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Fiscal Policy and Unemployment*

Abstract

This paper explores the interaction between fiscal policy and unemployment. It develops a dynamic economic model in which unemployment can arise but can be mitigated by tax cuts and public spending increases. Such policies are fiscally costly, but can be financed by issuing government debt. In the context of this model, the paper analyzes the simultaneous determination of fiscal policy and unemployment in long run equilibrium. Outcomes with both a benevolent government and political decision-making are studied. With political decision-making, the model yields a simple positive theory of fiscal policy and unemployment.

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1 Introduction

An important role for fiscal policy is the mitigation of unemployment and stabilization of the economy.[1] Despite sceptism from some branches of the economics profession, politicians and policy-makers tend to be optimistic about the potential fiscal policy has in this regard. Around the world, countries facing downturns continue to pursue a variety of fiscal strategies, ranging from tax cuts to public works projects. Nonetheless, politicians' willingness to use fiscal policy to aggressively fight unemployment is tempered by high levels of debt. The main political barrier to deficit-financed tax cuts and public spending increases appears to be concern about the long-term burden of high debt.

This extensive practical experience with fiscal policy raises a number of basic positive public finance questions. In general, how do employment concerns impact the setting of taxes and public spending? When will government employ fiscal stimulus plans? What determines the size of these plans and how does this depend upon the economy's debt position? What will be the mix of tax cuts and public spending increases in stimulus plans? What will be the overall effectiveness of fiscal policy in terms of reducing unemployment?

This paper presents a theory of the interaction between fiscal policy and unemployment that sheds light on these questions. The starting point for the theory is a simple dynamic economic model in which unemployment can arise but can be mitigated by tax cuts and public spending increases. Such policies are fiscally costly, but can be financed by issuing debt. This model is used to analyze the simultaneous determination of fiscal policy and unemployment in long run equilibrium. Outcomes with both a benevolent government and political decision-making are considered. With political decision-making, the model delivers a simple positive theory of fiscal policy and unemployment.

The economic model has a public and private sector. The private sector consists of entrepreneurs who hire workers to produce a private good. The public sector hires workers to produce a public good. Public production is financed by a tax on the private sector. The government can also borrow and lend in the bond market. The private sector is affected by exogenous shocks (oil price hikes, for example) which impact entrepreneurs' demand for labor. Unemployment can arise because of a downwardly rigid real wage. In the presence of unemployment, reducing taxes in-

[1] For an informative recent discussion of this role see Auerbach, Gale, and Harris (2010).

creases private sector hiring, while increasing public production creates public sector jobs. Thus, tax cuts and increases in public production reduce unemployment. However, both actions are costly for the government.

We show that in this model there would be no unemployment in the long run with a benevolent government. Moreover, the mix of public and private outputs would be optimal. The way in which the government achieves this first best outcome is by accumulating bond holdings. In the long run, in every period the government hires sufficient public sector workers to provide the Samuelson level of the public good and sets taxes so that the private sector has the incentive to hire the remaining workers. When the private sector is experiencing negative shocks, these taxes are sufficiently low that tax revenues fall short of the costs of public good provision. The earnings from government bond holdings are then used to finance this shortfall.

The benevolent government solution is provocative in showing how governments can use fiscal policy to completely circumvent the inefficiencies stemming from labor market frictions in the long run. The lesson suggested by the analysis is that no satisfactory theory of unemployment can abstract from how fiscal policy is chosen. Nonetheless, when interpreted as a positive theory, the solution is less interesting and this motivates considering political decision-making. To introduce this, we follow Battaglini and Coate (2007, 2008) in assuming that policy decisions are made in each period by a legislature consisting of representatives from different political districts. We also incorporate the friction that legislators can transfer revenues back to their districts.

With political decision-making, the government has no stock of bonds and, when the private sector experiences negative shocks, unemployment arises. Moreover, when these shocks occur, government mitigates unemployment with stimulus plans that are financed by increases in debt. These equilibrium stimulus plans typically involve both tax cuts and public production increases. When choosing such plans, the government balances the benefits of reducing unemployment with the costs of distorting the private-public output mix. In normal times, when the private sector is not experiencing negative shocks, the government reduces debt until it reaches a floor level. The existence of this floor level prevents bond accumulation as in the benevolent government solution. Even in normal times, the private-public output mix is distorted and unemployment can arise, depending on the economic and political fundamentals. With or without negative shocks, when there is unemployment, it will be higher the larger the government's debt level. High debt levels are therefore associated with high unemployment levels.

While there is a vast theoretical literature on fiscal policy, we are not aware of any work that systematically addresses the positive public finance questions that motivate this paper. Neoclassical theories of fiscal policy, such as the tax smoothing approach, assume frictionless labor markets and thus abstract from unemployment.[2] Traditional Keynesian models incorporate unemployment and allow consideration of the multiplier effects of changes in government spending and taxes. However, these models are static and do not incorporate debt and the costs of debt financing.[3] This limitation also applies to the literature in optimal taxation which has explored how optimal policies are chosen in the presence of involuntary unemployment.[4] The modern new Keynesian literature with its sophisticated dynamic general equilibrium models with sticky prices typically treats fiscal policy as exogenous.[5] Papers in this tradition that do focus on fiscal policy, analyze how government spending shocks impact the economy and quantify the possible magnitude of multiplier effects.[6]

Addressing the questions we are interested in requires a simple and tractable dynamic model. In creating such a model, we have made a number of strong assumptions. First, we employ a model without money and therefore abstract from monetary policy. This means that we cannot consider the important issue of whether the government would prefer to use monetary policy to achieve its policy objectives.[7] Second, we obtain unemployment by simply assuming a downwardly rigid real wage, as opposed to a more sophisticated micro-founded story.[8] This means that our analysis abstracts from any possible effects of fiscal policy on the underlying friction generating unemployment. Third, the source of cyclical fluctuations in our economy comes from the supply

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[2] The tax smoothing approach studies how governments use distortionary taxes and debt to finance government spending in the face of uncertainty. Key papers in the literature are Aiyagari et al (2002), Barro (1979), and Lucas and Stokey (1983).

[3] For a nice exposition of the traditional Keynesian approach to fiscal policy see Peacock and Shaw (1971). Blinder and Solow (1973) discuss some of the complications associated with debt finance and extend the IS-LM model to try and capture some of these.

[4] This literature includes papers by Bovenberg and van der Ploeg (1996), Dreze (1985), Marchand, Pestieau, and Wibaut (1989), and Roberts (1982).

[5] See, for example, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003).

[6] See, for example, Christiano, Eichenbaum, and Rebelo (2009), Hall (2009), Mertens and Ravn (2010), and Woodford (2010).

[7] In an interesting recent contribution, Mankiw and Weinzierl (2011) study optimal fiscal and monetary policy in a two period general equilibrium model with sticky prices. Their analysis of fiscal policy differs from ours because they assume lump sum taxation so that Ricardian Equivalence holds.

[8] There is a literature incorporating theories of unemployment into dynamic general equilibrium models (see Gali (1996) for a general discussion). Modelling options include matching and search frictions (Andolfatto 1996), union wage setting (Ardagna 2007), and efficiency wages (Burnside, Eichenbaum, and Fisher 1999).

rather than the demand side. In our model, recessions arise because negative shocks to the private sector reduce the demand for labor. Labor market frictions prevent the wage from adjusting and the result is unemployment. This vision differs from the traditional and new Keynesian perspectives that emphasize the importance of shocks to consumer demand.[9]

While these strong assumptions undoubtedly represent limitations of our analysis, we nonetheless feel that our model provides an interesting framework in which to study activist fiscal policy. First, the model incorporates the two broad ways in which government can create jobs: *indirectly* by reducing taxes on the private sector, or *directly* through increasing public production. Second, the model allows consideration of two conceptually different types of activist fiscal policy: *balanced-budget policies* wherein tax cuts are financed by public spending decreases or visa versa, and *deficit-financed policies* wherein tax cuts and/or spending increases are financed by increases in public debt. Third, the mechanism by which taxes influence private sector employment in the model is consonant with arguments that are commonplace in the policy arena. For example, the main argument behind objections to eliminating the Bush tax cuts for those making $250,000 and above, was that it would lead small businesses to reduce their hiring during a time of high unemployment. Fourth, the mechanism by which high debt levels are costly for the economy also captures arguments that are commonly made by politicians and policy-makers. Higher debt levels imply larger debt service costs which require either greater taxes on the private sector and/or lower public spending. These policies, in turn, have negative consequences for jobs and the economy.

The organization of the remainder of the paper is as follows. Section 2 outlines the model. Section 3 studies fiscal policy and unemployment with a benevolent government. Section 4 introduces political decision-making, and Section 5 concludes.

2 Model

The environment We consider an infinite horizon economy in which there are two final goods; a private good x and a public good g. There are two types of citizens; entrepreneurs and workers. Entrepreneurs produce the private good by combining labor l and an input z with their own effort. Workers are endowed with 1 unit of labor each period which they supply inelastically. The public

[9] In the new Keynesian literature demand shocks are created by stochastic discount rates (see, for example, Christiano, Eichenbaum, and Rebelo (2009)). In Mankiw and Weinzerl's (2011) two period model, a demand shock in period one arises from a reduction in period two productivity which causes households to have lower expectations about period two income.

good is produced by the government using labor.

There are n_e entrepreneurs and n_w workers where $n_e + n_w = 1$. Each entrepreneur produces with the Leontief production technology $x = A\min\{l, z, \epsilon\}$ where ϵ represents the entrepreneur's effort and A is a productivity parameter. The idea underlying this production technology is that when an entrepreneur hires more workers he must put in more effort to manage them. The public good production technology is $g = l$.

Workers' per period payoff function is $x + \gamma \ln g$, where γ measures the relative value of the public good. Entrepreneurs' per period payoff function is $x + \gamma \ln g - \xi\epsilon^2/2$ where the third term represents the disutility of providing entrepreneurial effort. All individuals discount the future at rate β.

There are markets for the private good, the input, and labor. The private good is the numeraire. The input is supplied by foreign suppliers and has an exogenous but variable price p_θ. We have in mind an input essential for production, such as energy. Each period, this price can take on one of two values p_L or p_H, where p_L is less than p_H and p_H is less than $A - \gamma/n_w$. We will say that the economy is in the *low cost* state when $\theta = L$ and the *high cost* state when $\theta = H$. The probability of the high cost state is α. The wage is denoted ω and the labor market operates under the constraint that the wage cannot go below an exogenous minimum $\underline{\omega}$.[10] This friction is the source of unemployment. There is also a market for risk-free one period bonds. The assumption that citizens have quasi-linear utility implies that the equilibrium interest rate on these bonds is $\rho = 1/\beta - 1$.

To finance its activities, the government taxes entrepreneurs' incomes at rate τ. It can also borrow and lend in the bond market. Government debt is denoted by b and new borrowing by b'. The government is also able to distribute surplus revenues to citizens via lump sum transfers.

Market equilibrium At the beginning of each period, the cost state of the economy is revealed. The government repays existing debt and chooses the tax rate, public good, new borrowing, and transfers. It does this taking into account how its policies impact the market and the need to

[10] We make this assumption to get a simple and tractable model of unemployment. While $\underline{\omega}$ could be literally interpreted as a statutory minimum wage, what we are really trying to capture are the sort of rigidities identified in the survey work of Bewley (1999). The assumption of some type of real wage rigidity is common in the macroeconomics literature (see, for example, Blanchard and Gali (2007), Hall (2005), and Michaillat (2011)) and a large empirical literature investigates the extent of real wage rigidity in practice (see, for example, Barwell and Schweitzer (2007), Dickens et al (2007), and Holden and Wulfsberg (2009)).

balance its budget.

To understand how policies impact the market, assume the cost state is θ, the tax rate is τ, and the public good level is g. Given a wage rate ω, each entrepreneur chooses hiring, the input, and effort, to maximize his utility

$$\max_{(l,z,\epsilon)} (1-\tau)(A\min\{l,z,\epsilon\} - p_\theta z - \omega l) - \xi\frac{\epsilon^2}{2}. \tag{1}$$

Obviously, the solution involves $z = \epsilon = l$. Substituting this into the objective function and maximizing with respect to l reveals that $l = (1-\tau)(A_\theta - \omega)/\xi$ where $A_\theta = A - p_\theta$. Aggregate labor demand from the private sector is therefore $n_e(1-\tau)(A_\theta - \omega)/\xi$. Labor demand from the public sector is g and labor supply is n_w. Setting demand equal to supply, the market clearing wage is

$$\omega = A_\theta - \xi(\frac{n_w - g}{n_e(1-\tau)}). \tag{2}$$

The minimum wage will bind if this wage is less than $\underline{\omega}$. In this case, the equilibrium wage is $\underline{\omega}$ and the unemployment rate is

$$u = \frac{n_w - g - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi}{n_w}. \tag{3}$$

To sum up, in cost state θ with government policies τ and g, the equilibrium wage rate is

$$\omega_\theta = \begin{cases} \underline{\omega} \quad \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \\ A_\theta - \xi(\frac{n_w-g}{n_e(1-\tau)}) \quad \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w-g}{n_e(1-\tau)}) \end{cases} \tag{4}$$

and the unemployment rate is

$$u_\theta = \begin{cases} \frac{n_w - g - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi}{n_w} \quad \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w-g}{n_e(1-\tau)}) \\ 0 \quad \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w-g}{n_e(1-\tau)}). \end{cases} \tag{5}$$

When the minimum wage is binding, the unemployment rate is increasing in τ. Higher taxes cause entrepreneurs to put in less effort and this reduces private sector demand for workers. The unemployment rate is also decreasing in g because to produce more public goods, the government must hire more workers. When the minimum wage is not binding, the equilibrium wage is decreasing in τ and increasing in g.

Each entrepreneur earns profits of $\pi_\theta = (1-\tau)(A_\theta - \omega_\theta)^2/\xi$. Assuming he receives no government transfers and consumes his profits, an entrepreneur obtains a period payoff of

$$v_{e\theta} = \frac{(A_\theta - \omega_\theta)^2(1-\tau)^2}{2\xi} + \gamma \ln g. \tag{6}$$

Jobs are randomly allocated among workers and so each worker obtains an expected period payoff

$$v_{w\theta} = (1 - u_\theta)\omega_\theta + \gamma \ln g. \tag{7}$$

Again, this assumes that the worker receives no transfers and simply consumes his earnings.

Aggregate output of the private good is $x_\theta = n_e A(1-\tau)(A_\theta - \omega_\theta)/\xi$. Substituting in the expression for the equilibrium wage, we see that

$$x_\theta = \begin{cases} n_e A(1-\tau)(A_\theta - \underline{\omega})/\xi & \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \\ A\,(n_w - g) & \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}). \end{cases} \tag{8}$$

Observe that the tax rate has no impact on private sector output when the minimum wage constraint is not binding. This is because labor is inelastically supplied and as a consequence the wage adjusts to ensure full employment. A higher tax rate just leads to an offsetting reduction in the wage rate. However, when there is unemployment, tax hikes reduce private sector output because they lead entrepreneurs to reduce effort. Public good production has no effect on private output when there is unemployment, but reduces it when there is full employment.

