Market vs. Design: Congestion and Evaluation Costs

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Mechanism Design

- Agents report private information, Center implements allocation
- **Revelation Principle:** Any feasible goals can be implemented with agents reporting all their information

Hayek's (1945) Critique

- Knowledge of "particular circumstances of time and place" too enormous to transmit to Center
- Modern Example: supermarket shopper reports values for all baskets, receives a basket and invoice?

1. Communication bandwidth: exponentially many bundles
   - Measured in bits (*communication complexity*) or real variables (*message space dimension*)
2. Cost of *evaluating* all possible products
3. Center's temptation to exploit information

Evidence of Costs of Matching

- NRMP: 20,000 positions, 30,000 residents
- Some observed mechanisms required very long communication
- Each player only interviews/forms preferences over a small number of potential partners
- "Appeals stage" allows additional preference evaluation (which would be too costly ex ante?)

Hayek's Solution: Free Market

- "Ultimate decisions must be left to the people familiar with these circumstances"
- Decisions are coordinated using *prices*:
  - Summarize all the relevant information
  - Guarantee efficiency
- "Nobody has yet succeeded in designing an alternative system"
- But now some very clever designers work in "non-classical" settings!
  - Do they have to use "market" mechanisms?
  - How complex do they have to be?

Is price mechanism *necessary*?

- **Fundamental Welfare Thms:** Supporting prices

  (1) Are *sufficient* to verify efficiency of allocation

  (2) Can be constructed *with full info* about the economy

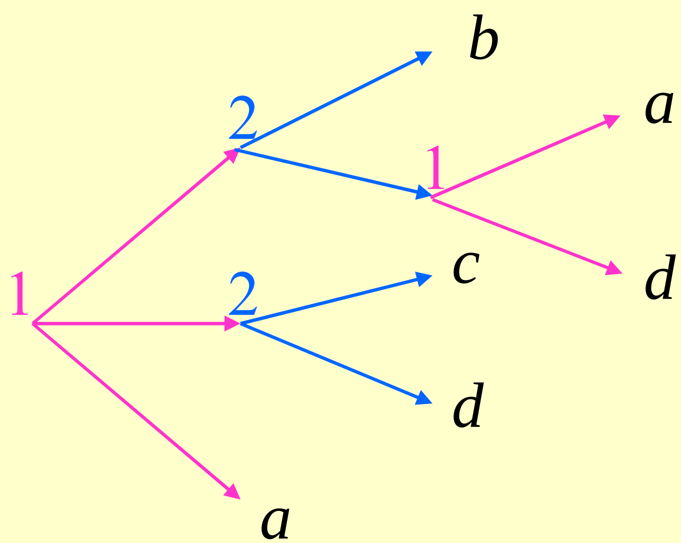
  - But then can compute efficient allocation directly
- Hurwicz, Mount-Reiter: In a convex economy *with distributed preference information*, Walrasian equilibrium is "dimensionally minimal" among "regular" mechanisms verifying efficiency
- Did not rule out mechanisms that don't use prices
- Inapplicable to non-classical settings:
  - Nonconvexities, discrete allocations
  - Other goals: approximation, group stability, fairness, …
  - Other costs: bits, evaluations, …

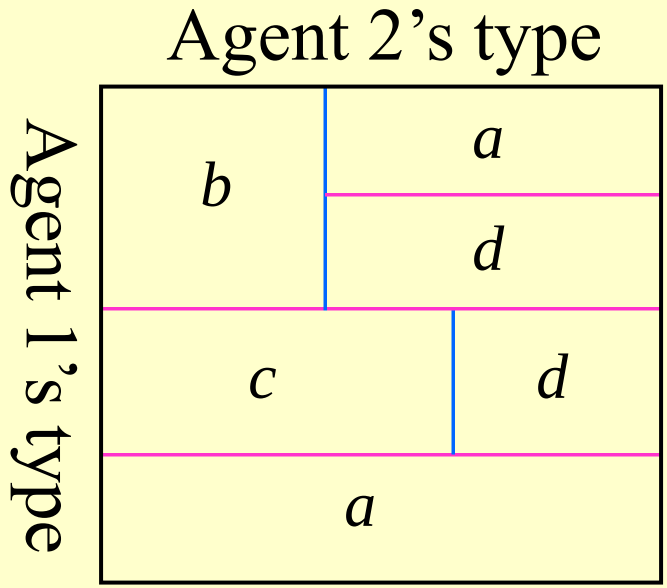
Communication

- Agents privately observe types (= preferences)
- State = profile of types
- Sequential mechanism may save over one-shot
- *Communication Protocol* =

