Discussion of "The College Admissions Problem with a Continuum of Students" and "Stability and Competitive Equilibrium in Trading Networks"

Eric Budish

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- It also seems of intrinsic intuitive appeal; and may be a useful input into subsequent work

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- Assumptions: quasi-linearity, full substitutability
  - N.B. paper’s discussion of various substitutes conditions in the literature, and how they relate, is a valuable contribution in its own right. In part a reminder of how strong an assumption substitutability is.

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  - Each contract specifies a unique buyer-seller pair: "doubly personalized" prices
  - Each contract specifies a unique good/service: allows e.g. for non-linear prices

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- Yet, in both papers prices play a familiar kind of role: identify and support efficient allocations (are prices / budgets informationally efficient in the sense of Nisan and Segal?)
- And in both papers the price-theoretic approach yields new theoretical intuitions and insights
  - Azevedo and Leshno: give a genuinely eye-opening take (to me at least) on the classical deferred acceptance algorithm.
  - Hatfield et al is an impressive work of unification, highlights some assumptions that really matter and some that are more dispensable for markets with indivisibilities

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So, the question I would like to pose for discussion is:

- How do works like these change one's basic thinking about the price system?
- How do "complex prices" fit into classical price theory?