trial value of the parameter vector using numerical integration methods. Thus we have 16 pieces of information in total from the sample wage distribution to use, along with other three sample moments, in an attempt to recover the 6 unknown parameters listed at the beginning of this paragraph. While we have not formally demonstrated that these particular probabilities are sufficient to identify the unknown parameters, Monte Carlo experiments and the results of our estimation seem to confirm that they are.

The final sample characteristic used in the estimator is the proportion of the sample who have completed college, \tilde{p}_2 . Under the model, the proportion of college completers is given by $\exp(-\kappa(a^* - 1))$. Thus this proportion, in conjunction with information from the wage distribution, aids in determining the values κ and a^* .

5.2 The Estimator

We conclude this section with a summary of the estimation method. After trimming the wage samples for low and high schooling types, we compute an estimate of θ^* using the minimum wage observed in the low schooling sample. We then form a vector of sample characteristics containing 19 elements, which we denote by \tilde{M}_0 . We then compute a weighting matrix W by resampling the original data matrix 5000 times, computing the value of $\tilde{M}(k)$ for each of the k resamples, and then defining the weighting matrix W as the inverse of the covariance matrix of the $\{\tilde{M}(1), \dots, \tilde{M}(5000)\}$.

Denote the corresponding model implied values of the sample characteristics by $M(\Omega)$, where Ω contains all of the primitive parameters and decision rule values described above, with the exception of θ^* . Then the MM estimator is given by

$$\hat{\Omega} = \arg \min_{\Omega} (\tilde{M}_0 - M(\Omega))' W (\tilde{M}_0 - M(\Omega)).$$

For reasons discussed in the following section, we expect that it will be difficult to precisely and reliably estimate the bargaining power parameter α using only sample information from the supply side of the market. Thus the second estimator we employ uses labor share information to pin down the bargaining power parameter in an extremely powerful manner. Now based on the model, the labor share is given by

$$LS = \frac{E(w|w \text{ acceptable})}{E(y|y \text{ acceptable})}.$$

Now in our case,

$$E(w|w \text{ acceptable}) = E(ah(\cdot)) \{ \alpha E(\theta|\theta \geq \theta^*) + (1 - \alpha)\theta^* \}$$

and

$$E(y|y \text{ acceptable}) = E(\alpha)E(\theta|\theta \geq \theta^*),$$

so that

$$\alpha = \frac{LS \times E(\theta|\theta \geq \theta^*) - \theta^*}{E(\theta|\theta \geq \theta^*) - \theta^*}. \quad (10)$$

The estimator that uses labor share (LS) information conditions on a value of LS , in our case 0.7, which is taken from the literature, and given $\hat{\theta}^*$ and the current estimates of μ and σ , uses (10) to determine α .

We use a Nelder-Mead simplex algorithm to locate the parameter vector that minimizes the distance function. Bootstrap standard errors have not yet been computed.

6 Empirical Results (Preliminary)

In this section we describe the data used to carry out this (quite) preliminary empirical analysis. Using the results of the estimation, we are able to conduct a first pass at answering the question of how sensitive are schooling investment decisions to the bargaining power possessed by the supply side of the market. In conducting the empirical exercises, we only consider the homogeneous schooling markets model, the estimation of the heterogeneous case is in progress.

6.1 Data

From previous experience (e.g., Eckstein and Wolpin (1995) and Flinn (2006)), we have learned that the estimation of the bargaining power parameter is fraught with difficulty, and that identification problems are exacerbated in small samples. For this reason, we have chosen to work with Current Population Survey (CPS) data, which enables us to amass large numbers of observations on wages and unemployment spell lengths for relatively tightly defined population subgroups. The price of acquiring so many sample cases is the limited amount of information available regarding labor market activities. The CPS is a household-based survey in which each household is interviewed for 4 consecutive months, then is rotated out of the sample for the following 8 months, and then is rotated back into the sample for its final 4 months of membership. Thus each household is, in principle, in the survey for a total of 8 months. In the 4th and 8th sample months, just before the household leaves the sample temporarily or permanently, detailed employment information, including wages, is ascertained. Households in their 4th and 8th sample months are referred to as the Outgoing Rotation Groups (ORG).

