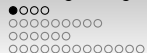




Decentralized Matching with Aligned Preferences

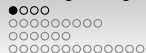
Muriel Niederle Leeat Yariv

May 7, 2011



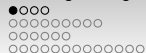
Motivation

- Much of the matching literature has focused on centralized markets



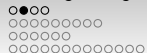
Motivation

- Much of the matching literature has focused on centralized markets
- Many real matching markets are decentralized: U.S. college admissions, market for law clerks, junior economists, etc.



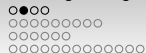
Motivation

- Much of the matching literature has focused on centralized markets
- Many real matching markets are decentralized: U.S. college admissions, market for law clerks, junior economists, etc.
- One aspect of decentralized markets we will focus on is the inherent dynamic interaction



The Goal

- Provide a framework to analyze a two-sided matching **market game** in which firms and workers interact over time.



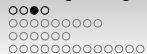
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- Provide a framework to analyze a two-sided matching **market game** in which firms and workers interact over time.
- Identify conditions under which decentralized markets and centralized markets produce identical outcomes



The Goal

- Provide a framework to analyze a two-sided matching **market game** in which firms and workers interact over time.
- Identify conditions under which decentralized markets and centralized markets produce identical outcomes
- Part of a general theoretical question - are there non-cooperative foundations for cooperative solutions (e.g., the core)?



Overview and Insights

- Main ingredients of market game:
 - preference distribution
 - information available



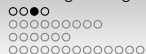
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Overview and Insights

- Main ingredients of market game:
 - preference distribution
 - information available
- Analyze **equilibrium outcomes** of this game
 - **Implementability:** sufficient preference richness allows stability
 - **Uniqueness:** complete information + aligned preferences + refinement



Related Literature

Empirical studies

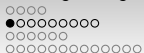
- Avery, Jolls, Posner, and Roth (2001), Niederle and Roth (2003, 2007), Echenique and Yariv (2011), Fox (2010)

Analysis of dynamic games (mostly complete information, restricted strategy spaces)

- Outcomes: Blum, Roth, and Rothblum (1997), Haeringer and Wooders (2009), Diamantoudi, Miyagawa, and Xue (2007)
- Implementation: Alcade (1996), Alcalde, Pérez-Castrillo, and Romero-Medina (1998), Alcalde and Romero-Medina (2000)

Strategic matching in markets with frictions

- Burdett and Coles (1997), Eeckhout (1999), Shimer and Smith (2000)



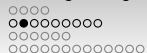
General Set Up

Economies and Preferences

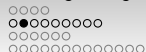
- A **market** is a triplet $\mathcal{M} = (\mathcal{F}, \mathcal{W}, U)$
 - Firms: $\mathcal{F} = \{1, \dots, F\}$
 - Workers: $\mathcal{W} = \{1, \dots, W\}$
 - Match utilities:

$$U = \left\{ \begin{array}{l} \infty \\ \succ \end{array} \right. \left. \begin{array}{l} u_{ij}^f \\ \{Z\} \end{array} \right\}, \quad \left\{ \begin{array}{l} \infty \\ \succ \end{array} \right. \left. \begin{array}{l} u_{ij}^w \\ \{Z\} \end{array} \right\}$$

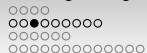
firm i 's utility from matching with j **worker** j 's utility from matching with i



- One-to-one matching with non-transferrable utilities
- Strict preferences, we say worker j is *unacceptable* to firm i if $u_{i\emptyset}^f > u_{ij}^f$. Similarly for workers.
- $u_{i\emptyset}^f, u_{\emptyset j}^w > 0$ for all i, j .

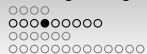


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- An **economy** is a quadruplet $(\mathcal{F}, \mathcal{W}, \mathcal{U}, G)$
 - Firms: $\mathcal{F} = \{1, \dots, F\}$
 - Workers: $\mathcal{W} = \{1, \dots, W\}$
 - \mathcal{U} is a *finite* collection of match utilities
 - G is a distribution over \mathcal{U}



Uniqueness

Assume every market $\mathcal{M} = (\mathcal{F}, \mathcal{W}, U)$ has a **unique stable matching** $\mu_{\mathcal{M}}$ (sidestep coordination).



General Set Up

Economies and Preferences

Game Structure

- Reminder: economy $(\mathcal{F}, \mathcal{W}, \mathcal{U}, G)$



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- $t = 0$: market is realized according to G
- $t = 1, 2, \dots$: two stages as follows



Game Structure

- $t = 0$: market is realized according to G
- $t = 1, 2, \dots$: two stages as follows

Stage 1: firms simultaneously decide whether and to whom to make an offer. Unmatched firm can have at most one offer out.