The government budget constraint Having understood how markets respond to government policies, we can now formalize the government's budget constraint. Tax revenue is

$$R_\theta(\tau, \omega_\theta) = \tau(n_e \pi_\theta) = \tau n_e (1-\tau)(A_\theta - \omega_\theta)^2/\xi. \tag{9}$$

Total government revenue is therefore $R_\theta(\tau, \omega_\theta) + b'$. The cost of public good provision and debt repayment is $\omega_\theta g + b(1+\rho)$. The budget surplus available for transfers is the difference between $R_\theta(\tau, \omega_\theta) + b'$ and $\omega_\theta g + b(1+\rho)$. The government budget constraint is that this budget surplus be non-negative, which requires that

$$R_\theta(\tau, \omega_\theta) - \omega_\theta g \geq b(1+\rho) - b'. \tag{10}$$

There is also an upper limit $\overline{b}$ on the amount of debt the government can issue. This limit is motivated by the unwillingness of borrowers to hold bonds that they know will not be repaid. If, in steady state, the government were borrowing an amount b such that the interest payments exceeded the maximum possible tax revenues in the high cost state; i.e., $\rho b > \max_\tau R_H(\tau, \underline{\omega})$, then, if the economy were in the high cost state, it would be unable to repay the debt *even if it provided no public goods or transfers.* The upper limit on debt is therefore $\overline{b} = \max_\tau R_H(\tau, \underline{\omega})/\rho$.

3 Benevolent government

It will prove instructive to break down the analysis of the benevolent government's solution into two parts. First, we study the static optimal policy problem for this economy. Thus, we ignore debt and, in the spirit of the optimal taxation literature, assume that the government faces an exogenous revenue requirement. Having understood how the static solution depends on the revenue requirement, we then introduce debt and study the dynamic policy choice problem. In the dynamic problem, the government's revenue requirement corresponds to the difference between debt repayment and new borrowing (as in (10)). Solving the dynamic model endogenizes the government's revenue requirement and completes the picture of the solution.

3.1 The static problem

The static optimal policy problem is to choose a tax rate τ and a level of public good g to maximize aggregate citizen utility subject to the requirement that revenues net of public production costs cover a revenue requirement r. To allow for the possibility of surpluses or deficits when debt is introduced, we assume that the revenue requirement can be positive or negative. Under the assumption that any surplus revenues are transferred to the citizens, this problem can be posed as:

$$\max_{(\tau,g)} \left\{ \begin{array}{c} R_\theta(\tau,\omega_\theta) - \omega_\theta g - r + n_e v_{e\theta} + n_w v_{w\theta} \\ \\ s.t.\ R_\theta(\tau,\omega_\theta) - \omega_\theta g \geq r \end{array} \right\}. \tag{11}$$

What makes this problem non-standard is the possibility of unemployment.[11] The problem is simplified by noting that there is no loss of generality in assuming that the government always sets taxes sufficiently high so that the equilibrium wage equals $\underline{\omega}$. As noted earlier, taxes are non-distortionary when the wage exceeds $\underline{\omega}$ and the government has the ability to make transfers. Thus, if the wage exceeded $\underline{\omega}$, there would be no change in aggregate utility if the government raised taxes and simply redistributed the additional tax revenues back to the citizens. This observation

[11] With no downward rigidity in the wage, the solution to this problem is very simple. The government provides the Samuelson level of the public good and taxes the private sector sufficient to finance it. The wage adjusts to ensure full employment and an efficient allocation of resources. In the dynamic policy choice problem, there is no role for government debt.

allows us to write problem (11) as:

$$\max_{(\tau,g)} \left\{ \begin{array}{c} x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma \ln g - r \\ s.t.\ R_\theta(\tau,\underline{\omega}) - \underline{\omega} g \geq r \ \&\ g + \frac{x_\theta(\tau)}{A} \leq n_w \end{array} \right\}, \tag{12}$$

where $x_\theta(\tau)$ is the output of the private good when the tax rate is τ and the wage rate is $\underline{\omega}$ (see the top line of (8)).

Problem (12) has a simple interpretation. The objective function is the aggregate surplus generated by outputs $x_\theta(\tau)$ and g, less the revenue requirement.[12] The first inequality is the government budget constraint under the assumption that the wage is $\underline{\omega}$. The second inequality is the *resource constraint*: it requires that the demand for labor at wage $\underline{\omega}$ is less than or equal to the number of workers n_w.[13]

A diagrammatic approach will be helpful in explaining the solution to problem (12). Without loss of generality, we assume that r is less than or equal to the maximum possible tax revenue which is $R_\theta(1/2,\underline{\omega})$.[14] We also assume that unemployment would result if the government faced the maximal revenue requirement.[15] To understand our diagrammatic approach, consider Fig. 1.A. The tax rate is measured on the horizontal axis and the public good on the vertical. The upward sloping line is the resource constraint. Using the expression for $x_\theta(\tau)$ from (8), this line is described by

$$g = n_w - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi. \tag{13}$$

At points along this line, there is full employment at the wage $\underline{\omega}$. Policies must be on or below this line and points below are associated with unemployment.

The upward sloping, convex curves represent the government's *indifference curves*. These curves tell us the government's preferences over different (τ, g) pairs. Indifference curves satisfy

[12] The expression for the surplus generated by $x_\theta(\tau)$ (the first two terms) reflects the fact that the surplus associated with the private good consists of the consumption benefits it generates less the costs associated with the input and entrepreneurial effort necessary to produce it.

[13] This constraint is required to ensure that the equilibrium wage is indeed $\underline{\omega}$.

[14] The revenue maximizing tax rate is 1/2 and the maximum revenue requirement is $n_e A_\theta(A_\theta - \underline{w})/4\xi$. Of course, if r were higher than this level, the problem would have no solution. In the dynamic model, however, this case will never arise.

[15] If the government faces the maximal revenue requirement it will set the tax rate equal to 1/2 and provide no public good. Private sector employment will be $n_e(A_\theta - \underline{\omega})/2\xi$ and there will be no public sector employment. Thus, this assumption amounts to the requirement that n_w exceeds $n_e(A_\theta - \underline{\omega})/2\xi$.

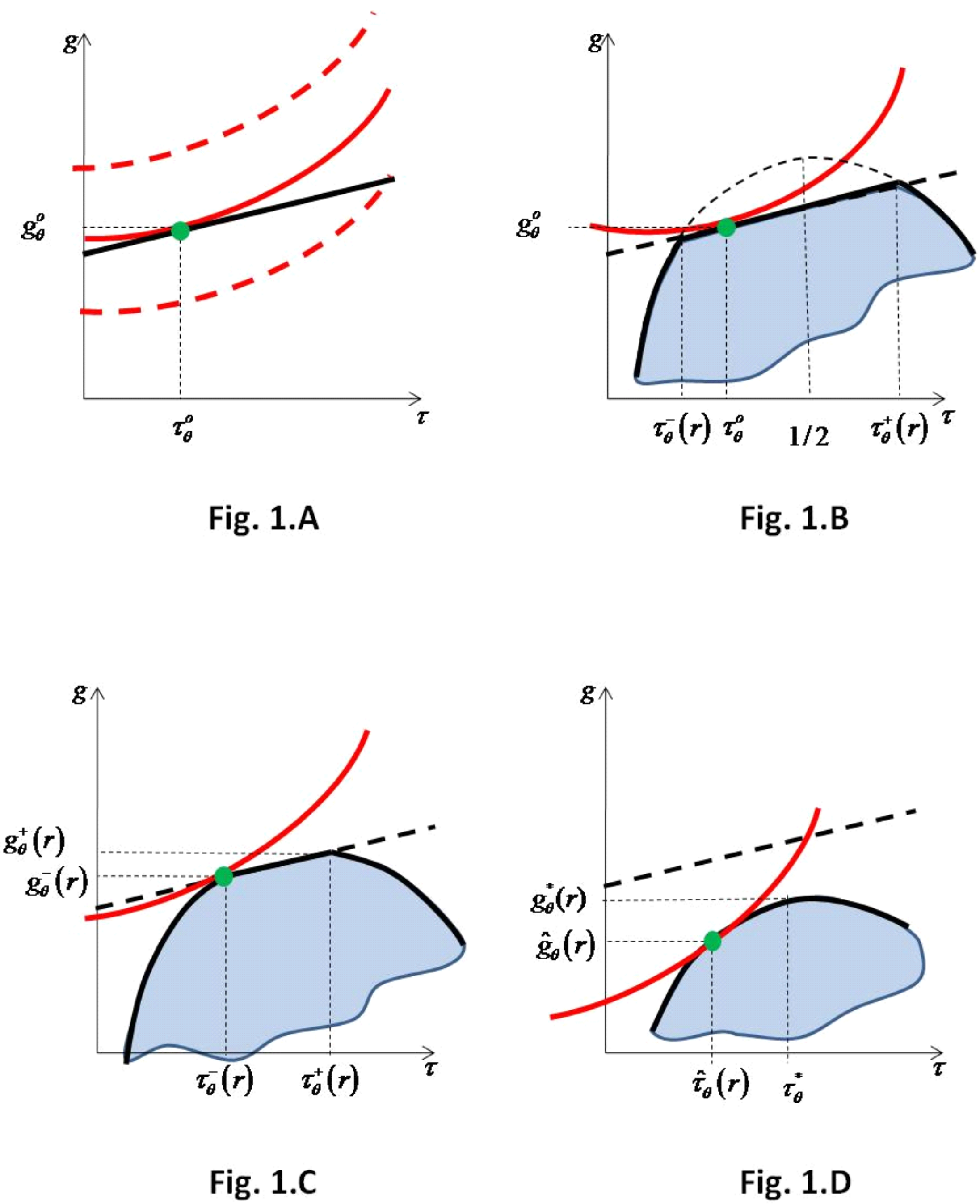
g
g_θ^o
τ_θ^o
τ
Fig. 1.A
g
g_θ^o
$\tau_\theta^-(r)$
τ_θ^o
1/2
$\tau_\theta^+(r)$
τ
Fig. 1.B
g
$g_\theta^+(r)$
$g_\theta^-(r)$
$\tau_\theta^-(r)$
$\tau_\theta^+(r)$
τ
Fig. 1.C
g
$g_\theta^*(r)$
$\hat{g}_\theta(r)$
$\hat{\tau}_\theta(r)$
τ_θ^*
τ
Fig. 1.D

Figure 1:

for some target utility level U

$$x_\theta(\tau)\left(\frac{A_\theta}{A}\right)-n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2}+\gamma\ln g=U. \tag{14}$$

Higher indifference curves are associated with higher utility levels, so utility is increasing as we move North-West. The indifference curves become flatter as we move South-East and the public good becomes more scarce.

The tangency point between the indifference curves and the full employment line defines the *first best* policies $(\tau_\theta^o, g_\theta^o)$. When these policies are in place, there is both full employment at wage $\underline{\omega}$ and the optimal mix of private and public outputs. The public good level g_θ^o is determined by the usual Samuelsonian considerations. The associated tax rate τ_θ^o provides entrepreneurs with just the right incentive to employ those workers not employed in the public sector at the wage rate $\underline{\omega}$. In Fig. 1.A, the tax rate τ_θ^o is positive, but there is nothing to prevent a subsidy being necessary to achieve full employment.

In the remaining panels of Figure 1, we add the government's *budget line* - the locus of points that satisfy the budget constraint with equality. The budget line associated with revenue requirement r can be solved to yield

$$g=\frac{R_\theta(\tau,\underline{\omega})}{\underline{\omega}}-\frac{r}{\underline{\omega}}. \tag{15}$$

Policies must be on or below this line and points below are associated with positive transfers. Each budget line is hump shaped, with peak at $\tau=1/2$. Increasing the revenue requirement shifts down the budget line but does not change the slope. Panels B, C, and D of Figure 1 represent increasing revenue requirements.

The feasible set of (τ, g) pairs for the optimal policy problem are those that lie below both the budget and resource constraints. This set is represented by the gray, cross hatched areas in Figure 1. Observe that the feasible set is (weakly) convex which makes the problem well-behaved.

With this diagrammatic apparatus in place, we can now explain the optimal policies. The government would ideally like to choose the first best policies $(\tau_\theta^o, g_\theta^o)$. This is feasible when the revenue requirement r is less than $r_\theta^o=R_\theta(\tau_\theta^o,\underline{\omega})-\underline{\omega}g_\theta^o$ as in Fig. 1.B. The surplus revenues r_θ^o-r can be rebated back to citizens in the form of transfers. In this range of revenue requirements, the optimal tax and public good levels are independent of r and changes in r are absorbed by changes in transfers.

When r is higher than r_θ^o, the first best policies are too expensive for the government. To meet its revenue requirement, the government must reduce public good provision and/or raise taxes. In making this decision, the government balances the costs of two types of distortions: unemployment and having the wrong mix of public and private outputs. Starting from the first best position of full employment and the optimal output mix, the costs of distorting the output mix are second order, while the costs of unemployment are first order. As a result, for revenue requirements only slightly higher than r_θ^o, the government will preserve full employment by appropriate adjustment of the output mix. The nature of this adjustment depends on the location of the first best policies $(\tau_\theta^o, g_\theta^o)$.

The situation is illustrated in Fig. 1.C. The government can achieve full employment by choosing any tax rate in the range $[\tau_\theta^-(r), \tau_\theta^+(r)]$ with associated level of public good given by (13), where $\tau_\theta^-(r)$ and $\tau_\theta^+(r)$ are defined by the left and right intersections of the budget line and resource constraint. When τ_θ^o lies to the left of the interval $[\tau_\theta^-(r), \tau_\theta^+(r)]$, as in Fig. 1.C, the optimal policies are the tax rate $\tau_\theta^-(r)$ with associated public good level $g_\theta^-(r)$. This policy combination involves the least distortion in the output mix consistent with achieving full employment. When τ_θ^o lies to the right of the interval $[\tau_\theta^-(r), \tau_\theta^+(r)]$, as in Fig. 2.B, the optimal policies are the tax rate $\tau_\theta^+(r)$ with associated public good level $g_\theta^+(r)$. Notice that in the former case, the government distorts the output mix *towards* public production and in the latter it distorts *away* from public production. These cases also generate different comparative static implications. In the former case, as the revenue requirement increases, taxes and public production increase, as illustrated in Fig. 2.A. In the latter, taxes and public production decrease as illustrated in Fig. 2.B. Intuitively, in the former case, the government generates a fiscal surplus by raising taxes on the private sector and hiring the displaced workers in the public sector. In the latter, it generates a fiscal surplus by laying off public sector workers and using tax cuts to incentivize the private sector to hire them.