We selected males between the ages of 30 to 34, inclusive, within the ORG samples during all of the months of 2005. We made no further restrictions on sample inclusion that were unrelated to missing information. In particular, we did not exclude individuals

based on race, ethnicity, or region of residence. Thus, while our sample inclusion criteria are relatively restrictive, a considerable amount of heterogeneity remains.

To be included in the final sample, an individual had to either be employed or unemployed, and have valid schooling completion information. If an individual was employed, to be included in the final sample there had to have been enough information available that would allow an hourly wage rate to at least be imputed.⁶ If an individual was unemployed, we required that they report the weeks of the on-going search spell to be included in the estimation sample. Our final sample consists of 9,985 individuals.

After experimenting with various schooling classifications systems, we determined the one that, informally, seemed to maximize differences in schooling group outcomes. This involved assigning all those who have completed college to the high schooling group and all those with partial college or less to the low schooling group. We began by assigning all of those with any college to one group, but found that those with less than four years of college were far more similar in their labor market outcomes to those who had not attended college at all than they were to those who had completed four years of college.

As noted in our discussion of the estimator used with these data, we trimmed the wage data to eliminate extreme observations at the upper and lower tails of each school-specific wage distribution. Originally, there were 6,416 employed individuals at the low schooling level and 3,238 at the high schooling level. From each set, we eliminated the top and bottom 2.5 percent of wage observations. All the wage information used in the estimator was taken from the trimmed samples. However, when computing employment and unemployment probabilities, the total number of employed individuals by schooling class (that is, prior to trimming) was used.

6.2 Estimates of the Homogeneous Model

Table 1 contains estimates of the primitive parameters of the model both with and without labor share information. When labor share information was used, we imposed a value of 0.7, which seemed to be intermediate in the range of values presented and discussed in Krueger (1999).

Column 1 contains the estimates without using labor share information. From the experiences reported in Flinn (2006), we know that the bargaining power parameter α is in principle identified as long as the matching distribution does not belong to a location-scale family, if there is no heterogeneity in ability, and if a maximum likelihood estimator is employed. While the lognormal belongs to a log location-scale family, therefore satisfying the first condition, the other two diverge from the situation analyzed in Flinn (2006). His Monte Carlo experiments revealed a tendency for the maximum likelihood estimate of α

⁶That is, an individual who reported being paid on an hourly basis and who reported their hourly wage rate would be included in the sample. Most males of this age range are not paid on an hourly basis, however. In these cases, if usual weekly hours and usual weekly earnings were reported, we could impute a “usual” hourly wage rate. Thus both types of individuals were included in employed subsample.

to approach the bounds of the parameter space, $[0, 1]$, even in relatively large samples. We find a similar kind of behavior in the estimates we now discuss, which is what explains our preference for the estimates obtained using share information in column 2.

The time unit in which our event occurrence rate parameters are expressed is the month. The estimate of λ of 0.201 in column 1 implies that offers are received every 5 months, on average. From our estimates of the matching distribution and the critical value of match acceptance, which are presented in Figure 2, we know that virtually all offers are accepted, so that the average duration of an unemployment spell is slightly greater than 5 months, as is found in the data. The rate of job termination is low, at 0.007, which implies that the average length of a job is almost 12 years. This low rate is explicable in terms of the age and gender composition of the sample we are using, and the fact that cyclically speaking, 2005 was not a “bad” year. Using more recent CPS data would undoubtedly yield higher estimates of η and perhaps lower estimates of λ .