  1. Extensive-form message game
  2. Outcomes assigned to terminal nodes
  3. Agents' strategies (type-contingent)
- Ignore incentives (e.g., agents are robots)

Communication



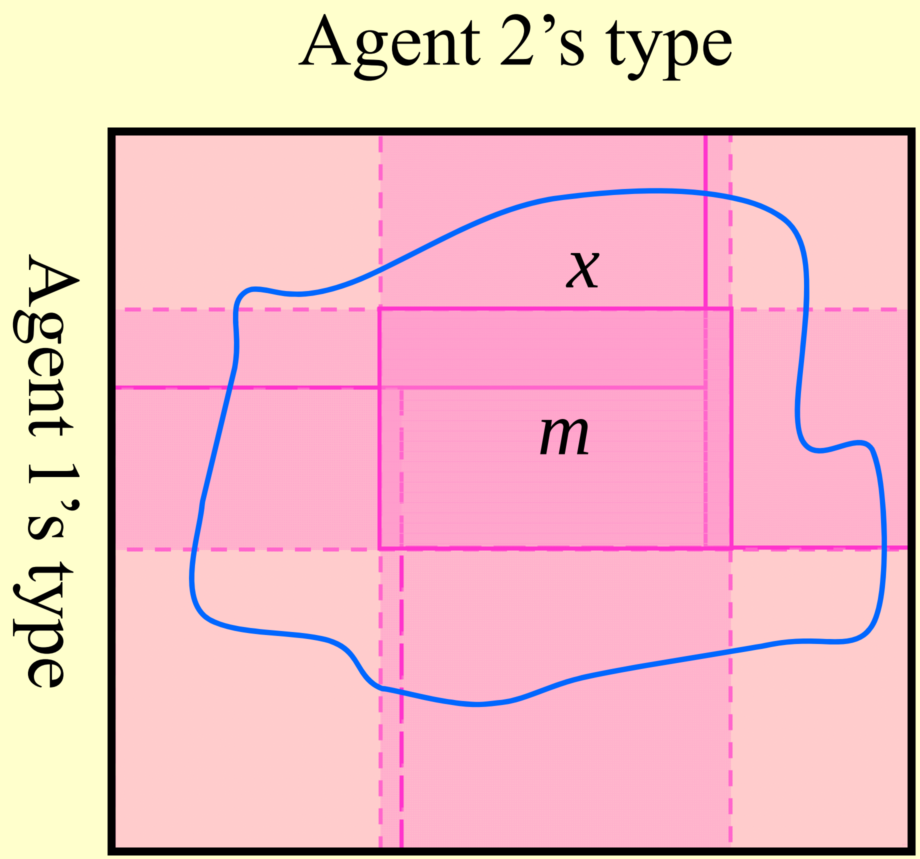


- State space partitioned into product sets
- In every state must implement an optimal outcome for this state
- Characterizing such protocols is hard

Verification

- Omniscient oracle knows the state but must prove to an outsider that outcome x is optimal

- Announces message $m \in M$
- Each agent accepts or rejects m based on his own type
- Acceptance by all agents verifies x
- Verification in each state