For revenue requirements significantly higher than r_θ^o, maintaining full employment requires a larger distortion in the output mix. Indeed, for sufficiently high revenue requirements, as in Fig. 1.D, maintaining full employment is not possible. Eventually, therefore, increased fiscal pressure must result in unemployment. In the Appendix, we prove that there is a cut point $r_\theta^* > r_\theta^o$ at which the government abandons the effort to maintain full employment. For revenue requirements higher than r_θ^*, the resource constraint is not binding and the optimal policies are at the tangency of the budget line and the indifference curve. These policies are denoted by $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ and are

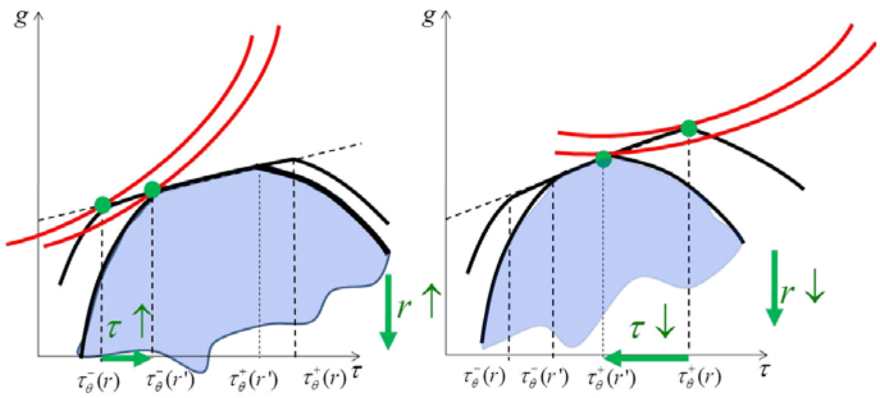

Fig. 2.A **Fig. 2.B**

Figure 2:

illustrated in Fig. 1.D. As the revenue requirement climbs above r_θ^*, the tax rate increases and the public production level decreases, so $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ moves to the South-East. Unemployment also increases.

Summarizing this discussion, we have the following description of the optimal policies.

Proposition 1 *There exists a revenue requirement $r_\theta^* > r_\theta^o$ such that the solution to problem (12) has the following properties.*

- *If $r \leq r_\theta^o$, the optimal policies are $(\tau_\theta^o, g_\theta^o)$ and involve full employment with the optimal output mix. In this range, the optimal policies are independent of the revenue requirement and increases in r are absorbed by reductions in government transfers.*

- *If $r \in (r_\theta^o, r_\theta^*]$, the optimal policies involve full employment with a distorted output mix. If $\tau_\theta^o < \tau_\theta^-(r)$, the optimal policies are $(\tau_\theta^-(r), g_\theta^-(r))$ and the output mix is distorted in favor of the public good. In this range, as r increases, both public production and the tax rate increase. If $\tau_\theta^o > \tau_\theta^+(r)$, the optimal policies are $(\tau_\theta^+(r), g_\theta^+(r))$ and the output mix is distorted in favor of the private good. As r increases, both public production and the tax rate decrease.*

- *If $r > r_\theta^*$, the optimal policies are $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ and involve unemployment. In this range, as r increases, public production decreases, the tax rate increases, and unemployment increases.*

Proposition 1 tells us how taxes, public good production, and employment in each state depend on the government's revenue requirement.[16] A premise of the analysis is that the revenue requirement is exogenous. In a dynamic model, however, r is endogenous, depending on the amount of government debt that needs to be repaid and new borrowing.[17] Proposition 1, therefore, leaves a key question unanswered. In which of the three cases described should we expect the government to be in the long run?

3.2 Dynamics

We now bring debt into the picture. Intuitively, debt should be helpful since it allows government to transfer revenues from good times in which low costs create robust private sector profits and labor demand, to bad times in which high costs result in a depressed private sector. In bad times, the revenues transferred will reduce fiscal pressure and permit policy changes which reduce unemployment and distortions in the output mix. The benefits from these changes will exceed the costs associated with raising revenue in good times because the distortions created by tax increases and public good reductions are lower in good times. Indeed, as noted earlier, taxation is non-distortionary when the minimum wage constraint is not binding.

The dynamic problem is to choose a time path of policies to maximize aggregate lifetime citizen utility. Since in equilibrium citizens are indifferent as to their allocation of consumption across time, their lifetime utility will equal the value of their initial bond holdings plus the payoff they would obtain if they simply consumed their net earnings and transfers in each period. Ignoring these initial bond holdings, the problem can therefore be formulated recursively as

$$V_\theta(b) = \max_{(\tau,g,b')} \left\{ \begin{array}{c} B_\theta(\tau, g, b', b, w_\theta) + n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b') \\ \\ s.t.\ B_\theta(\tau, g, b', b, w_\theta) \geq 0 \ \&\ b' \leq \overline{b} \end{array} \right\}, \tag{16}$$

where $V_\theta(b)$ is aggregate lifetime citizen utility in state θ with initial debt level b and $B_\theta(\cdot)$ denotes the budget surplus available for transfers.[18] Under this recursive formulation, in each period,

[16] It is important to note that the solution described in Proposition 1 reflects our (w.l.o.g.) assumption that the government sets a tax rate such that the wage is $\underline{\omega}$. When r is less than r_θ^o, the government could equally well reduce the tax rate and let the wage rate rise above $\underline{\omega}$, compensating for the lost tax revenues by reducing transfers. Thus, in the case of full employment with no distortions, the optimal tax rate and level of transfers are not uniquely defined. In all the other cases, the solution must be exactly as described in Proposition 1.

[17] Specifically, from (10), we see that r will equal $(1+\rho)b - b'$.

[18] That is, $B_\theta(\tau, g, b', b, \omega_\theta) = R_\theta(\tau, \omega_\theta) + b' - \omega_\theta g - b(1+\rho)$.

given the cost state θ and initial debt level b, the government chooses the current tax rate τ, the public good level g, and new borrowing b'. Transfers are determined residually by $B_\theta(\tau, g, b', b, w_\theta)$.

As in the static problem, there is no loss of generality in assuming that the government always sets taxes sufficiently high that the wage is equal to $\underline{w}$. Thus, proceeding as in the static case and substituting $b(1+\rho) - b'$ for r, we may rewrite the government's problem as

$$V_\theta(b) = \max_{(\tau,g,b')} \left\{ \begin{array}{c} x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma \ln g + b' - b(1+\rho) + \beta EV_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \underline{w}) \geq 0,\ g + \frac{x_\theta(\tau)}{A} \leq n_w\ \&\ b' \leq \overline{b} \end{array} \right\}. \tag{17}$$

To focus the analysis on the natural case of interest, we assume that when debt is zero the government is able to achieve the first best without borrowing in the low but not the high cost state. More precisely, we make:[19]

Assumption 1

$$R_H(\tau_H^o, \underline{w}) - \underline{w}g_H^o < 0 < R_L(\tau_L^o, \underline{w}) - \underline{w}g_L^o.$$

Recalling the definition of r_θ^o, the critical revenue requirement delineating the first and second cases of Proposition 1, Assumption 1 implies that r_H^o is negative and r_L^o is positive.

A solution to problem (17) consists of optimal policy functions $\{\tau_\theta(b), g_\theta(b), b'_\theta(b)\}$ for each state θ and value functions $V_H(b)$ and $V_L(b)$. By standard methods, it can be shown that there exists a solution and that the associated value functions are concave and differentiable. Corresponding to any solution, we can define $r_\theta(b) = (1+\rho)b - b'_\theta(b)$ to be the revenue requirement implied by the optimal policies in state θ with initial debt level b. Letting $(\tau_\theta^s(r), g_\theta^s(r))$ denote the optimal static policies described in Proposition 1, it is clear that $(\tau_\theta(b), g_\theta(b))$ will equal $(\tau_\theta^s(r_\theta(b)), g_\theta^s(r_\theta(b)))$. As discussed above, therefore, the key issue is to identify how the revenue requirement behaves in the long run. This will tell us which of the three cases described in Proposition 1 will arise.

To study the long run, note that given a solution to problem (17), for any initial debt level b_0 and sequence of shocks $\langle\theta_t\rangle$, we can associate a sequence of policies $\langle\tau_t, g_t, b'_t\rangle$.[20] The associated sequence of revenue requirements is then $\langle r_t\rangle$ where for all t, $r_t = (1+\rho)b'_{t-1} - b'_t$. The question

[19] In terms of the fundamental parameters of the model this assumption can be shown to be equivalent to:

$$n_eA_H - \xi n_w < n_e\left(\underline{\omega} + \frac{\gamma}{n_w}\right) - \frac{\xi\gamma}{\underline{\omega} + \frac{\gamma}{n_w}} < n_eA_L - \xi n_w.$$

[20] This sequence is defined inductively as follows: $(\tau_0, g_0, b'_0) = \left(\tau_{\theta_0}(b_0), g_{\theta_0}(b_0), b'_{\theta_0}(b_0)\right)$ and for all $t \geq 1$, $(\tau_t, g_t, b'_t) = \left(\tau_{\theta_t}(b'_{t-1}), g_{\theta_t}(b'_{t-1}), b'_{\theta_t}(b'_{t-1})\right)$.

is how these sequences behave as t becomes large. In fact, we can show that the probability that r_t is less than or equal to r_H^o converges to one as t becomes large. From Proposition 1, we may conclude that, in the long run, the relevant case is the first. Thus we have:

Proposition 2 *With a benevolent government, the economy converges to full employment with the optimal output mix.*

In the long run, therefore, in cost state θ, taxes and public production are $(\tau_\theta^o, g_\theta^o)$. In the high cost state ($\theta = H$) public production is higher and tax revenues are lower. The increase in public production occurs because, while the benefit of public goods is state independent, the cost of the private good is higher. Lower tax revenues also reflect the fact that the private sector is less profitable.[21] Despite lower net tax revenues, the government is able to implement the first best policies in the high cost state in the long run because it has accumulated sufficient bond holdings.

Precisely how the government finances its activities is not tied down by the theory because there are multiple solutions to problem (17) and financing will depend on the details of the solution. The simplest solution is that the government gradually accumulates bonds until its debt level reaches r_H^o/ρ (recall that r_H^o is negative by Assumption 1). Once debt reaches this level, the steady state revenue requirement is r_H^o. This negative revenue requirement reflects the fact that the government is earning interest on its bond holdings. In the high cost state, these interest earnings are just sufficient to finance the shortfall in net tax revenues. In the low cost state, the interest earnings are rebated back to the citizens in a transfer along with the surplus net tax revenues r_L^o.[22] Intuitively, other solutions are possible because once debt has reached r_H^o/ρ, the government can further reduce it temporarily with no effect on citizens' long run utility.

Proposition 2 strikes us as an interesting result. The conclusion that in the long run a benevolent government can employ fiscal policy to achieve full employment seems likely to hold in many models of unemployment. After all, in environments where unemployment is caused by a rigidity, it can typically be overcome by appropriate corrective subsidies or taxes. If government can finance

[21] The impact on the tax rate of moving from the low cost to the high cost state is ambiguous. On the one hand, to hire any given number of workers, entrepreneurs need to be provided with lower taxes in the high cost state since workers are less profitable. On the other hand, entrepreneurs need to hire fewer workers because public production increases.

[22] As noted in footnote #16, in the low cost state, the government could equally well reduce the tax rate and let the wage rate rise above $\underline{w}$, compensating for the lost tax revenues by reducing transfers. In this case, we would observe wage reductions rather than transfer reductions when the economy moves from the low to the high cost state.

such corrective programs with lump sum taxation, unemployment can be eliminated immediately. But even when only distortionary taxes are feasible, unemployment can still be eliminated in the long run if the government can accumulate assets as illustrated in Proposition 2. This suggests that in many environments the real cause of long run unemployment lies in the policy-making process rather than frictions in the market. The general lesson hinted at, therefore, is that no satisfactory theory of unemployment can abstract from how fiscal policy is chosen.

The logic underlying Proposition 2 is similar to that arising in tax smoothing models. In these models, government finances spending with a distortionary tax and uses debt to smooth tax rates across periods. The need to smooth is created by shocks to government spending needs and/or by cyclical variation in revenue yields. In the case of spending shocks, for example, tax smoothing requires the government to transfer revenues from times of low spending needs (peacetime) to times of high spending needs (wartime). The revenues transferred permit tax reductions and the benefits from these reductions exceed the costs associated with higher taxes in peacetime because the distortionary costs of taxation are convex. Under certain conditions, the optimal policy in these models is for the government to accumulate a stock of bonds which eventually allows it to finance government spending with no taxation (Aiyagari, et al 2002). All distortions are therefore eliminated in the long run. While the nature of the distortions arising in our model are very different from those arising in a tax smoothing model, the basic forces driving optimal policy are the same.[23] The role for debt is to smooth distortions across periods and, in the long run, it is optimal to completely eliminate distortions.

4 Political decision-making

The lesson from the previous section is that, at least in this model, introducing political decision-making is necessary to obtain an interesting positive theory of fiscal policy and unemployment. Our strategy for doing this follows Battaglini and Coate (2007, 2008). Thus, we assume that the economy is divided into N identically sized political districts, each a microcosm of the economy as a whole. In each period, policy decisions are made by a legislature consisting of N representatives, one from each district. Each representative wishes to maximize the aggregate utility of the citizens in his district. In addition to choosing taxes, public goods, and borrowing, the legislature must

[23] The distortions arising in a tax smoothing model are the standard deadweight costs of taxation.

also choose how to divide any budget surplus B_θ between the districts.

The decision-making process in the legislature follows a simple sequential protocol. At stage $j = 1, 2, ...$ of this process, a representative is randomly selected to make a proposal to the floor. A proposal consist of policies (τ, g, b') and district-specific transfers $(s_i)_{i=1}^N$ satisfying the constraints that $\sum_i s_i$ does not exceed the budget surplus $B_\theta(\tau, g, b', b, \omega_\theta)$ and b' does not exceed the debt limit $\bar{b}$. If the proposal receives the votes of $Q < N$ representatives, then it is implemented and the legislature adjourns until the following period. If the proposal does not pass, then the process moves to stage $j + 1$, and a representative is selected again to make a new proposal.[24]

Following the analysis in Battaglini and Coate (2008), it can be shown that in cost state θ with initial debt level b, the equilibrium policies $\{\tau_\theta(b), g_\theta(b), b'_\theta(b)\}$ are chosen to solve the maximization problem:

$$\max_{(\tau,g,b')} \left\{ \begin{array}{c} qB_\theta(\tau, g, b', b, \omega_\theta) + n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \omega_\theta) \geq 0 \ \&\ b' \leq \bar{b} \end{array} \right\}, \tag{18}$$

where $q = N/Q$ and $V_{\theta'}(b')$ is equilibrium aggregate lifetime citizen expected utility in state θ' with debt level b'. The equilibrium value functions $V_H(b)$ and $V_L(b)$ are defined recursively by:

$$V_\theta(b) = B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \omega_\theta) + n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b'_\theta(b)) \tag{19}$$

for $\theta \in \{L, H\}$. Representatives' value functions, which reflect only aggregate utility in their respective districts, are given by $V_H(b)/N$ and $V_L(b)/N$.

Since it is not the focus of this work, we omit here the formal proof of this characterization of equilibrium.[25] The underlying intuition, however, is easily described. In the legislative bargaining process, the proposer chooses policies (τ, g, b') and transfers $(s_i)_{i=1}^N$ to maximize the expected welfare of his own district. The proposer may have to offer transfers to other districts to get his proposal approved. In this case, in a symmetric equilibrium, he offers the same transfer s to $Q - 1$ other districts. At stage j of the process, the transfer s must be such that:

$$s + \frac{1}{N}\left[n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b')\right] \geq \frac{V_\theta^{j+1}(b)}{N}, \tag{20}$$

[24] This process may either continue indefinitely until a proposal is chosen, or may last for a finite number of stages as in Battaglini and Coate (2008): the analysis is basically the same. In Battaglini and Coate (2008) it is assumed that in the last stage, one representative is randomly picked to choose a policy; this representative is then required to choose a policy that divides the budget surplus evenly between districts.