Estimates of the parameters describing the match value distribution, the distribution of ability, and bargaining power, are highly dependent on whether share information is utilized, which explains the large change in these parameters across the two columns. In column 2, when the bargaining power is “anchored” to an external estimate of labor’s share, the estimated value of α comes down markedly, and, consequently, the distribution of match productivity tends to shift toward the right. The estimated value of the (negative) exponential distribution parameter κ is relatively constant across the two specifications, with the estimated mean of ability in specification 1 being 1.187. Recall that we have assumed that this distribution is truncated from below at 1. We think of the ability distribution, in the absence of schooling investment, as simply “shifting out” the match draw distribution vis-a-vis the generation of “total” match productivity. This estimated parameter indicates that the individual specific heterogeneity contribution to total match productivity is not huge, at least before its contribution to human capital investment is factored in. The lowest ability level is 1, as we have said. Under our estimate of this distributional parameter, we find that the probability that a randomly selected individual is 50 percent more able than the least able distribution is only 0.07. Thus, the individual-specific ability distribution displays much less dispersion than does the match distribution. Of course, these interpretations must be considered with a large degree of caution, since identification of all distributional parameters is strictly via functional form assumptions. In particular, an identification analysis built on the use of multiple employment spells in which the ability and schooling components were assumed constant would be much stronger clearly, but this is not possible given the data to which we have access.

We note that the estimate of the “return” to college completion is approximately 22 percent across both specifications. We have to point out that the estimate is largely determined through the imposition of the necessary condition for college attendance by anyone in the population, which is that

$$\exp(-\rho\tau_2)h_2 - 1 > 0.$$

In particular, in estimating the model we imposed this condition by writing

$$h_2 = \exp(\rho\tau_2) + \exp(\gamma),$$

where γ was a free parameter to be estimated. Since we assume a value for ρ , and considered the time to complete college, τ_2 , equal to 48, the term $\exp(\rho\tau_2)$ is known. Under both model specifications, the estimate of γ was extremely small, so that h_2 was approximately equal to $\exp(\rho\tau_2)$ in both cases. Our feeling is that this issue could be rectified by making alternative assumptions regarding the ability distribution, which we are pursuing in our on-going empirical analysis. That said, our feeling is that a 22 percent return on college completion (in comparison with a high school diploma, say), is not terribly unreasonable.

As mentioned above in our discussion of identification, we are able to estimate θ^* and a^* as free “parameters” since the model is essentially just-identified (after assuming a value of the discount rate ρ). In particular, given an estimate of θ^* and all of the other primitive parameters, along with the assumed value of ρ , we are able to “back out” the baseline flow utility in unemployed search, b_0 . In a parallel manner, given the estimated θ^* , h_2 , and a^* , we are able to back out the flow cost of schooling, \tilde{c}_2 . In order to minimize numerical problems in estimation, it is typically vastly preferable to estimate the decision rules directly and then to invert them using point estimates of other primitive parameters to recover other parameters that appear exclusively within the decision rules. As was discussed above, after trimming the sample of wage observations for each schooling level, we used the smallest wage observation in the low schooling subsample as a superconsistent estimator of θ^* . Our estimate of θ^* is 6.00. Our “direct” estimate of a^* is 1.207. Given our estimated ability distribution, this estimate essentially results in a perfect fit of the proportion of the sample who completed a four year college program.

The estimates from our preferred specification in column 2 of Table 1 are quite similar to those reported in column 1 after account is taken of the large adjustment in the estimate of α induced by using the labor share information. In particular, the rates of event occurrences are quite similar, as is the estimate of h_2 (for reasons given above), θ^* (by construction), κ , and a^* . Only the estimated distribution of G varies markedly across the two specifications, in response to the much lower value of $\hat{\alpha}$ that is essentially imposed when we incorporate the labor share value of 0.7.

In Table 2 we present some evidence regarding model fit. This table also shows what sample characteristics were used in the method of moments stage of the estimation. Most of the information utilized relates to the schooling-specific wage distributions, with 8 wage probabilities computed for each of the schooling classes. The other 3 sample characteristics utilized in estimation are the proportion unemployed, the average duration of search among the currently unemployed, and the proportion of the sample with $s = 2$. In all, we use 19 sample characteristics (in addition to the lowest accepted wage in the low schooling group and the labor share value of 0.7) to estimate 9 parameters, of which two are actually decision rule values.

The fit to the nonwage sample characteristics is extremely good, as is often the case in this class of models. The model is able to effectively reproduce the unemployment rate in the sample and the proportion who complete college. The fit to the average length of an ongoing unemployment spell is not perfect, but differs by only about one-third of a month.