Stage 2: each worker j who has received an offer from i can accept, reject, or hold the offer.

- Once an offer is accepted, worker j is matched to firm i irreversibly.



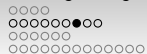
Payoffs

- Firm i matched to worker j at time $t \rightarrow$ payoffs $\delta^t u_{ij}^f$ and $\delta^t u_{ij}^w$, where $\delta \leq 1$ is the *market discount factor*. Unmatched agents receive 0.



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- To ease getting stable matching: focus on high δ



General Set Up

Economies and Preferences

Game Structure

Information



General Set Up

Economies and Preferences

Game Structure

Information

- $t = 0$: underlying structure (particularly G) is common knowledge. Two information structures:



General Set Up

Economies and Preferences

Game Structure

Information

- $t = 0$: underlying structure (particularly G) is common knowledge. Two information structures:
 - **Complete Information:** all agents are informed of realized U .
 - **Private Information:** each agent is informed of their own realized match utilities.



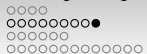
Market Monitoring

- Firms and workers observe receipt, rejection, and deferral only of own offers. When an offer is accepted, the whole market is informed of the match. Similarly, when there is market exit.



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- Firms and workers observe receipt, rejection, and deferral only of own offers. When an offer is accepted, the whole market is informed of the match. Similarly, when there is market exit.
- **Equilibrium notion:** Bayesian Nash equilibrium.



Setup Summary



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- **Strategic dynamic game:** Two important components
 - Preference distribution (unique stable outcome)
 - Information: complete or private

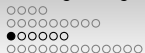


Setup Summary

- **Strategic dynamic game:** Two important components
 - Preference distribution (unique stable outcome)
 - Information: complete or private
- **Assumptions making stability easier to achieve:**
 - In any market, unique stable matching
 - High discount factor

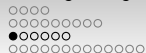


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Proposition 1: *For any economy in the market game there exists a Nash equilibrium in strategies that are not weakly dominated that generates the unique stable matching.*



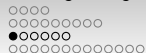
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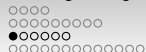
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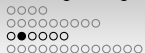
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But there can be other (unstable) equilibrium outcomes...



Example: Multiplicity

F1 : $W2 \succ W1 \succ W3$

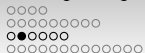
F2 : $W1 \succ W2 \succ W3$,

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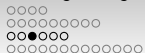


Example: Multiplicity

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F2 : $W1 \succ W2 \succ W3$, **W2** : **F2** $\succ F1 \succ F3$.
F3 : $W1 \succ W2 \succ W3$ **W3** : $F1 \succ F3 \succ F2$

$$\mu_M = \begin{array}{ccc} F1 & F2 & F3 \\ W1 & W2 & W3 \end{array} , \quad \tilde{\mu} = \begin{array}{ccc} F1 & F2 & F3 \\ W2 & W1 & W3 \end{array} .$$

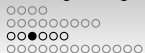
μ_M unique stable matching, can implement $\tilde{\mu}$.



In “sub-market” without $(F3, W3)$, multiple stable matchings:

$$\begin{array}{ll} \mathbf{F1} : & W2 \succ W1 \\ \mathbf{F2} : & W1 \succ W2 \end{array} , \quad \begin{array}{ll} \mathbf{W1} : & F1 \succ F2 \\ \mathbf{W2} : & F2 \succ F1 \end{array} .$$

$$\mu = \begin{array}{lll} F1 & F2 & F3 \\ W1 & W2 & W3 \end{array} , \quad \tilde{\mu} = \begin{array}{lll} F1 & F2 & F3 \\ W2 & W1 & W3 \end{array} .$$



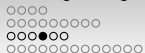
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Stage 1 : $F3$ and $W3$ match, Stage 2: follow $\tilde{\mu}$.

$\tilde{\mu}$ induces the firm preferred stable matching in stage 2.



Aligned Preferences

Aligned preferences: [Today] $u_{ij}^w = \alpha u_{ij}^f$ for some $\alpha > 0$ for i, j mutually acceptable



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- *When preferences are aligned, there is a unique stable matching μ_M (cf. Clark, 2006).*



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- *When preferences are aligned, there is a unique stable matching μ_M (cf. Clark, 2006).*

Intuition: Construct stable match recursively:

- top match of entire market must be part of stable match
- then top match of remaining market must be part of stable match



Aligned Preferences – Uniqueness

Proposition 2 (Complete Information - Alignment): *With complete information, when all supported preferences are aligned, the stable matching of each realized market is the unique Nash equilibrium outcome surviving iterated elimination of weakly dominated strategies.*



Complete Information - Interim Summary

- Stable matching is always an equilibrium outcome
- Aligned Preferences: All equilibria surviving iterated elimination of weakly dominated strategies yield stability.
- In general: There may be equilibria that yield unstable outcomes.