[25] See Battaglini and Coate (2008) for a more extensive discussion.

where $V_{\theta}^{j+1}(b)$ is equilibrium aggregate lifetime citizen expected utility in state θ with initial debt level b at the beginning of stage $j+1$. This reflects the fact that a representative would vote for the proposal only if the associated utility, the left hand side of (20), is at least as high as the utility of his outside option, $V_{\theta}^{j+1}(b)/N$.[26] In equilibrium, (20) must be satisfied with equality, implying that, at the margin, transfers depend on $\frac{1}{N}\left[n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b')\right]$. This forces the proposer to internalize the opportunity cost of the policies (τ, g, b') for a fraction Q/N of the population. The resulting objective function for the proposer,

$$B_{\theta}(\tau, g, b', b, \omega_{\theta}) + \frac{Q}{N}\left[n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b')\right], \tag{21}$$

is equivalent to the objective function in (18) (just multiply through by $q = N/Q$). The fact that representatives' value functions are described by (19) (divided by $1/N$) then follows from the fact that each representative is ex ante equally likely to be the proposer or, if not the proposer, to be included in the coalition whose districts receive transfers.

A convenient short-hand way of understanding the equilibrium is to imagine that in each period a minimum winning coalition (mwc) of Q representatives is randomly chosen and that this coalition collectively chooses policies to maximize its aggregate utility. Problem (18) reflects the coalition's maximization problem and, because membership in this coalition is random, all representatives are ex ante identical and have a common value function given by (19) (divided by $1/N$). In what follows, we will use this way of understanding the equilibrium and speak *as if* a randomly drawn mwc is choosing policy in each period.

For the purposes of the rest of this paper, we define a *political equilibrium* as consisting of policy functions $\{\tau_{\theta}(b), g_{\theta}(b), b'_{\theta}(b)\}$ for each state θ and value functions $V_H(b)$ and $V_L(b)$ such that: (i) the policy functions solve (18) given the value functions, and, (ii) the value functions satisfy (19) given the policy functions. An equilibrium is said to be *well-behaved* if the associated value functions $V_H(b)$ and $V_L(b)$ are concave for debt levels below $\overline{b}$. In the Appendix, we show:

Proposition 3 *There exists a well-behaved equilibrium.*

The equilibrium policies are characterized by solving problem (18). Again, there is no loss of generality in assuming that the mwc will always set taxes sufficiently high that the wage is $\underline{\omega}$. Indeed, because $q > 1$, it *must* be the case that the wage is $\underline{\omega}$, for the mwc would always raise

[26] The term $V_{\theta}^{j+1}(b)/N$ is endogenous in equilibrium, but from the point of view of the proposer it is given and so irrelevant for his policy choice.

taxes if it could extract more revenue with no deadweight cost. Thus, we can rewrite (18) as:

$$\max_{(\tau,g,b')}\left\{\begin{array}{c} x_\theta(\tau)\left(\frac{A_\theta}{A}\right)-n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2}+\gamma\ln g+(q-1)\left(R_\theta(\tau,\underline{\omega})-\underline{\omega}g\right)+q\left(b'-(1+\rho)b\right)+\beta EV_{\theta'}(b') \\ s.t.\ B_\theta(\tau,g,b',b,\underline{\omega})\geq 0,\ g+\frac{x_\theta(\tau)}{A}\leq n_w\ \&\ b'\leq\overline{b} \end{array}\right\}, \tag{22}$$

which is the equilibrium analog of (17). To understand the equilibrium policies we follow the procedure used for the benevolent government case. First, we investigate the equilibrium tax and public good levels for a given revenue requirement. Then we understand the revenue requirements that arise in the long run by characterizing the equilibrium debt distribution.

4.1 The static problem

The equilibrium version of static problem (12) is

$$\max_{(\tau,g)}\left\{\begin{array}{c} x_\theta(\tau)\left(\frac{A_\theta}{A}\right)-n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2}+\gamma\ln g+(q-1)\left(R_\theta(\tau,\underline{\omega})-\underline{\omega}g\right)-qr \\ s.t.\ R_\theta(\tau,\underline{\omega})-\underline{\omega}g\geq r\ \&\ g+\frac{x_\theta(\tau)}{A}\leq n_w \end{array}\right\}. \tag{23}$$

The key difference between this and problem (12) is that, since $q>1$, the mwc cares directly about net tax revenues $R_\theta(\tau,\underline{\omega})-\underline{\omega}g$. This makes the mwc's indifference curves more convex than those of the benevolent government, steeper at high tax rates and flatter at low tax rates. Moreover, rather than preferences being always increasing in g and decreasing in τ, there are interior optimal levels of both g and τ. Thus, the mwc's preferences exhibit an interior satiation point in (τ,g) space. As q increases, this point converges to $(1/2,0)$, the revenue maximizing policies.

Following the strategy used to study the static problem, first consider what happens when the revenue requirement is so low that the budget constraint is not binding. Let the optimal policies in this situation be denoted $(\tau_\theta^q,g_\theta^q)$ and let r_θ^q to be the revenue requirement equal to $R_\theta(\tau_\theta^q,\underline{\omega})-\underline{\omega}g_\theta^q$. If the revenue requirement is less than r_θ^q, the mwc will choose $(\tau_\theta^q,g_\theta^q)$ and use the surplus revenues r_θ^q-r to finance transfers to their districts. There are two possibilities for the optimal policies, depending on whether the mwc's satiation point lies outside or inside the resource constraint. The first possibility is illustrated in Fig. 3.A. In this case, the optimal policies $(\tau_\theta^q,g_\theta^q)$ correspond to the point of tangency between the indifference curve and the resource constraint. This case is similar to the situation illustrated in Fig. 1.B, although the optimal policies differ and hence the output mix is distorted. The second possibility is illustrated in Fig. 3.B. In this

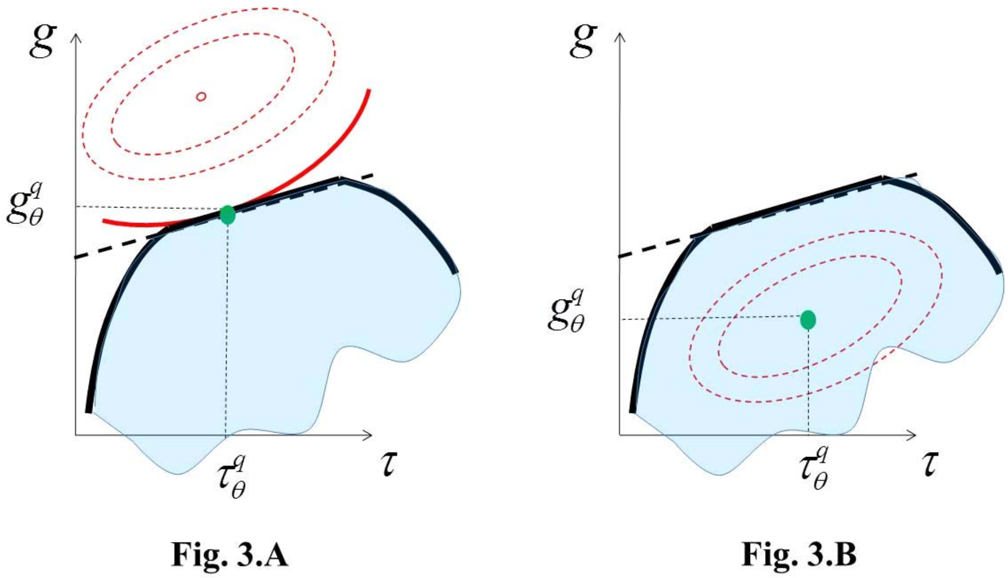

Figure 3:

case, the optimal policies $(\tau_\theta^q, g_\theta^q)$ are just equal to the mwc's satiation point. Obviously, since they lie inside the resource constraint, these policies involve unemployment.

As we show in the Appendix, the mwc's satiation point lies outside the resource constraint if and only if q is less than q_θ^*, where q_θ^* is defined by:

$$n_e \left[\frac{q(A_\theta - \underline{\omega}) + \underline{\omega}}{\xi(2q-1)} \right] + \frac{\gamma}{(q-1)\,\underline{\omega}} = n_w. \tag{24}$$

Intuitively, higher values of q increase the mwc's preference for net revenues. When q exceeds q_θ^*, the mwc's preferred tax rate is sufficiently high and its preferred public good level sufficiently low, that unemployment arises.

If the revenue requirement exceeds r_θ^q the mwc will not make transfers to their districts and the budget constraint will bind. The optimal policies will then solve:

$$\max_{(\tau,g)} \left\{ \begin{array}{c} x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma \ln g - qr \\ s.t.\ R_\theta(\tau, \underline{\omega}) - \underline{\omega} g \geq r \ \&\ g + \frac{x_\theta(\tau)}{A} \leq n_w \end{array} \right\}. \tag{25}$$

This is equivalent to the problem studied in Section 3 and thus the solution will be as described by Proposition 1.[27] When q is less than q_θ^*, there will be two regions, one with full employment

[27] The fact that r is multiplied by q in the objective function has no effect on the optimal policies, since qr is

with a distorted output mix, $r \leq r^*_\theta$; and one with unemployment, $r > r^*_\theta$. The threshold r^*_θ is the same as the threshold identified in Proposition 1. When q exceeds q^*_θ, there will be just one region with unemployment (since $r^q_\theta > r^*_\theta$).

Summarizing this discussion, we have the following analogy to Proposition 1.

Proposition 4 *There exists a revenue requirement $r^*_\theta > r^o_\theta$ such that the solution to problem (23) has the following properties.*

- *If $r \leq r^q_\theta$, the optimal policies are $(\tau^q_\theta, g^q_\theta)$. When $q < q^*_\theta$, these policies involve full employment with a distorted output mix. When $q > q^*_\theta$, these policies involve unemployment.*

- *If $r \in (r^q_\theta, \max\{r^q_\theta, r^*_\theta\}]$, the optimal policies involve full employment with a distorted output mix. If $\tau^o_\theta < \tau^-_\theta(r)$, the optimal policies are $(\tau^-_\theta(r), g^-_\theta(r))$, and, if $\tau^o_\theta > \tau^+_\theta(r)$, the optimal policies are $(\tau^+_\theta(r), g^+_\theta(r))$. This case only arises when $q < q^*_\theta$, since $r^q_\theta > r^*_\theta$ when $q > q^*_\theta$.*

- *If $r > \max\{r^q_\theta, r^*_\theta\}$, the optimal policies are $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ and involve unemployment. In this range, as r increases, unemployment increases.*

4.2 Dynamics

As for the benevolent government case, define $r_\theta(b) = (1+\rho)b - b'_\theta(b)$ to be the revenue requirement implied by the equilibrium policies in state θ with initial debt level b. Letting $(\tau^e_\theta(r), g^e_\theta(r))$ denote the static equilibrium policies described in Proposition 4, it is clear that $(\tau_\theta(b), g_\theta(b))$ will equal $(\tau^e_\theta(r_\theta(b)), g^e_\theta(r_\theta(b)))$. As in the previous section, therefore, the key issue is to identify which of the cases described in Proposition 4 will arise in the long run. This requires understanding the long run behavior of debt.

Given the equilibrium policy functions, for any initial debt level b, we can define $H(b, b')$ to be the probability that next period's debt level will be less than b'. Given a distribution $\psi_{t-1}(b)$ of debt at time $t-1$, the distribution at time t is $\psi_t(b') = \int_b H(b, b') d\psi_{t-1}(b)$. A distribution $\psi^*(b')$ is said to be an *invariant distribution* if $\psi^*(b') = \int_b H(b, b') d\psi^*(b)$. If it exists, the invariant distribution describes the steady state of the government's debt distribution. We now have:

Proposition 5 *With political decision-making, there exists a debt level $b^q \in (r^o_H/\rho, \overline{b})$ such that the equilibrium debt distribution converges to a unique, non-degenerate, invariant distribution with*

just a constant.

full support on $[b^q, \overline{b})$. The dynamic pattern of debt is counter-cyclical: the government expands debt when private sector costs are high and contracts debt when costs are low until it reaches the floor level b^q.

The logic underlying the counter-cyclical behavior of debt is straightforward. As explained earlier, in this economy, debt allows government to transfer revenues from good times to bad times which permits the smoothing of distortions. With a benevolent government, debt does not play this role in the long run because the government accumulates sufficient assets to completely eliminate distortions. With political decision-making, the floor debt level b^q described in the proposition limits government asset accumulation. Intuitively, once the debt level has reached b^q, the mwc prefers to divert surplus revenues in good times to transfers rather than to paying down more debt. As a result, the need to smooth distortions remains in the long run and debt exhibits a counter-cyclical pattern. This finding is analogous to the results of Battaglini and Coate (2008) and Barshegyan, Battaglini, and Coate (2010) for the tax smoothing model. The debt level b^q depends on the fundamentals of the economy and can be characterized following the approach in Battaglini and Coate (2008), but these details are not central to our mission here and so we relegate them to the Appendix. For now, we will simply assume that b^q is positive, which seems the empirically relevant case.

Higher debt levels translate into higher revenue requirements for the government. Thus, Proposition 5 implies that the range of revenue requirements arising in equilibrium in state θ are $[r_\theta(b^q), r_\theta(\overline{b})]$. By comparing these ranges with the thresholds in Proposition 4, we obtain the following result.

Proposition 6 *With political decision-making, the following is true in the long run.*

- *If $q > q_L^*$, there is always unemployment in both states. Unemployment is weakly increasing in the economy's debt level, strictly so in the high cost state and in the low cost state for sufficiently high debt levels.*

- *If $q \in (q_H^*, q_L^*)$, there is always unemployment in the high cost state. In the low cost state, there is full employment with a distorted output mix for low debt levels and unemployment for high debt levels. Unemployment is weakly increasing in the economy's debt level, strictly so in the high cost state and in the low cost state for sufficiently high debt levels.*

- *If $q < q_H^*$, in the high cost state, there is full employment with a distorted output mix for low debt levels and unemployment for high debt levels. In the low cost state, there is full employment with a distorted output mix for low debt levels and either full employment with a distorted output mix or unemployment for high debt levels. Unemployment is weakly increasing in the economy's debt level, strictly so in the high cost state for sufficiently high debt levels.*

To understand the result, recall from Proposition 4 that when q is less than q_θ^*, there will be full employment with a distorted output mix if the revenue requirement is less than r_θ^* and unemployment if r exceeds r_θ^*. This unemployment will be increasing in the revenue requirement. When q exceeds q_θ^*, there will always be unemployment. This unemployment will be constant in the revenue requirement when r is less than r_θ^q and increasing when r exceeds r_θ^q. Now note that, for the high cost state, it can be shown that $r_H(b^q)$ exceeds r_H^q and that $r_H(\bar{b})$ exceeds r_H^*. The latter implies that when q is less than q_H^*, there will be unemployment in the high cost state for sufficiently large debt levels. The former implies that when q exceeds q_H^*, unemployment will be increasing in r. For the low cost state, it can be shown that $r_L(b^q)$ is less than r_L^q. Depending on the parameters, $r_L(\bar{b})$ may or may not exceed r_L^*. These facts imply that when q is less than q_L^*, there will be full employment in the low cost state for low debt levels and there may or may not be unemployment for sufficiently large debt levels. When q exceeds q_L^*, unemployment in the low cost state will be constant in r for low debt levels and increasing for sufficiently high debt levels. Pulling all this information together, yields Proposition 6.

