Comments
Panel on Open Questions

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Conference on Matching and Price Theory

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Bounds

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  assume it is of rank r. Use the singular value decomposition (spectral decomposition).
- Let A be square (not really needed). See C. R. Rao (1971)
  $\underset{\sim}{\lambda}$ is matrix of eigenvalues of A. Then we know from linear algebra that there exists $\begin{matrix} P & (M & r) & \lambda(r & r) \\ Q & M & r & A = P\lambda Q' \end{matrix}$ where
  columns of P are orthonormal (mutually orthogonal L unit length)
  true even if A is m n $m \neq n$
  P is m r λ is r r Q is n r.

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- Rank one case is Cobb-Douglas (assuming *9* a cardinal scale)

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- The spectral decomposition assigns a Cobb-Douglas interaction to each component:

P_{ik} is quality k of worker i
q_{jk} is quality k of firm j.

- $\therefore$ implicitly we have a Cobb-Douglas technology in qualities

$$\begin{pmatrix} P_{11} & P_{1r} \\ \vdots & \vdots \\ P_{n1} & P_{nr} \end{pmatrix} \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_r \end{pmatrix}$$

$$\begin{pmatrix} q_{11} & q_{n1} \\ \vdots & \vdots \\ q_{1r} & q_{nr} \end{pmatrix}.$$

- Observe $a_{ii} = w_i + \pi_i$. Now $a_{ji} \quad w_j + \pi_i$

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- $\therefore w_i \quad w_j \quad a_{ii} \quad a_{ji}$.

- Similarly we have that

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- Now use spectral decomposition

$$a_{ij} \quad a_{jj} \;=\; \sum_{k=1}^{r} \lambda_k \left(P_{ik} \quad P_{jk} \right) q_{jk}$$

like marginal product

$$\therefore \sum_{k=1}^{r} \lambda_k (P_{ik} \quad P_{jk}) q_{jk} \quad w_i \quad w_j \quad \sum_{k=1}^{r} \lambda_k (P_{ik} \quad P_{jk}) q_{ik}$$

- Left and right hand sides are differential marginal product using firm j and firm i^s attributes, respectively. Suppose all firms alike: $q_{jk} = q_{ik}$ (firms possess no identity). Then we get Gorman-Lancaster form of the model

$$w_i \quad w_j = \sum_{k=1}^{r} \lambda_k (P_{ik} \quad P_{jk}) q_k$$

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  (pure factor structure model).
- Otherwise, we get the notion that workers have different productivities depending on properties of firms). (Then characteristics payment will depend on distributions of firms.

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- The population distribution of s is $g(s \mid \Theta)$ where Θ is a vector of parameters.
- Agents cannot invest in order to change skills s.
- $t_i(s)$ is a function that expresses the amount of sector i specific tasks a worker with endowment of skills s can perform.

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- P_i is the price of sector i output.
- π_i is the price of one unit of sector i specific task.

Optimization implies:

(1)
$$\pi_i = P_i \frac{\partial F^{(i)}}{\partial T_i}$$

An agent with endowment s works in sector i if:

(2)
$$\pi_i t_i(s) \quad \pi_j t_j(s), \ i, j = 1, 2 \text{ and } i \neq j.$$

Let L_i denote the set of agents working in sector i :

$$L_i = \{s : \pi_i t_i(s) \quad \pi_j t_j(s), i \neq j\}.$$

The log wage in sector i of an individual with endowment s is:

$$\ln w_i(s) = \ln \pi_i + \ln t_i(s) \tag{3}$$

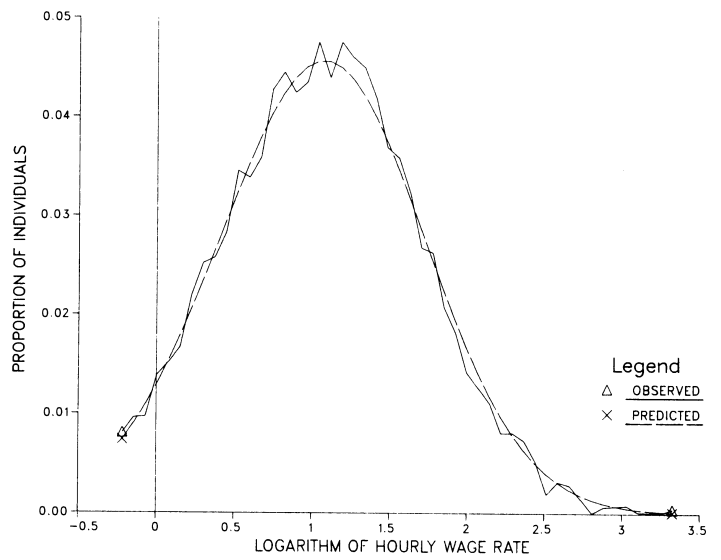
The proportion of the population working in sector i is:

$$pr(i) = \int_{\mathcal{L}_i} g(sj\Theta)\, ds, \quad i = 1, 2$$

Roy model assumes that $g(sj\Theta)$ and $t_i(s)$ are such that:

$$\begin{bmatrix} \ln t_1(s) \\ \ln t_2(s) \end{bmatrix} \quad N\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \Sigma\right]$$