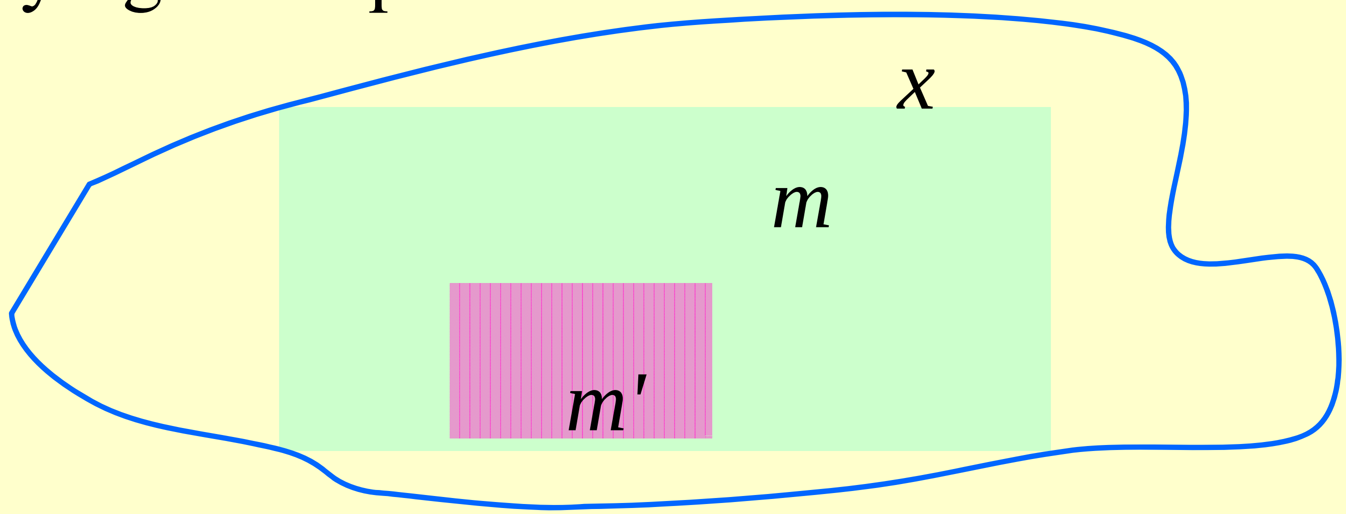
Why Verification?

- Any communication protocol can be verified by oracle sending messages in agents' stead (i.e., announcing terminal node)

  $\Rightarrow$ verification costs $\leq$ communication costs
- Economic example – Walrasian equilibrium: message = (allocation, prices)
- Steady state of a communication process (e.g., tatonnement, auctions, deferred acceptance algorithms)?

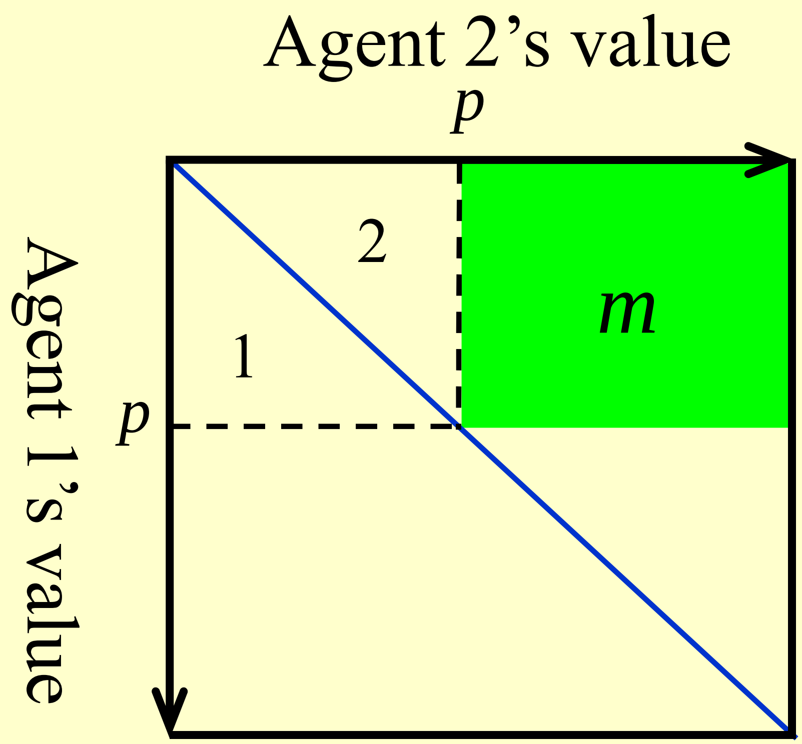
Informativeness

- Message m is *less informative than* message m' if m' accepted $\Rightarrow$ m accepted.
- m is *a minimally informative* message verifying outcome x if any less informative message verifying x is equivalent to m.