To illustrate the workings of the model, consider the case in which q is between q_L^* and q_H^*. In this case, there is always unemployment in high cost states but full employment in low cost states for suitably low debt levels. The government mitigates unemployment in high cost states by issuing debt. If the economy's debt level is low enough, then a return to the low cost state will be sufficient to restore full employment. If the economy is in the high cost state for a sufficiently long period of time, however, debt will get sufficiently high that unemployment will persist even when the low cost state returns. When the low cost state returns, the legislators reduce debt. If the low cost state persists, debt will fall below the level at which full employment is achieved. Debt will eventually fall to the floor level b^q, at which point the mwc will divert surplus revenues to transfers rather than debt reduction.

4.3 The equilibrium policy mix

Having established the basic patterns of fiscal policy and unemployment arising in equilibrium, we now offer some observations on the policy mix the government chooses. We first discuss the case in which there is full employment and then turn to unemployment.

Full employment Proposition 6 tells us that when there is full employment, the output mix will always be distorted. This means that either the public sector is too large or too small. The direction of the distortion turns out to depend on the underlying parameters of the economy in a relatively simple way. Recall that full employment arises in state θ when q is less than q_θ^* and the revenue requirement is less than r_θ^*. There are two possibilities: the revenue requirement can be above or below r_θ^q. In the first possibility, there are no surplus revenues; in the second, the mwc is making transfers to their districts. The second possibility can only arise in the low cost state since equilibrium revenue requirements in the high cost state are larger than r_H^q. We discuss each possibility in turn.

No suplus revenues When the revenue requirement exceeds r_θ^q, we know from Propositions 1 and 4 that the output mix is distorted towards public production when τ_θ^o is less than $\tau_\theta^-(r)$ and towards the private good otherwise. In the Appendix, we show that τ_θ^o is less than $\tau_\theta^-(r)$ if and only if

$$n_e < \frac{1 - 2\gamma/A_\theta}{1 + A_\theta/2\xi}. \tag{26}$$

This condition is more likely to hold the smaller is the number of entrepreneurs n_e and the larger is the economy's preference for public goods γ. To gain intuition, recall that with respect to the first best policies, the policies in this range of revenue requirements maintain full employment but are biased in the direction of raising revenue. The government is therefore seeking changes in tax rates and public production that keep employment constant but generate more revenue. Keeping employment constant requires that if taxes are raised, any private sector workers laid off are employed in the public sector. Conversely, if public production is reduced, entrepreneurs must be incentivized to hire the displaced public sector workers. Clearly, if entrepreneurs can be induced to hire more workers for only a very small tax cut, then it makes sense to reduce public production. The savings from reducing public production will exceed the loss in tax revenues. The employment response for any given tax cut will be greater, the higher are the first best taxes. Accordingly, when first best taxes are high, reducing public production will be the optimal way to distort the output mix. First best taxes will be high when the first best public good level is high (high γ) and when the size of the private sector is large (high n_e).

Transfers When the revenue requirement is less than r_L^q, the mwc chooses the policies (τ_L^q, g_L^q) and uses the surplus revenues $r_L^q - r$ to finance transfers. The equilibrium policies correspond to the point of tangency between the indifference curve and the resource constraint as illustrated in Fig. 3.A. In this case, it again turns out that the output mix is biased towards public production (i.e., $g_\theta^q > g_\theta^o$) when condition (26) is satisfied and towards the private good otherwise. To understand this, note that relative to a benevolent government, the mwc is putting more weight on raising revenue for transfers but is still choosing to preserve full employment. It is therefore choosing tax rates and public production that keep employment constant but generate more revenue. The logic discussed above therefore applies.[28]

Unemployment When there is unemployment, condition (26) also turns out to play a role in determining how the equilibrium policies compare with those that minimize unemployment. The unemployment minimizing policies when the revenue requirement is r involve the tax rate τ_θ^* at which the slope of the budget line is equal to the slope of the full employment line with associated public good level $g_\theta^*(r)$ given by (15) (see Fig. 1.D).[29] In general, the equilibrium policies will not minimize unemployment. They could involve a lower or higher tax rate depending upon the parameters of the economy and the size of the revenue requirement. When they involve a lower tax rate, increasing the size of government would create jobs but legislators hold back because the lost private output is more valuable than the additional public output. When they involve a higher tax rate, reducing the size of government would create jobs but legislators hold back for the opposite reason. If condition (26) is not satisfied, the equilibrium tax rate is greater than τ_θ^*.[30] If condition (26) is satisfied, matters depend on the revenue requirement. For sufficiently high revenue requirements, the equilibrium tax rate in the high cost state must again be greater

[28] Indeed, it is the case that $(\tau_\theta^q, g_\theta^q)$ equals $(\tau_\theta^-(r_\theta^q), g_\theta^-(r_\theta^q))$ if τ_θ^o is less than $\tau_\theta^-(r_\theta^q)$ and $(\tau_\theta^+(r_\theta^q), g_\theta^+(r_\theta^q))$ otherwise.

[29] In the Appendix, we show that $\tau_\theta^* = (A_\theta - 2\underline{\omega})/2(A_\theta - \underline{\omega})$. This discussion assumes that $g_\theta^*(r) = \frac{R_\theta(\tau_\theta^*, \underline{w})}{\underline{w}} - \frac{r}{\underline{w}}$ is non-negative. If this is not the case, the unemployment minimizing tax rate is such that $R_\theta(\tau, \underline{w}) = r$ and the associated public good level is 0.

[30] To see this, observe that since there is unemployment, the revenue requirement r must exceed r_θ^* and hence $\widehat{\tau}_\theta(r)$ must be greater than $\widehat{\tau}_\theta(r_\theta^*)$. But if condition (26) is not satisfied, then $\widehat{\tau}_\theta(r_\theta^*)$ must equal $\tau_\theta^+(r_\theta^*)$ which is greater than τ_θ^*.

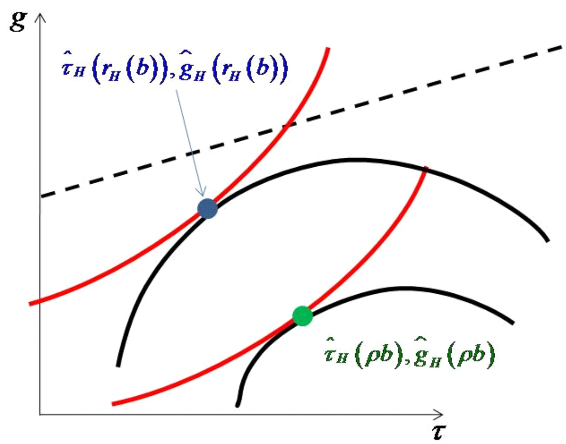

Figure 4:

than τ_H^*. This is because as r_H approaches $r_H(\bar{b})$, the equilibrium tax rate approaches the revenue maximizing level 1/2, which exceeds τ_H^*. However, in the low cost state or in the high cost state for sufficiently low revenue requirements, the equilibrium tax rate can be less than τ_θ^*.

4.4 Equilibrium stimulus plans

In the steady state of the political equilibrium, when private sector costs are high, the government expands debt and the funds are used to mitigate unemployment.[31] The government therefore employs fiscal stimulus plans, as conventionally defined. By studying the size of these stimulus plans and the changes in policy they finance, we can obtain a positive theory of fiscal stimulus. More specifically, in the high cost state, we can interpret $\rho b - r_H(b)$ as the magnitude of the stimulus, since this measures the amount of additional resources obtained by the government to finance fiscal policy changes (i.e., the debt increase $b'_H(b) - b$). An understanding of how the stimulus funds are used can be obtained by comparing the equilibrium tax and public good policies with the policies that would be optimal if the debt level were held constant.

The simplest case to consider is when the stimulus package does not completely eliminate unemployment. From Proposition 6, this must be the case when q exceeds q_H^*. Moreover, even when this is not the case, unemployment will remain post-stimulus whenever the economy's debt

[31] Even when q is less than q_H^* and the economy's debt is low, Assumption 1 implies that there will be unemployment prior to government stimulus if the floor debt level b^q is positive.

level is sufficiently high (i.e., $r_H(b) > r_H^*$). Drawing on the analysis in Section 3, Figure 4 illustrates what happens in this case. From Proposition 4, the policies that would be chosen if the debt level were held constant are $(\widehat{\tau}_H(\rho b), \widehat{g}_H(\rho b))$. The reduction in the revenue requirement made possible by the stimulus funds, shifts the budget line up and permits a new policy choice $(\widehat{\tau}_H(r_H(b)), \widehat{g}_H(r_H(b)))$. As discussed in Section 3, in the unemployment range, the tax rate is increasing in the revenue requirement and public production is decreasing. Thus, we know that $\widehat{\tau}_H(r_H(b))$ is less than $\widehat{\tau}_H(\rho b)$ and that $\widehat{g}_H(r_H(b))$ exceeds $\widehat{g}_H(\rho b)$, implying that stimulus funds will be used for both tax cuts and increases in public production.[32]

In terms of the effectiveness of equilibrium stimulus plans, we know from the previous sub-section that if $\widehat{\tau}_H(r_H(b))$ is less than τ_H^* (the tax rate at which the slope of the budget line equals that of the resource constraint) then reducing the tax cut slightly and using the revenues to finance a slightly larger public production increase will produce a bigger reduction in unemployment. Conversely, if $\widehat{\tau}_H(r_H(b))$ exceeds τ_H^* then reducing the public production increase and using the revenues to finance a slightly larger tax cut will produce a bigger reduction in unemployment. As discussed in the previous sub-section, both situations are possible depending on the parameters and the economy's debt level.

One way of thinking about these results concerning the comparison of $\widehat{\tau}_H(r_H(b))$ and τ_H^* is in terms of multipliers. It is commonplace in the empirical literature to try to evaluate the multipliers associated with different stimulus measures.[33] The multiplier associated with a particular stimulus measure is defined to be the change in GDP divided by the budgetary cost of the measure. In our model, measuring GDP is more problematic than in the typical macroeconomic model because output is produced by both the private and public sectors, and there is no obvious way to value public sector output. Perhaps the simplest approach is to define GDP as equalling

[32] It should be stressed that the purpose of the tax cuts is to incentivize the private sector to hire more workers. This is logically distinct from the idea that tax cuts return purchasing power to citizens and stimulate demand, thereby creating jobs. Both types of arguments for tax cuts arise in the policy debate and it is important to keep them distinct. Similarly, the purpose of the increase in government spending is to hire more public sector workers, not to increase transfers to citizens. Notice that while the model allows the government to use stimulus funds to increase transfers, it chooses not to do so. Such transfers would have no aggregate stimulative effect because they must be paid for by future taxation. Taylor (2011) argues that the 2009 American Recovery and Reinvestment Act largely consisted of increases in transfers. Moreover, he argues that these transfer increases had little impact on household consumption since they were saved.

[33] Papers trying to measure the multiplier impacts of different policies include Alesina and Ardagna (2010), Barro and Redlick (2011), Blanchard and Perotti (2002), Mountford and Uhlig (2009), Ramey (2011a), Romer and Romer (2010), Serrato and Wingender (2011), and Shoag (2010). A central issue in this literature is the relative size of tax cut and public spending multipliers. For overviews and discussion of the literature see Auerbach, Gale, and Harris (2010), Parker (2011), and Ramey (2011b).

private sector output plus the cost of public production. With this definition, when there is unemployment, the public production multiplier is 1 and the tax cut multiplier is approximately $A_H / (1 - 2\widehat{\tau}_H(r_H(b)))(A_H - \underline{\omega})$.[34] The tax cut multiplier will exceed the public production multiplier if $\widehat{\tau}_H(r_H(b))$ exceeds τ_H^* and be less than the public production multiplier if $\widehat{\tau}_H(r_H(b))$ is less than τ_H^*. The analysis illustrates why we should not expect the government to choose policies in such a way as to equate multipliers across instruments.[35] Tax cuts and public production increases have different implications for the mix of public and private outputs. A further important point to note is that the tax multiplier is highly non-linear.[36] Tax cuts will be more effective the larger is the tax rate and the tax rate will be higher the larger the economy's initial debt level.

When the stimulus package eliminates unemployment, as would be the case when q exceeds q_H^* and the economy's debt level is low ($r_H(b) < r_H^*$), matters are more complicated. This is because of the non-monotonic behavior of policies in the second case identified in Proposition 1. In particular, we will not necessarily get both tax cuts and an increase in public production. Fig. 5.A illustrates a case in which the stimulus package involves not only using all the stimulus funds to fund tax cuts but also reducing public production to supplement the stimulus funds. Fig. 5.B illustrates a case in which the stimulus package involves increases in both taxes and public production. The model is therefore consistent with a variety of possible stimulus plans.

It is interesting to understand how the magnitude of the stimulus as measured by $\rho b - r_H(b)$ depends on the initial debt level b. Note first that as b approaches its maximum level $\overline{b}$, the size of the stimulus must converge to zero. Interpreting the distance $\overline{b} - b$ as the economy's *fiscal space*, this result is simply saying that when the economy's fiscal space becomes very small (as a result, say, of a sequence of negative shocks or weak political institutions), its efforts to fight further negative shocks with fiscal policy will necessarily be limited.[37] It is tempting to conclude more generally, that the size of the stimulus as measured by $\rho b - r_H(b)$ should depend negatively on the initial debt level b. While we conjecture that this will typically be the case, it is not something

[34] This definition of GDP becomes more problematic when there is full employment and the minimum wage constraint is not binding. This is because τ and g impact the wage and hence the costs of public production. Perhaps the key point to note concerning multipliers when the minimum wage constraint is not binding, is that the level of employment is independent of τ and g.

[35] This point is also made by Mankiw and Weinzierl (2011).

[36] The importance of non-linearities and the difficulties this creates for measurement is a theme of Parker (2011).

[37] For more on the concept of fiscal space and an attempt to measure it see Ostroy, Ghosh, Kim and Qureshi (2010).

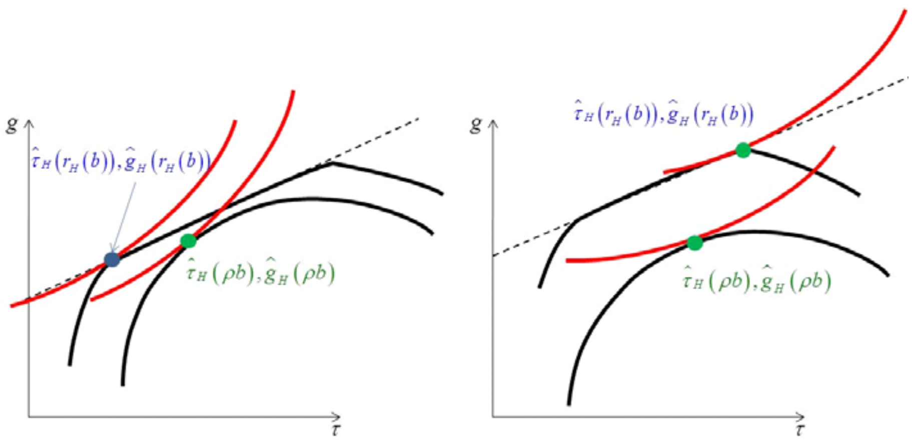

Fig. 5.A **Fig. 5.B**

Figure 5:

that we have been able to show analytically. It should also be noted that even if this were the case, the effectiveness of stimulus plans will not necessarily be decreasing in the economy's fiscal space. This is because as the economy's fiscal space contracts, taxes on the private sector increase to finance debt repayment. When taxes are high, the tax multiplier is high, meaning that small tax cuts can create large gains in employment.