Non-manufacturing sector: predicted vs. observed log wage distribution



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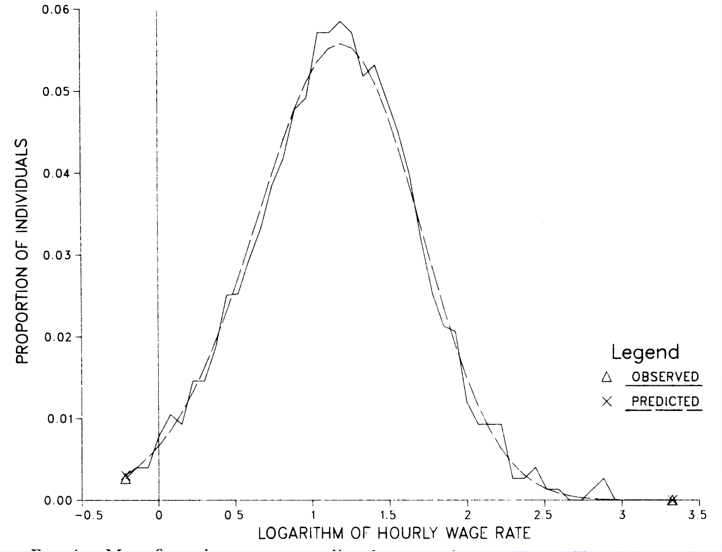

Simulation of a 1% increase in the energy price index

	Manufacturing Sector	Nonmanufacturing Sector	U.S. Aggregate
Year: 1972:			
1. Percentage change in persons employed	−1.854	1.320	...
2. Percentage change in mean task or quality level for the employed population	.919	−1.496	...
3. Percentage change in task price	−1.480	.471	−.062*
4. Percentage change in observed average wage (2 + 3)	−.561	−1.025	−.950
Year: 1976:			
1. Percentage change in persons employed	−2.007	1.371	...
2. Percentage change in mean task or quality level for the employed population	.886	−1.461	...
3. Percentage change in task price	−1.480	.471	−.063*
4. Percentage change in observed average wage (2 + 3)	−.594	−.990	−.939
Year: 1980:			
1. Percentage change in persons employed	−1.993	1.244	...
2. Percentage change in mean task or quality level for the employed population	.953	−1.568	...
3. Percentage change in task price	−1.480	.471	−.034*
4. Percentage change in observed average wage (2 + 3)	−.527	−.997	−.949

NOTE.—The data sets on which the simulations are performed are defined in App. C.
* This is a weighted average of the task price change in each sector using the relative proportions employed in the sector in the year.

Assessing the impact of self-selection on the means and variances of log wage rates for white males, 1980

	Prediction of Extended Roy Model	Actual 1980 Value	Random Assignment Economy Using 1980 Equilibrium Task Prices
	Nonmanufacturing Sector		
Mean of log wages (M_1)	1.054	1.040	.651
Variance of log wages (σ_1)	.319	.323	.344
Proportion of population in sector (P_1)	.619*	.630	.619*
	Manufacturing Sector		
Mean of log wages (M_2)	1.199	1.202	.968
Variance of log wages (σ_2)	.192	.201	.211
Proportion of population in sector (P_2)	.200*	.206	.200*
	Economywide		
Mean of log wages $\left(\frac{P_1 M_1 + P_2 M_2}{P_1 + P_2}\right)$	1.089	1.079	.728
Sum of within-sector variance $\left(\frac{P_1\sigma_1 + P_2\sigma_2}{P_1 + P_2}\right)$	.288	.293	.311
Between-sector variance $\left[\frac{P_1 P_2 (M_1 - M_2)^2}{(P_1 + P_2)^2}\right]$	.003	.004	.018
Total variance†	.291	.297	.329

* The random assignment economy is restricted to have the proportion of people in each of the three sectors predicted by our model using 1980 equilibrium values.

† Total variance = within-variance + between-variance

$$= \left(\frac{P_1\sigma_1 + P_2\sigma_2}{P_1 + P_2}\right) + \left[\frac{P_1 P_2 (M_1 - M_2)^2}{(P_1 + P_2)^2}\right].$$