- Such messages minimize communication costs:
  - Size of M - number of bits or reals to encode a message
  - Preference evaluation costs, etc.

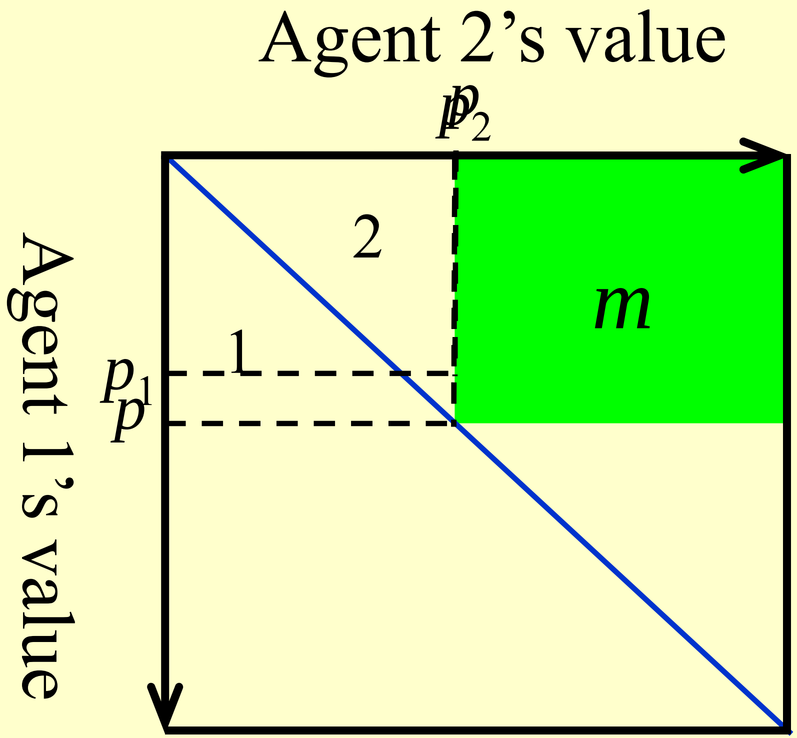
Allocate an Object between 2 Agents



- Messages verifying "2"
- *Minimally informative* messages verifying "2"

- Equivalent to announcing supporting equilibrium price *p*
- Each *p* must be used (in a diagonal state) ⇒ need infinitely many messages (continuum)

What is the “right” price space?



- Verify “2” with personalized prices $p_1 < p_2$?
  - would be “too informative,” require two numbers
- Minimal informative verification = minimal budget sets – here maximal p_1, minimal p_2

General Social Choice Problems (Segal JET 2007)

- Characterize *social choice problems* (*social goals* and *preference domains*) for which it is *necessary* to reveal supporting prices
- Algorithm deriving the *form of prices* (budget sets) that verify solutions to a given problem with minimal information (= minimal budget sets)
- Size of price space ⇒ communication cost
- Some applications:
  - Pareto efficiency in convex economies ⇒ Walrasian prices (linear and anonymous)
  - Efficient or approximately efficient combinatorial auctions ⇒ combinatorial and personalized prices
  - Stable many-to-one matchings ⇒ non-combinatorial but personalized budget sets/prices

Stable Many-to-one Matching

- Firms (F) and workers (W)
- Matching = binary relation from F to W in which a worker matches with at most one firm
- Each agent has preferences over partners