4.5 Empirical implications

The model has two unambiguous qualitative implications. The first is that debt and unemployment levels should be positively correlated. This follows from Proposition 6. Since we are not aware of any other theoretical work that links debt and unemployment, we believe this is a novel prediction. While we not aware of any empirical work that looks at this issue, it is certainly something that could be tested.

The second implication is that the dynamic pattern of debt is counter-cyclical. More precisely, increases in debt should be positively correlated with reductions in output and visa versa. This follows from Proposition 5. This implication also emerges from tax smoothing models and simple Keynesian theories of fiscal policy, so there is nothing particularly distinctive about it. Empirical support for this prediction for the U.S. comes from the work of Barro (1986).

The model has no robust implications for the cyclical behavior of taxes and public spending.[38]

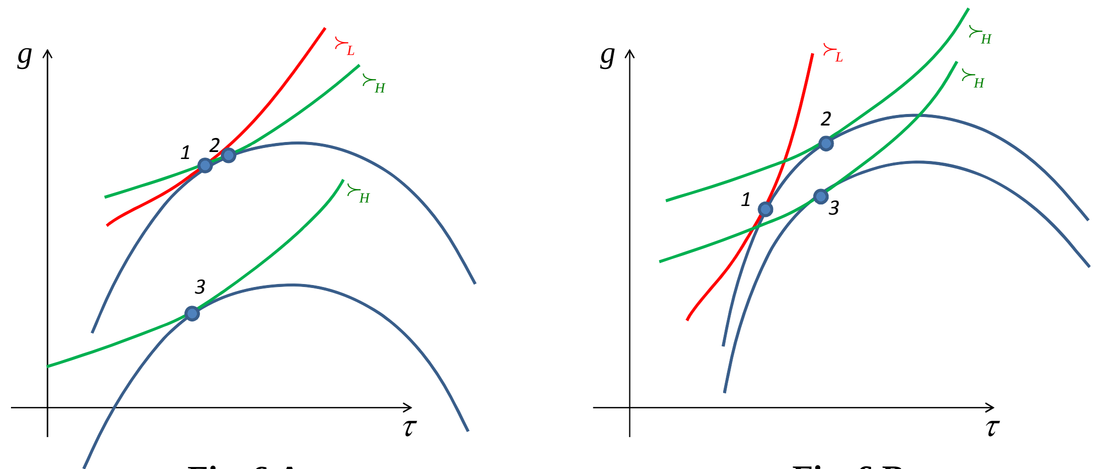

Fig. 6.A **Fig. 6.B**

Figure 6:

Depending on the parameters, when the economy experiences a high cost shock, public spending could increase or decrease, and tax rates could increase or decrease. To illustrate, consider a situation in which there is unemployment pre and post-shock. Two effects are at work. First, when A_θ decreases the mwc's indifference curve becomes flatter, so if the budget line did not change, τ and g would increase (this is represented by the move from point 1 to point 2 in Fig. 6). Intuitively, the marginal cost of raising taxes is lower because the private sector is less productive and therefore taxation results in a lower output response. It therefore becomes optimal to increase the size of the public sector. The reduction in private sector productivity, however, does impact the budget line. Specifically, it both shifts downward and becomes flatter. Intuitively, any given tax raises less revenue and any given increase in taxes results in a smaller revenue increase. Although the downward shift is partially compensated by an increase in debt, the combination of the downward shift and the flattening makes the net effect on taxes and public spending ambiguous. This is illustrated in Fig. 6. When the economy experiences a high cost shock, taxes and public spending move from point 1 to point 3. In Fig. 6.A, public spending and taxes decrease and in Fig. 6.B, public spending and taxes increase. Matters are only made more

[38] There is an extensive literature on the cyclical behavior of public spending and taxes. See, for example, Alesina, Campante, and Tabellini (2008), Barro (1986), Barshegyan, Battaglini, and Coate (2010), Furceri and Karras (2011), Gavin and Perotti (1997), Lane (2003), and Talvi and Vegh (2005). In light of the variety of empirical correlations found in the literature, the fact that the model predicts no clear pattern of behavior is perhaps a virtue.

complicated if there is full employment pre-shock.

In terms of political variables, the logic of the model certainly suggests that weaker political institutions (as proxied by higher q) should be positively correlated with high average levels of debt and unemployment. However, actually proving this comparative static result is difficult to do analytically. Exploring this therefore requires computing a calibrated version of the equilibrium of the model, which is a task we leave for future work.

5 Conclusion

This paper has developed a simple dynamic model in which to explore the interaction between fiscal policy and unemployment. Two distinct scenarios have been considered, one in which policies are chosen by a benevolent government and the other with political decision-making. The benevolent government solution provides an interesting normative benchmark, but has clearly counter-factual predictions. The equilibrium with political decision-making offers a more appealing account of the behavior of fiscal policy and unemployment.

With political decision-making, unemployment will arise when the private sector experiences negative shocks. To mitigate this unemployment, the government will employ debt-financed fiscal stimulus plans, which will generally involve both tax cuts and public production increases. In normal times, the government will contract debt until it reaches a floor level. Depending on the extent of political frictions, unemployment can arise even in normal times. At all times, unemployment levels are increasing in the economy's debt level. When there is full employment, the mix of public and private output is distorted. The direction of distortion in terms of whether the size of government is too large or too small, depends upon the underlying parameters of the economy.

There are many different directions in which the ideas presented here might usefully be developed. In terms of the basic model, it would be desirable to incorporate a richer model of unemployment into the analysis. The search theoretic approach of Michaillat (2011) would seem promising in this regard since it allows for both rationing unemployment (as in this paper) and frictional unemployment. This would permit us to move beyond the sharp distinction between full employment and unemployment, which would be helpful for developing empirical predictions. With respect to political decision-making, it would be interesting to introduce class conflict into the analysis. The current model limits the conflict among citizens to disagreements concerning the allocation of transfers between districts. This is made possible by assuming that each legislator behaves so as to maximize the aggregate utility of the citizens in his district. Alternatively, we could assume that legislators either represent workers or entrepreneurs in their districts. This would introduce an additional conflict over policies in the sense that workers prefer policies that keep wages and employment high, while entrepreneurs prefer policies which keep profits high. Such class conflict may have important implications for the choice of fiscal policy. Finally, on the empirical front, it would be interesting to explore the model's predictions concerning the relationship between unemployment and debt.

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6 Appendix

6.1 Proof of Proposition 1

The Lagrangian for Problem (12) is

$$L = x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma\ln g - r + \lambda\left(R_\theta(\tau,\underline{\omega}) - \underline{\omega}g - r\right) + \mu\left(n_w - g - \frac{x_\theta(\tau)}{A}\right).$$

Thus, λ is the multiplier on the budget constraint and μ is the multiplier on the resource constraint. Using the expressions for $x_\theta(\tau)$ and $R_\theta(\tau,\underline{\omega})$ in (8) and (9), the first order conditions with respect to g and τ are

$$\frac{\gamma}{g} = \lambda\underline{\omega} + \mu, \tag{27}$$

and

$$\lambda(1-2\tau)(A_\theta - \underline{\omega}) = \tau A_\theta + (1-\tau)\underline{\omega} - \mu. \tag{28}$$

There are three cases to consider.

Case 1. Assume first that the budget constraint is not binding, so that $\lambda = 0$. In this case (27) implies that $\mu = \frac{\gamma}{g} > 0$ and hence there is no unemployment. Substituting this expression for μ into (28), we obtain

$$\tau A_\theta + (1-\tau)\underline{\omega} = \frac{\gamma}{g}. \tag{29}$$

Combining this equation with the resource constraint and solving, we find that the solution is

$$g_\theta^o = \frac{\sqrt{(A_\theta n_e - \xi n_w)^2 + 4\xi n_e\gamma} - (A_\theta n_e - \xi n_w)}{2\xi}. \tag{30}$$

and

$$\tau_\theta^o = 1 - \frac{\xi\left(n_w - g_\theta^o\right)}{n_e(A_\theta - \underline{\omega})}. \tag{31}$$

Recalling that $r_\theta^o = R_\theta(\tau_\theta^o,\underline{\omega}) - \underline{\omega}g_\theta^o$, we need that $r \le r_\theta^o$ for the budget constraint not to be binding.

Case 2. Assume next that both the budget constraint and the resource constraints are binding, so that $\lambda > 0$ and $\mu > 0$. In this case, we have that

$$R_\theta(\tau,\underline{\omega}) - \underline{\omega}g = r, \tag{32}$$

and

$$g + \frac{x_\theta(\tau)}{A} = n_w. \tag{33}$$

Substituting (33) into (32), we obtain

$$R_\theta(\tau,\underline{\omega}) - \underline{\omega}\left(n_w - \frac{x_\theta(\tau)}{A}\right) = r. \tag{34}$$

Assuming that (34) has a solution, it will have two solutions $\tau_\theta^-(r)$ and $\tau_\theta^+(r)$, which correspond to the points illustrated in Fig. 1.C. The associated public good levels, $g_\theta^-(r)$ and $g_\theta^+(r)$, are then obtained from (33).

It remains to describe when (34) has a solution and also whether $\left(\tau_\theta^-(r), g_\theta^-(r)\right)$ or $\left(\tau_\theta^+(r), g_\theta^+(r)\right)$ provide a higher value of the objective function. In order for (34) to have a solution, the budget line must lie above the resource constraint for some range of taxes. Let τ_θ^* denote the tax rate at which the slope of the budget line is equal to the slope of the full employment line and let $g_\theta^*(r)$ denote the level of public good that satisfies the budget constraint (32) given this tax rate. Now define r_θ^{**} to be that revenue requirement at which the point $(\tau_\theta^*, g_\theta^*(r))$ is tangent to the full employment line. Then, (34) has a solution if and only if $r \leq r_\theta^{**}$. Moreover, $\tau_\theta^-(r_\theta^{**}) = \tau_\theta^+(r_\theta^{**}) = \tau_\theta^*$.

Turning to the issue of whether $\left(\tau_\theta^-(r), g_\theta^-(r)\right)$ or $\left(\tau_\theta^+(r), g_\theta^+(r)\right)$ provide a higher value of the objective function, assume that $r \in (r_\theta^o, r_\theta^{**})$. There are two possibilities: i) $\tau_\theta^o < \tau_\theta^-(r)$ in which case the first best policy $(\tau_\theta^o, g_\theta^o)$ lies to the left of $\left(\tau_\theta^-(r), g_\theta^-(r)\right)$, and ii) $\tau_\theta^o > \tau_\theta^+(r)$ in which case the first best policy $(\tau_\theta^o, g_\theta^o)$ lies to the right of $\left(\tau_\theta^+(r), g_\theta^+(r)\right)$. Since the objective function is concave, in case i) the optimal choice is $\left(\tau_\theta^-(r), g_\theta^-(r)\right)$ and in case ii), the optimal policy choice is $\left(\tau_\theta^+(r), g_\theta^+(r)\right)$. In case i), taxes and public goods are increasing in r and in case ii) taxes and public goods are decreasing in r. We conclude that a necessary condition to be in Case 2 is that $r \in (r_\theta^o, r_\theta^{**}]$ and that in Case 2 the solution is $\left(\tau_\theta^-(r), g_\theta^-(r)\right)$ if $\tau_\theta^o < \tau_\theta^-(r)$ and $\left(\tau_\theta^+(r), g_\theta^+(r)\right)$ if $\tau_\theta^o > \tau_\theta^+(r)$. We will return to provide a necessary and sufficient condition to be in Case 2 after analyzing Case 3.

Case 3. Finally, assume that only the budget constraint binds, so that $\lambda > 0$ and $\mu = 0$. Substituting (27) into (28), we obtain

$$g = \frac{\gamma(1-2\tau)(A_\theta - \underline{\omega})}{\underline{\omega}\left(\tau(A_\theta - \underline{\omega}) + \underline{\omega}\right)}. \tag{35}$$

Substituting (35) into the budget constraint (32), we obtain:

$$\tau n_e(1-\tau)(A_\theta - \underline{\omega})^2/\xi - \left(\frac{\gamma(1-2\tau)(A_\theta - \underline{\omega})}{\tau(A_\theta - \underline{\omega}) + \underline{\omega}}\right) = r \tag{36}$$

This equation has a unique solution $\widehat{\tau}_\theta(r)$ in the relevant range for τ, i.e. $[0, 1/2]$. Since the right hand side of (36) is always increasing for τ less than $1/2$, $\widehat{\tau}_\theta(r)$ is increasing in r. The associated value of g, $\widehat{g}_\theta(r)$, is obtained from (35). Since the right hand side of (35), is decreasing in τ, $\widehat{g}_\theta(r)$ is decreasing in r. Furthermore, note that unemployment is an increasing function of τ and a decreasing function of g, so it is increasing in r as well. Now define the revenue requirement r^*_θ to be such that the resource constraint is satisfied with equality at the policies $(\widehat{g}_\theta(r), \widehat{\tau}_\theta(r))$; that is, which satisfies

$$\widehat{g}_\theta(r^*_\theta) = n_w - n_e(1 - \widehat{\tau}_\theta(r^*_\theta))(A_\theta - \underline{\omega})/\xi. \tag{37}$$

It is clear that this revenue requirement exists, is unique, and satisfies $r^*_\theta \in (r^o_\theta, r^{**}_\theta]$. Moreover, if $r < r^*_\theta$, then it must be the case that the resource constraint binds while if $r > r^*_\theta$, then the resource constraint does not bind. Thus, as claimed in the Proposition, we are in Case 2 if $r \in (r^o_\theta, r^*_\theta]$ and Case 3 if $r > r^*_\theta$. ■

6.2 Proof of Proposition 2

Using the Envelope Theorem and the first order condition for b', it is straightforward to show that for each cost state θ and any initial debt level b, the optimal level of borrowing $b'_\theta(b)$ is such that:

$$1 + \lambda_\theta(b) - \psi_\theta(b) = 1 + E\left[\lambda_{\theta'}(b'_\theta(b))\right], \tag{38}$$

where $\lambda_\theta(b)$ and $\psi_\theta(b)$ are, respectively, the Lagrange multipliers of the budget constraint and of the upperbound on debt in (17). We now proceed in two steps.

Step 1. We first prove that for each cost state θ and any initial debt level b such that $b < \overline{b}$ there is an $\phi(\theta, b) > 0$ such that $b'_\theta(b) < \overline{b} - \phi(\theta, b)$. Consider the optimal policy $\{\tau_\theta(b), g_\theta(b), b'_\theta(b)\}$ with associated Lagrange multipliers $\lambda_\theta(b)$, $\psi_\theta(b)$, and $\mu_\theta(b)$, where $\mu_\theta(b)$ is the multiplier on the resource constraint in (17). As noted in the text, if $(\tau^s_\theta(r), g^s_\theta(r))$ denote the optimal static policies described in Proposition 1, it is clear that $(\tau_\theta(b), g_\theta(b))$ will equal $(\tau^s_\theta(r_\theta(b)), g^s_\theta(r_\theta(b)))$ where $r_\theta(b) = (1+\rho)b - b'_\theta(b)$ is the revenue requirement implied by the optimal borrowing level. Assume, by contradiction, that $b'_\theta(b)$ is arbitrarily close to $\overline{b}$; that is, $b'_\theta(b) = \overline{b} - \zeta$, where ζ is arbitrarily small. There are three cases to consider.

