Decentralized Matching with Aligned Preferences

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- Suppose all follow 'DA'

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- Will Worker 1 accept?

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- When Firm 1 observes $(3, 6)$,
  - Follows MDA $\Rightarrow$ payoff: $6(1-p) + 3p\delta$
  - Deviate to an immediate offer to $W1 \Rightarrow$ payoff: $6(1-p)\delta + 3p$
  - If $p > 2/3$ the deviation is profitable.

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- No equilibrium (mixed or pure) generates the stable match always.

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Main Issue: The timing of offers in and of itself is informative

Example: Assume labels of workers and firms are fully randomized:

$$\begin{array}{lll} \mathbf{F1}: & W3 \succ \mathbf{W1} \succ W2 \\ \mathbf{F2}: & W1 \succ \mathbf{W2} \succ W3 \\ \mathbf{F3}: & W1 \succ \mathbf{W3} \succ W2 \end{array} , \qquad \begin{array}{lll} \mathbf{W1}: & \mathbf{F1} \succ F2 \succ F3 \\ \mathbf{W2}: & \mathbf{F2} \succ F3 \succ F1 \\ \mathbf{W3}: & \mathbf{F3} \succ F1 \succ F2 \end{array}$$

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- Suppose $F2$ gets much higher match utility for $W1$ than from $W2, W3$.
- $F2$ can benefit from delaying offer till period 2.

Similarly, need to know that the offer made to a new worker.

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- Crucial difference in information transmission:
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- Stable outcome may not be achievable with conditions analogous to above

Example: Suppose there are the following two preference realizations, with identities randomized.

$$M_1 \quad \begin{array}{ll} \mathbf{F1}: & \mathbf{W1} \succ W2 \succ W3 \\ \mathbf{F2}: & W1 \succ \mathbf{W2} \succ W3 \\ \mathbf{F3}: & \mathbf{W3} \succ W2 \succ W1 \end{array}, \quad \begin{array}{ll} \mathbf{W1}: & F3 \succ \mathbf{F1} \succ F2 \\ \mathbf{W2}: & F1 \succ \mathbf{F2} \succ F3 \\ \mathbf{W3}: & F1 \succ \mathbf{F3} \succ F2 \end{array}$$

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In M_1 and M_2, $W1$ receives offers from $F1$ and $F2$, and $W3$ receives an offer from his second choice firm $\Longrightarrow$ **no information transmitted**.