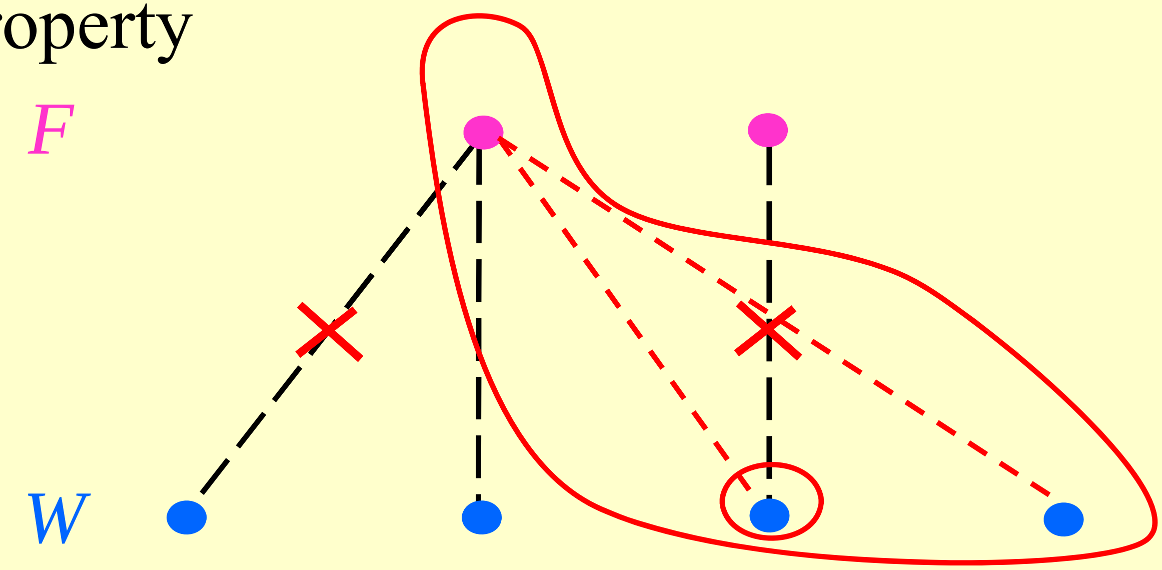
Stable Many-to-one Matching

- Matching is *stable* if these coalitional deviations are not strictly Pareto improving:

  (i) Firm hires some new workers (and fires some)

  (ii) Worker quits to become unemployed

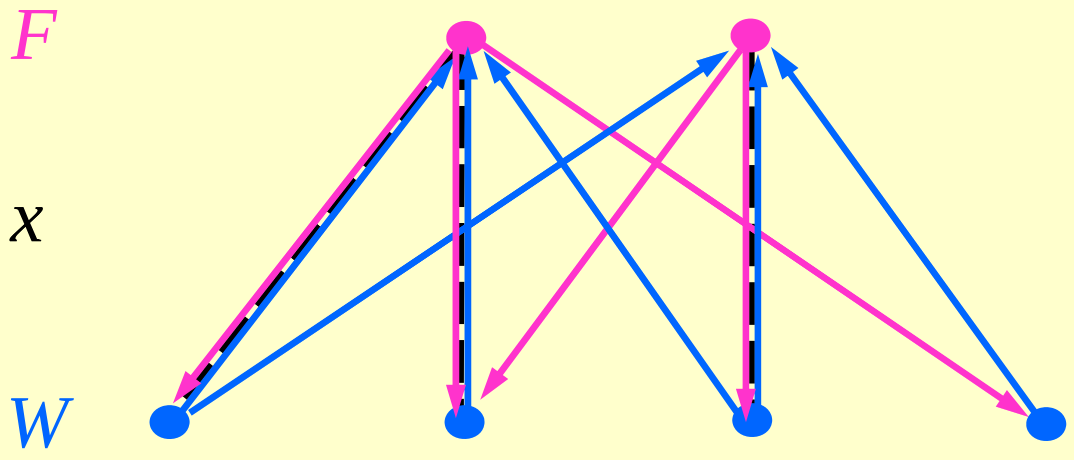
– This choice rule has budget equilibrium revelation property



Minimally Informative Equilibria

- A worker's budget set described by available firms
- A firm's budget set: available *groups* of workers
- In a minimally informative equilibrium verifying stability, the groups consist of
  - The firm's current workers
  - Workers who don't have the firm in their budget sets

$\Rightarrow$ firms' budget sets, non-combinatorial



Lemma. *m* is a minimally informative message verifying a stable matching

$\Leftrightarrow$ *m* is equivalent to a *partitional equilibrium*, which allocates each off-equilibrium match to either partner's budget set (not both).