Case 1.1. Assume that $\lambda_\theta(b) > 0$ and $\mu_\theta(b) > 0$. Suppose first that (26) is satisfied. In this case, from Proposition 1, taxes and public goods are given by $(\tau^-_\theta(r(b,\zeta)), g^-_\theta(r(b,\zeta)))$, where $r(b,\zeta) = (1+\rho)b - \overline{b} + \zeta$. Combining the first order conditions (27) and (28), we have:

$$\lambda_\theta(b,\zeta) = \frac{\tau^-_\theta(r(b,\zeta)A_\theta + (1 - \tau^-_\theta(r(b,\zeta))\underline{\omega} - \frac{\gamma}{g^-_\theta(r(b,\zeta))}}{(1 - 2\tau^-_\theta(r(b,\zeta))(A_\theta - \underline{\omega}) - \underline{\omega}} \tag{39}$$

Consider now $\lambda_{\theta'}(\overline{b}-\zeta)$, for $\theta' \in \{H, L\}$. As $\zeta \to 0$, we must have $\tau_H(\overline{b}-\zeta) \to 1/2$ and $g_H(\overline{b}-\zeta) \to 0$. By Assumption 1, this implies that $\mu_H(\overline{b}-\zeta) = 0$ for ζ sufficiently small.[39] We therefore have from (28) that

$$\lambda_H(\overline{b}-\zeta) = \frac{\tau_H(\overline{b}-\zeta)A_H + (1-\tau_H(\overline{b}-\zeta))\underline{\omega}}{(1-2\tau_H(\overline{b}-\zeta))(A_H-\underline{\omega})}$$

for ζ small. It follows that as $\zeta \to 0$, $\lambda_H(\overline{b}-\zeta)$ becomes arbitrarily large. Since $\lambda_L(\overline{b}-\zeta) \geq 0$, this implies that $E\left[\lambda_{\theta'}(\overline{b}-\zeta)\right]$ becomes arbitrarily large. Since $\lim_{\zeta\to 0} r(b,\zeta) < \rho\overline{b}$, on the contrary, condition (39) shows that $\lambda_\theta(b,\zeta)$ is bounded as $\zeta \to 0$.[40] This generates a contradiction since by (38) we must have $\lambda_\theta(b) \geq E\left[\lambda_{\theta'}(b'_\theta(b))\right]$. The case in which (26) is not satisfied is completely analogous.

Case 1.2. Assume that $\lambda_\theta(b) > 0$ and $\mu_\theta(b) = 0$. In this case, from Proposition 1, taxes and public goods are given by $(\widehat{\tau}_\theta(r_\theta(b)), \widehat{g}_\theta(r_\theta(b)))$. We can write $g_\theta(b) = f(\tau_\theta(b))$ where $f(\tau)$ is a continuous function defined by the right hand side of equation (35). Since $\lambda_\theta(b) > 0$, we must also have:

$$B_\theta(\tau_\theta(b), f(\tau_\theta(b)), b'_\theta(b), b, \underline{\omega}) = 0. \tag{40}$$

It follows that if $b'_\theta(b) = \overline{b}-\zeta$, we can express all the policy choices as a function of ζ. Thus, setting $b'_\theta(b) = \overline{b}-\zeta$, we have $\tau_\theta(b) = \tau(\zeta)$ where $\tau(\zeta)$ solves (40) and $g_\theta(b) = f(\tau(\zeta)) = g(\zeta)$. Note that as $\zeta \to 0$, we have $\tau(\zeta) \to \widetilde{\tau} < 1/2$. For if $\tau(\zeta) \to 1/2$, then $g(\zeta) \to 0$ and (40) would not be satisfied since $b < \overline{b}$. Moreover, $\tau(\zeta) \to \widetilde{\tau}$ implies $g(\zeta) \to \widetilde{g} > 0$. From (28) and (38), we have that:

$$\lambda_\theta(b) = \frac{\tau_\theta(b)A_\theta + (1-\tau_\theta(b))\underline{\omega}}{(1-2\tau_\theta(b))(A_\theta-\underline{\omega})} \geq E\left[\lambda_{\theta'}(\overline{b}-\zeta)\right] \geq \alpha\left[\frac{\tau_H(\overline{b}-\zeta)A_H + (1-\tau_H(\overline{b}-\zeta))\underline{\omega}}{(1-2\tau_H(\overline{b}-\zeta))(A_\theta-\underline{\omega})}\right], \tag{41}$$

where the last equality follows from the fact that, as in Case 1.1, we must have $\tau_H(\overline{b}-\zeta) \to 1/2$ and $g_H(\overline{b}-\zeta) \to 0$ as $\zeta \to 0$: and this implies that $\mu_H(\overline{b}-\zeta) = 0$ for ζ sufficiently small. As in Case 1.1, the right hand side of (41) diverges to infinity, while the left hand side converges to a finite value: so we again have a contradiction.

Case 1.3. Assume $\lambda_\theta(b) = 0$. In this case (38) implies that $\psi_\theta(b) = 0$ and $\mu_\theta(b) > 0$ as well. This implies that taxes and public goods are given by $(\tau^o_\theta, g^o_\theta)$. The first order condition with respect

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[39] This follows from the fact that Assumption 1 implies $n_w > n_e(A_H - \underline{\omega})/2\xi$.

[40] If $\lambda_\theta(b,\zeta) \to \infty$ as $\zeta \to 0$, then τ would have to converge to $1/2$ and g to 0: but then neither the resource constraint, nor the budget constraint would be binding, a contradiction.

to b' is $1 = E\left[\lambda_{\theta'}(\overline{b} - \zeta)\right]$, which is impossible since by an argument similar to the argument of the previous two cases, the right hand side becomes arbitrarily large as $\zeta \to 0$.

Step 2. Step 1 implies that $b'_\theta(b) < \overline{b}$ and that $\psi_\theta(b) = 0$ for any $b < \overline{b}$. From (38) we therefore conclude that the Lagrange multiplier $\lambda_\theta(b)$ is a non-negative martingale. Defining the sequences $\langle \tau_t, g_t, b'_t \rangle$ as in the text, Corollary 2 of Shiryaev (1991) (p. 508) implies that $\Pr\left(\lim_{t\to\infty} \lambda_{\theta_t}(b'_{t-1}) \text{ exists}\right) = 1$. Define $\underline{b} = r^o_H/\rho$ to be the level of assets that in steady state generate interest earnings just sufficient to cover the discrepancy between $\underline{\omega} g^o_H$ and $R_H(\tau^o_H, \underline{\omega})$. When $b \leq \underline{b}$, by holding asset levels constant the government can achieve full employment and the optimal output mix in both the high and low cost state in the current and all future periods. So $\lambda_\theta(b) = 0$ for any θ if and only if $b \leq \underline{b}$. Assume by contradiction that $\Pr\left(\lim_{t\to\infty} r_t > r^o_H\right) > 0$. In this case, there must be an $\varepsilon > 0$, and a set of sequences $\langle r_t \rangle$ with positive probability such that, for each sequence in the set, we can find a subsequence $\langle r_{n(t)} \rangle$ with $r_{n(t)} > r^o_H + \varepsilon$ for any arbitrarily large $n(t)$. Note that $r_t > r^o_H + \varepsilon$ implies that $\lambda_H(b'_{t-1}) > 0$ and $\lambda_H(b'_{t-1}) > \lambda_L(b'_{t-1})$. Along these sequences, therefore, $\lambda_H(b'_{n(t)-1}) - \lambda_L(b'_{n(t)-1}) > \eta$ for some positive η: this contradicts the fact that $\lambda_{\theta_{n(t)}}(b'_{n(t)-1})$ converges with probability one. We conclude that $\Pr\left(\lim_{t\to\infty} r_t \leq r^o_H\right) = 1$. The result now follows from Proposition 1. ■

6.3 Proof of Proposition 3

We will demonstrate the existence of a pair of concave value functions $V_H(b)$ and $V_L(b)$ that satisfy (19) given the policy functions they generate via (18). Note first that in political equilibrium, the minimum winning coalition will always choose a higher debt level than $\underline{b} \equiv r^o_H/\rho$. If the debt level were below $\underline{b}$, the mwc could marginally increase debt, use the proceeds to finance transfers, and increase its current period payoff by q (see (18)). The cost of such an action would be going into the next period with a marginally higher debt level. But the only effect of this would be to reduce the transfers received by next period's mwc (see Propositions 4 and 5 below). Since members of the current mwc will not necessarily belong to next period's mwc, the discounted expected reduction in future transfer payoffs is $1 < q$. Thus, increasing debt is optimal.

Given this observation, we look for a pair of equilibrium value functions in the compact space of continuous and concave functions defined on the interval $\left[\underline{b}, \overline{b}\right]$. Let F be the metric space of real valued continuous and bounded, weakly concave functions of b in $\left[\underline{b}, \overline{b}\right]$ endowed with the sup norm, $\|f\| = \sup_{b\in\left[\underline{b},\overline{b}\right]} |f|$. Let F^2 be the Cartesian product of two such spaces endowed with the maximum norm, $\left\|f^2\right\| = \max\left\{\left\|f_1^2\right\|, \left\|f_2^2\right\|\right\}$, for any $f^2 \in F^2$.

As noted in the paragraph following the statement of Proposition 3, the policies in a political equilibrium solve the following problem:

$$\max_{(\tau,g,b')}\left\{\begin{array}{c} b' - (1+\rho)b + x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma\ln g + (q-1)\,B_\theta(\tau,g,b',b,\underline{\omega}) + \beta EV_{\theta'}(b') \\ s.t.\ B_\theta(\tau,g,b',b,\underline{\omega}) \geq 0,\ g + \frac{x_\theta(\tau)}{A} \leq n_w\ \&\ b \in \left[\underline{b},\overline{b}\right] \end{array}\right\}. \tag{42}$$

To compact notation let $p = (\tau, g, b')$ denote a generic policy and let $P = [\underline{\tau}, 1] \times [0, \overline{g}] \times \left[\underline{b}, \overline{b}\right]$ denote the feasible policy space, where $\overline{g}$ is some sufficiently large upper bound on public goods and $\underline{\tau}$ is some sufficiently low lower bound on taxes. Note that we can impose an upper bound on public good levels without loss of generality since in every period revenues are bounded by $\max_\tau R_L(\tau, \underline{\omega}) + \overline{b}$; similarly there is no loss of generality in assuming that the tax rate is bounded below since it would never be optimal nor feasible to provide unbounded subsidies to the entrepreneurs. Define a *policy function* to be a function $p_\theta(b)$ which associates a policy with any given initial debt level b and cost state θ. Let $P_\theta(b; V) \subseteq P$ be the set of optimal policies for (42) when the initial debt level is b, the state is θ, and the value functions are $V = (V_H, V_L)$. Let $P_\theta(V)$ be the set of optimal policy functions for the problem with value functions V; that is, $p_\theta(b) \in P_\theta(V)$ if and only if $p_\theta(b) \in P_\theta(b; V)$ for all $b \in \left[\underline{b}, \overline{b}\right]$.

Now, for any $V \in F^2$, define $P^*_\theta(V) \subset P_\theta(V)$ to be the subset of optimal policy functions which are (i) continuous in b, and, (ii) which generate a net of transfer budget surplus function $B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \underline{\omega})$ that is weakly convex in b. Observe that, when viewed as a function of V, $P^*_\theta(V)$ is a correspondence from F^2 into the set of policy functions.

Lemma A.1. *For each state θ, the correspondence $P^*_\theta(V)$ is non empty, compact, and convex valued.*

Proof. Available from the authors on request. ■

For a given $V \in F^2$ and cost state θ, let $\Upsilon_\theta(V)$ be defined by:

$$\Upsilon_\theta(V) = \left\{ \widetilde{V}(b) \;\middle|\; \begin{array}{c} \exists\ p(b) \in P^*_\theta(V)\ s.t.\ \widetilde{V}(b) = \left[\begin{array}{c} b'(b) - (1+\rho)b + x_\theta(\tau(b))\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau(b))}{An_e}\right)^2}{2} \\ +\gamma\ln g(b) + \beta EV_{\theta'}(b'(b)) \end{array}\right] \\ for\ (\tau(b), g(b), b'(b)) = p(b) \end{array} \right\}.$$

This expression defines a correspondence from F^2 into the set of real-valued functions defined on $[\underline{b}, \overline{b}]$:

Lemma A.2. *For each cost state θ, $\Upsilon_\theta(V)$ is a non empty, convex, and compact valued correspondence from F^2 into F with a closed graph.*

Proof. Available from the authors on request. ■

Define the correspondence from F^2 into F^2:

$$T(V) = \left\{ (\widetilde{V}_H(b), \widetilde{V}_L(b)) \middle| \widetilde{V}_\theta(b) \in \Upsilon_\theta(V) \quad \theta \in \{H, L\} \right\}.$$

We have:

Lemma A.3. *The correspondence $T(V)$ has a fixed point $V^* = T(V^*)$.*

Proof. It can be verified that Lemma A.2 implies that $T(V)$ is a non empty, compact, and convex-valued correspondence form F^2 to F^2 with a closed graph. The result therefore follows from the Glicksberg-Fan Theorem (see Theorem 9.2.2 in Smart (1974)). ■

Let $(V_H(b), V_L(b))$ be a fixed point of the correspondence $T(V)$. Then $V_H(b)$ and $V_L(b)$ are concave functions that by construction satisfy (19) given the policy functions they generate via (18). Proposition 3 is therefore established. ■

6.4 Proof of Proposition 4

Given the discussion in the text, it suffices to show that the solution to Problem (23) when the budget constraint is not binding, which is denoted $(\tau_\theta^q, g_\theta^q)$, is given by the tangency between the indifference curve and resource constraint when $q < q_\theta^*$ and by the satiation point when $q > q_\theta^*$. The Lagrangian for Problem (23) (ignoring the budget constraint) is

$$L = x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma \ln g + (q-1)\left(R_\theta(\tau, \underline{\omega}) - \underline{\omega} g - r\right) - qr + \mu\left(n_w - g - \frac{x_\theta(\tau)}{A}\right),$$

where μ is the multiplier on the resource constraint. Using the expressions for $x_\theta(\tau)$ and $R_\theta(\tau, \underline{\omega})$ in (8) and (9), the first order conditions with respect to g and τ respectively are

$$\frac{\gamma}{g} = (q-1)\underline{\omega} + \mu, \tag{43}$$

and

$$(q-1)(1-2\tau)(A_\theta - \underline{\omega}) = \tau A_\theta + (1-\tau)\underline{\omega} - \mu. \tag{44}$$

It is easy to show that if $\mu = 0$, these equations imply that the solution is given by

$$(\tau_\theta^q, g_\theta^q) = \left(\frac{(q-1)\,A_\theta - q\underline{\omega}}{(A_\theta - \underline{\omega})\,(2q-1)}, \frac{\gamma}{(q-1)\,\underline{\omega}} \right). \tag{45}$$

It follows that the resource constraint is not binding if at these values of $(\tau_\theta^q, g_\theta^q)$, $g_\theta^q + \frac{x_\theta(\tau_\theta^q)}{A} \leq n_w$. This implies:

$$n_e \left[\frac{q(A_\theta - \underline{w}) + \underline{w}}{\xi(2q-1)} \right] + \frac{\gamma}{(q-1)\,\underline{w}} \leq n_w. \tag{46}$$

Note that the left hand side of (46) is decreasing in q, so it is satisfied if and only if $q > q_\theta^*$, where q_θ^* is defined by (24).