In terms of the wage distribution information, the model performs well here as well, even under rather severe restrictions on the data generation process. The model does a slightly better job at fitting the low schooling wage distribution; there is never more of a difference of 0.02 between a cell's actual proportion of cases and the predicted proportion under the model. The model has a harder time fitting the high schooling wage distribution, particularly at low wage values, which have zero or low probability under the model. For example, the predicted probability of a wage between 8 and 11 dollars to a college graduate is 0.031 under the model but is 0.069 in the data. Nonetheless, for most of the wage intervals we find the fit adequate. Our general conclusion is that for such a stylized model, we are able to capture the main features of the data reasonably well.

6.3 The Impact of Bargaining Power on Educational Investment

The last exercise we conduct goes to the heart of the question being addressed in the paper: How sensitive is educational investment to bargaining power? Using our preliminary estimates of the model, we are able to provide a heavily-qualified response.

In order to answer the question, we first must use the estimates reported in the second column of Table 1 to retrieve the remaining primitive parameters required to solve the educational choice problem, namely b_0 and \tilde{c} . We do this by first solving for b_0 , which is given by

$$\hat{b}_0 = \hat{\theta}^* - \frac{\hat{\lambda}\hat{\alpha}}{\lambda}$$

of college completers, which is 0.330. A reduction of approximately 13 percent in the bargaining power parameter is sufficient to shut down college completion completely. In terms of the impact of increases in bargaining power on college completion, the results are at least as striking. By increasing bargaining power by a bit more than 11 percent, all individuals would complete college.

Now this preliminary exercise has unattractive features, most importantly the absence of any general equilibrium mechanisms that might dampen such extreme sensitivity to α . For example, the costs of attending college may well depend on the number attending; the payoff to a particular level of schooling investment may depend on the economy-wide

Table 1
 Estimates of Homogeneous Markets Model
 With and Without Labor Share Information

<i>Parameter</i>	<i>No Share Information</i>	<i>Share Information</i>
λ	0.201	0.188
η	0.007	0.006
μ	2.643	2.850
σ	0.486	0.542
κ	5.326	5.398
h_2	1.222	1.222
α	0.800	0.575
θ^*	6.000	6.000
a^*	1.207	1.206
Distance Value	247.276	305.492

Table 2
Homogeneous Markets Model with Labor Share
Actual and Predicted Sample Characteristics

<i>Sample Characteristic</i>	<i>Actual</i>	<i>Model</i>
Proportion Unemployed	0.033	0.032
Average Duration Unemployed	5.122	5.464
Proportion College	0.330	0.329
Proportion Employed with:	$s = 1$	
$w \leq 8$	0.092	0.071
$8 < w \leq 11$	0.205	0.224
$11 < w \leq 14$	0.216	0.225
$14 < w \leq 17$	0.175	0.168
$17 < w \leq 20$	0.133	0.113
$20 < w \leq 23$	0.065	0.073
$23 < w \leq 26$	0.054	0.045
$26 < w \leq 29$	0.029	0.029
Proportion Employed with:	$s = 2$	
$w \leq 8$	0.012	0
$8 < w \leq 11$	0.069	0.031
$11 < w \leq 14$	0.102	0.114
$14 < w \leq 17$	0.114	0.149
$17 < w \leq 20$	0.123	0.147
$20 < w \leq 23$	0.095	0.126
$23 < w \leq 26$	0.110	0.103
$26 < w \leq 29$	0.085	0.079

Table 3
Changes in Bargaining Power and College Completion

α	θ^*	a^*	P_2
0.50	4.081	1.773	0.016
0.51	4.364	1.658	0.029
0.52	4.638	1.560	0.049
0.53	4.903	1.476	0.077
0.54	5.160	1.402	0.114
0.55	5.410	1.337	0.162
0.56	5.651	1.280	0.220
0.57	5.886	1.229	0.290
0.58	6.115	1.183	0.372
0.59	6.337	1.142	0.465
0.60	6.553	1.104	0.570
0.61	6.764	1.070	0.687
0.62	6.970	1.038	0.814
0.63	7.170	1.009	0.952
0.64	7.366	0.982	1.000

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