- Describing partitional equilibrium takes ~ *FW* bits
- *Every* equilibrium must be used for verification (it is a *unique* partitional equilibrium in some state)
- With transfers and quasilinear payoffs, minimally informative verification is with *FW* real prices
- Combinatorial prices/budget sets aren't needed
  - conditional on existence of a stable match

What makes "matching markets" harder than others?

- Prices/budget sets are *doubly* personalized
- Prices are not reducible to hedonic characteristics (cf. e.g. housing)
- Players /goods change over time

$\Rightarrow$ New prices must be discovered each time

Price *discovery* is harder than *verification*

- Extra communication, evaluation costs, delays
- Incentives for price manipulation when agents don't have many close substitutes ($\Rightarrow$ large core)
  - E.g. "demand reduction" - not listing acceptable matches
  - Unraveling

Deferred Acceptance Algorithm

- Monotone tatonnement, finds a partitional equilibrium (for substitutable preferences)
- Close to minimal number of bits:
  - Each match offered, kept/rejected: $\leq 2FW$ messages, of $\log_2(FW)$ bits each
  - Exponentially less than full revelation of firms' preferences over 2^W subsets of workers
- But number of stages (W) may be too long in practice
- Preference evaluation cost:
  - Responding agents evaluate only partners from their (minimal possible) budget sets
  - But in worst case, must reveal ranking of all partners in budget set

Example: One-to-One Matching with Common Ranking of Firms

- DAA = serial dictatorship by firms:
- In round $f = 1,\ldots F$, firm f chooses from its budget set = all workers not taken by the previous firms
- This finds the *unique* stable match (= unique partitional equilibrium) with minimal preference evaluation
  - Each agent knows budget set when choosing; no need to defer acceptance
- But takes F rounds - congestion

Single-Round Protocol

- Not knowing its budget set, firm *f* must rank its top *f* workers, to guarantee getting one of them
  - onerous for low-ranked firms
- Let's have a model where ranking top *f* workers is more costly than just choosing one best worker

Evaluation Cost Model

- Each firm has a "prior ranking" of all workers
- Each worker is a "match" only with prob. $q < 1$ (iid) – checking requires an evaluation (interview)
  - E.g. $q \approx 1$ – but still must interview to hire
- Choose the best match from a budget set: interview in the prior ranking order, expected number of interviews $<1/q$ (= if N large)
- Choose f best matches: need at least f interviews
  - This is what firm f must do in a one-round mechanism

Optimal Mechanism with *K* rounds

Proposition: Any mechanism minimizing (total weighted) evaluation costs partitions the firms in K “tiers”, with tier k comprised of firms f_k+1 to f_{k+1} ($f_0=0$, $f_{K+1}=K$). In round $k = 1,\ldots,K$, each firm f of tier k submits its ranking of $f-f_k$ of the remaining workers, serial dictatorship is run on those rankings, and the tier’s firms leave with their matched workers.

Proof sketch:

- No more than f workers should be ranked by a top-f firm
- We want to minimize size b of each firm f’s “potential” budget set = workers not ranked by any higher firm when it evaluates:
  - Its “true” budget set has $F - f$ workers, but it must rank $b-(F-f)$ workers
- **Order preservation:** firm f should not start evaluating in an earlier stage than when all higher-ranked firms finish evaluations
  - Waiting would reduce firm f’s potential budget set without increasing others’
- **Optimal sizes of tiers** depend on firms’ relative evaluation costs
- Equal costs for all firms $\Rightarrow$ each tier has $\approx F/K$ firms, the tier’s bottom firm ranks F/K candidates $\Rightarrow$ average evaluation cost $\approx F/(2K)$

Possible Extensions

- Relax common ranking of firms – workers also face evaluation costs
- Approximate stability
  - Guarantee some fraction of stable matches? – similar results
  - Stability with high enough probability?
  - Stability only given formed preferences? Too easy
- Incentives (so far trivial)
- "Large" markets: approximate equilibrium with little communication and good incentives?