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Centralized clearinghouse with complete information: All Nash equilibria in weakly undominated strategies yield the stable outcome.

In decentralized markets: Firms can condition their second round offers on the first period matches, and more outcomes can be achieved in equilibrium.



Economies with Uncertainty

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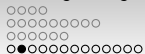
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 - Match formation or market exit
 - Making offers
 - Reacting to offers: acceptance, rejection, or holding



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- Need to find the stable matching, then implement it.
- **Transmission of information:**
 - Match formation or market exit
 - Making offers
 - Reacting to offers: acceptance, rejection, or holding
- For the rest of the talk, assume preferences are aligned.



Aligned Economies: No Frictions

Suppose agents follow **deferred acceptance strategies**.



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Suppose agents follow **deferred acceptance strategies**.

- Firms make offers to workers according to their ordinal preferences.
- Firms exit when all acceptable workers rejected them or exited.
- Workers hold most preferred acceptable offer, **accept an offer from most preferred unmatched firm**.
- **Workers exit as soon as no acceptable firm is unmatched.**

Proposition 3: *Suppose preferences are aligned, and $\delta = 1$. Deferred acceptance strategies constitute a Bayesian Nash equilibrium in weakly undominated strategies and yield the stable matching μ_M .*



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Note: Alignment – In every period some information becomes public.



Aligned Economies: Adding Frictions

Will agents use deferred acceptance strategies even with discounting (frictions)?



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Example: one market economy

$$U_1 =$$

	$W1$	$W2$
$F1$	3	6
$F2$	4	5



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$$U_1 = \begin{array}{|c|c|c|} \hline & W1 & W2 \\ \hline F1 & 3 & \mathbf{6} \\ \hline F2 & \mathbf{4} & 5 \\ \hline \end{array}$$

- $F2$ knows $W1$ will accept an offer immediately.
- $F2$ will not make an offer to $W2$.



Incentive Issues with Alignment

In general, this sort of skipping can lead to economies in which no equilibrium implements the stable matching.



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- Firm 1 and Worker 1 cannot tell U_1 and U_2 apart.
- Suppose all follow deferred acceptance, with appropriate skipping.



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- Firm 1 makes an offer to Worker 2, then Worker 1



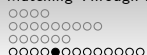
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- Firm 1 makes an offer to Worker 2, then Worker 1
- Firm 2 makes an offer to Worker 2 in U_1 , to Worker 1 in U_2



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- Suppose 1 accepts a period 1 offer (add more markets...).



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When Firm 1 observes $(3, 6)$,

- Follows deferred acceptance \Rightarrow payoff: $6(1 - p) + 3p\delta$
- Deviate to an immediate offer to $W1 \Rightarrow$ payoff:
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- If $p > 2/3$ the deviation is profitable.



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Main Issue: The timing of offers in and of itself is informative



Simply Use Gale-Shapley?

- Two potential problems:
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 - The example illustrated one incentive issue: the incentive to *speed up* matches.
 - Another issue: the incentive to *alter* final match.
- For ‘deferred acceptance’ to be incentive compatible, learning must be limited:
 - ‘Rich’ economies...



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An economy is **rich** if:

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- Generation by a two-stage randomization.



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Corollary: *In a rich economy, for sufficiently high δ , the stable match is implementable through a Bayesian Nash equilibrium in strategies that are not weakly dominated.*

In general: Can define 'learning free' economies that rule out possibility to speed up or alter matches using deferred acceptance-type of strategies.



How Alignment Helps

- At every stage some information becomes public.
- No incentive to reject a firm in order to trigger a chain leading to a superior offer.



Conclusions

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- With **complete information**, the unique stable match is always implementable
 - generally not uniquely
- With **incomplete information**,
 - Without frictions ($\delta = 1$), can always implement the stable matching
 - With frictions, implementability for sufficiently high δ when the economy is **Wic9cm10d9cm10d@enougm10d0) □le8(ys)r1(s)-**



Extensions

Some market attributes that make achieving stability more difficult:

- General preferences



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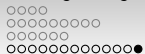
- General preferences
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Extensions

Some market attributes that make achieving stability more difficult:

- General preferences
- Wages
- Exploding offers



T H E E N D