It is straightforward to solve for $(\tau_\theta^q, g_\theta^q)$ when the resource constraint is binding. In this case, (43) implies that $\mu = \frac{\gamma}{g} - (q-1)\underline{\omega} > 0$. Substituting this expression for μ into (44), we obtain

$$\tau A_\theta + (1-\tau)\underline{\omega} = \frac{\gamma}{g} + (q-1)\left[(1-2\tau)(A_\theta - \underline{\omega}) - \underline{\omega}\right]. \tag{47}$$

Combining this equation with the resource constraint and solving we find that the solution is

$$(\tau_\theta^q, g_\theta^q) = \left(1 - \frac{\xi\,(n_w - g_\theta^q)}{n_e(A_\theta - \underline{\omega})}, \frac{\sqrt{(qA_\theta n_e - (2q-1)\xi n_w)^2 + (2q-1)4\xi n_e \gamma} - (qA_\theta n_e - (2q-1)\xi n_w)}{(2q-1)2\xi} \right). \tag{48}$$

■

6.5 Proof of Proposition 5

The proof is broken into three parts. In Section 7.5.1 we characterize b^q - the lower bound of the equilibrium debt distribution. In Section 7.5.2 we prove that debt behaves in a counter-cyclical way. In Section 7.5.3 we prove that a non-degenerate stable distribution exists and has full support in $\left[b^q, \overline{b}\right)$.

6.5.1 The lowerbound b^q

Consider problem (22). When the budget constraint is not binding, the mwc will choose a debt level from the set[41]

$$\mathcal{X}(V) = \arg \max_{b' \in [\underline{b}, \overline{b}]} \left\{ qb' + \beta E V_{\theta'}(b') \right\}.$$

We will show that $\mathcal{X}(V)$ consists of just a single point. We first need:

[41] As explained in the proof of Proposition 3, there is no loss of generality in assuming that $b' \geq \underline{b}$.

Lemma A.4. $r_L^q > r_H^q$.

Proof. Available from the authors on request. ■

We can now show that:

Lemma A.5. *In any equilibrium, the set $\mathcal{X}(V)$ is a singleton.*

Proof. Available from the authors on request. ■

Given Lemma A.5, we define b^q to be the unique element of the set $\mathcal{X}(V)$. We also define b_θ^q to be the value of debt such that the triple $(\tau_\theta^q, g_\theta^q, b^q)$ satisfies the constraint that $B_\theta(\tau_\theta^q, g_\theta^q, b^q, b_\theta^q, \underline{w})$ equal 0. This is given by:

$$b_\theta^q = \frac{r_\theta^q + b^q}{1+\rho}. \tag{49}$$

Then, if the debt level b is such that $b \leq b_\theta^q$ the tax-public good-debt triple is $(\tau_\theta^q, g_\theta^q, b^q)$ and the mwc uses the budget surplus $B_\theta(\tau_\theta^q, g_\theta^q, b^q, b_\theta^q, \underline{\omega})$ to finance transfers. If $b > b_\theta^q$ the budget constraint binds so that no transfers are given. Tax revenues net of public good costs strictly exceed r_θ^q and the debt level strictly exceeds b^q. In this case, the policies solve the problem

$$\max_{(\tau,g,b')} \left\{ \begin{array}{c} b' - (1+\rho)b + x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma \ln g + \beta E V_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0,\ g + \frac{x_\theta(\tau)}{A} \leq n_w \ \&\ b \in [\underline{b}, \overline{b}] \end{array} \right\} \tag{50}$$

Note also that by Lemma A.4., it must be the case that $b_L^q > b_H^q$.

Further information on the debt level b^q can be obtained by using a first order condition to characterize it. However, before we can do this, we must first establish that the value function is differentiable. We have:

Lemma A.6. *(i) If $q > q_\theta^*$, the equilibrium value function $V_\theta(b)$ is differentiable for all $b \neq b_\theta^q$. Moreover,*

$$-\beta V_\theta'(b) = \begin{cases} 1 & if\ b < b_\theta^q \\ 1 + \frac{\tau_\theta(b)A_\theta + (1-\tau_\theta(b))\underline{\omega}}{(1-2\tau_\theta(b))(A_\theta - \underline{\omega})} & if\ b > b_\theta^q \end{cases}.$$

(ii) If $q < q_\theta^*$, there exists a unique debt level $b_\theta^{**} \in (b_\theta^q, \overline{b}]$ such that the resource constraint is binding if and only if $b \leq b_\theta^{**}$. The equilibrium value function $V_\theta(b)$ is differentiable for all $b \neq b_\theta^q$

and

$$-\beta V_\theta'(b) = \left\{ \begin{array}{cc} 1 & if\ b < b_\theta^q \\ 1 + \frac{\tau_\theta(b)A_\theta + (1-\tau_\theta(b))\underline{\omega} - \frac{\gamma}{g_\theta(b)}}{(1-2\tau_\theta(b))(A_\theta - \underline{\omega}) - \underline{\omega}} & b \in (b_\theta^q, b_\theta^{**}) \\ 1 + \frac{\tau_\theta(b)A_\theta + (1-\tau_\theta(b))\underline{\omega}}{(1-2\tau_\theta(b))(A_\theta - \underline{\omega})} & b \geq b_\theta^{**} \end{array} \right\}.$$

Proof. (i) Suppose first that $q > q_\theta^*$. From the discussion presented above, we know that if the debt level b is such that $b \leq b_\theta^q$ the optimal policies are $(\tau_\theta^q, g_\theta^q, b^q)$, and, if $b > b_\theta^q$, the budget constraint $B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0$ will be binding and the policies solve (50). Moreover, we know from Proposition 5 that the resource constraint will not be binding. Consider some debt level b_o. Assume first that $b_o < b_\theta^q$. Then, we know from (19) that in a neighborhood of b_o it must be the case that

$$V_\theta(b) = b^q - (1+\rho)b + x_\theta(\tau_\theta^q)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau_\theta^q)}{An_e}\right)^2}{2} + \gamma \ln g_\theta^q + \beta E V_\theta(b^q).$$

Thus, it is immediate that the value function $V_\theta(b)$ is differentiable at b_o and that

$$V_\theta'(b_o) = -(1+\rho) = -1/\beta.$$

Assume now that $b_o > b_\theta^q$. Consider the function

$$\varphi_\theta(b) = \max_{(\tau,g)} \left\{ \begin{array}{c} b_\theta'(b_o) - (1+\rho)b + x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma \ln g + \beta E V_{\theta'}(b_\theta'(b_o)) \\ s.t.\ B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) = 0. \end{array} \right\} \quad (51)$$

Since the equilibrium policies are such that the budget constraint binds and the resource constraint does not bind, it follows that $V_\theta(b_o) = \varphi_\theta^o(b_o)$ and $V_\theta(b) \geq \varphi_\theta^o(b)$ for all b in a neighborhood of b_o. By the Envelope Theorem, the function $\varphi_\theta(b)$ is differentiable in b and its derivative is equal to $-(1+\rho)[1+\lambda_\theta^o(b)]$, where $\lambda_\theta^o(b)$ is the Lagrange multiplier on the constraint $B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) = 0$ in (51) at b. It is also the case that $\varphi_\theta^o(b)$ is concave. To see this note that we may write:

$$\varphi_\theta(b) = \max_{(\tau,g)} \left\{ \begin{array}{c} b_\theta'(b_o) - (1+\rho)b + x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma \ln g + \beta E V_{\theta'}(b_\theta'(b_o)) \\ s.t.\ B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) \geq 0. \end{array} \right\} \quad (52)$$

The objective function in (52) is concave in τ, g and b, and the constraint set is convex in τ, g and b: so by a standard argument $\varphi_\theta(b)$ is concave. It now follows from Theorem 4.10 of

Stokey, Lucas and Prescott (1989) that $V_\theta(b)$ is differentiable at b_o with derivative $V'_\theta(b_o) = \varphi'_\theta(b_o) = -(1+\rho)[1+\lambda^o_\theta(b_o)] = -[1+\lambda^o_\theta(b_o)]/\beta$. To complete the proof, consider the first order conditions for (51) at b_o:

$$\frac{\gamma}{g^o_\theta(b_o)} = \lambda^o_\theta(b_o)\underline{\omega} \tag{53}$$

and

$$\lambda^o_\theta(b_o)(1-2\tau^o_\theta(b_o))(A_\theta - \underline{\omega}) = \tau^o_\theta(b_o)A_\theta + (1-\tau^o_\theta(b_o))\underline{\omega}. \tag{54}$$

The second condition implies

$$\lambda^o_\theta(b_o) = \frac{\tau_\theta(b_o)A_\theta + (1-\tau_\theta(b_o))\underline{\omega}}{(1-2\tau_\theta(b_o))(A_\theta - \underline{\omega})}$$

where we are using the fact that the solution of (51) at b_o, $(\tau^o_\theta(b_o), g^o_\theta(b_o))$, must equal the equilibrium policies $(\tau_\theta(b_o), g_\theta(b_o))$. We conclude that

$$-\beta V'_\theta(b_o) = 1 + \frac{\tau_\theta(b_o)A_\theta + (1-\tau_\theta(b_o))\underline{\omega}}{(1-2\tau_\theta(b_o))(A_\theta - \underline{\omega})}. \tag{55}$$

(ii) Suppose now that $q < q^*_\theta$. Then we know from Proposition 4 that the resource constraint is binding for $b \leq b^q_\theta$. Consider the problem

$$\max_{(\tau,g,b')} \left\{ \begin{array}{c} b' - (1+\rho)b + x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma\ln g + (q-1)B_\theta(\tau,g,b',b,\underline{\omega}) + \beta EV_\theta(b') \\ s.t.\ B_\theta(\tau,g,b',b,\underline{\omega}) \geq 0 \ \&\ b \in [\underline{b},\overline{b}] \end{array} \right\}. \tag{56}$$

This is the mwc's problem but ignoring the resource constraint. Let $(\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b))$ denote the optimal tax and public good levels for this problem. It is easy to show that $\widetilde{\tau}_\theta(b)$ is non-decreasing and $\widetilde{g}_\theta(b)$ is non-increasing in b, implying that $\widetilde{g}_\theta(b) + \frac{x_\theta(\widetilde{\tau}_\theta(b))}{A}$ is non-increasing in b. From Proposition 4, we know that $\widetilde{g}_\theta(b^q_\theta) + \frac{x_\theta(\widetilde{\tau}_\theta(b^q_\theta))}{A} > n_w$. If $\widetilde{g}_\theta(\overline{b}) + \frac{x_\theta(\widetilde{\tau}_\theta(\overline{b}))}{A} < n_w$, define b^{**}_θ to be the minimal level of b such that $\widetilde{g}_\theta(b) + \frac{x_\theta(\widetilde{\tau}_\theta(b))}{A} \leq n_w$. Otherwise let $b^{**}_\theta = \overline{b}$. Then the resource constraint in the mwc's problem is binding if and only if $b \leq b^{**}_\theta$.

Turning to differentiability, consider some debt level b_o. If $b_o < b^q_\theta$ or $b_o > b^{**}_\theta$, the argument follows exactly that in part (i). Suppose that $b_o \in (b^q_\theta, b^{**}_\theta)$. It follows that both the budget and resource constraints are binding in a neighborhood of b_o. Consider the function

$$\varphi_\theta(b) = \max_{(\tau,g)} \left\{ \begin{array}{c} b'_\theta(b_o) - (1+\rho)b + x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{An_e}\right)^2}{2} + \gamma\ln g + \beta EV_{\theta'}(b'_\theta(b_o)) \\ s.t.\ B_\theta(\tau,g,b'_\theta(b_o),b,\underline{\omega}) = 0,\ g + \frac{x_\theta(\tau)}{A} = n_w. \end{array} \right\} \tag{57}$$

Clearly, $V_\theta(b_o) = \varphi_\theta^o(b_o)$ and $V_\theta(b) \geq \varphi_\theta^o(b)$ for all b in a neighborhood of b_o. The function $\varphi_\theta(b)$ is differentiable in b and its derivative is equal to $-(1+\rho)[1+\lambda_\theta^o(b)]$, where $\lambda_\theta^o(b)$ is the Lagrangian multiplier of the constraint $B_\theta(\tau, g, b'_\theta(b_o), b, \underline{\omega}) = 0$ in (57) at b. By a similar argument to that just used, $\varphi_\theta(b)$ is concave. It again follows from Theorem 4.10 of Stokey, Lucas and Prescott (1989) that $V_\theta(b)$ is differentiable at b_o with derivative $V'_\theta(b_o) = \varphi'_\theta(b_o) = -(1+\rho)[1+\lambda_\theta^o(b_o)] = -[1+\lambda_\theta^o(b_o)]/\beta$. To complete the proof, consider the first order conditions of (57) at b_o:

$$\frac{\gamma}{g_\theta^o(b_o)} = \lambda_\theta^o(b_o)\underline{\omega} + \mu_\theta^o(b_o) \tag{58}$$

and

$$\lambda_\theta^o(b_o)(1-2\tau_\theta^o(b_o))(A_\theta - \underline{\omega}) = \tau_\theta^o(b_o)A_\theta + (1-\tau_\theta^o(b_o))\underline{\omega} - \mu_\theta^o(b_o), \tag{59}$$

where $\mu_\theta^o(b_o)$ is the Lagrange multiplier of the resource constraint. Solving the system, we have:

$$\lambda_\theta(b_o) = \frac{\tau_\theta(b_o)A_\theta + (1-\tau_\theta(b_o))\underline{\omega} - \frac{\gamma}{g_\theta(b_o)}}{(1-2\tau_\theta(b_o))(A_\theta - \underline{\omega}) - \underline{\omega}}$$

where we are using the fact that the solution of (57) at b_o, $(\tau_\theta^o(b_o), g_\theta^o(b_o))$ equals the equilibrium policies $(\tau_\theta(b_o), g_\theta(b_o))$. We therefore conclude that:

$$-\beta V'_\theta(b_o) = 1 + \frac{\tau_\theta(b_o)A_\theta + (1-\tau_\theta(b_o))\underline{\omega} - \frac{\gamma}{g_\theta(b_o)}}{(1-2\tau_\theta(b_o))(A_\theta - \underline{\omega}) - \underline{\omega}}. \tag{60}$$

Finally, suppose that $b = b_\theta^{**}$. It is easy to see that in a neighborhood of b_θ^{**} the function $\varphi_\theta(b)$ defined in (57) satisfies the properties: $V_\theta(b_o) = \varphi_\theta^o(b_o)$ and $V_\theta(b) \geq \varphi_\theta^o(b)$. As noted above, moreover, the function $\varphi_\theta(b)$ is differentiable and concave in b; its derivative is equal to $-(1+\rho)[1+\lambda_\theta^o(b)]$, where $\lambda_\theta^o(b)$ is the Lagrangian multiplier of the constraint $B_\theta(\tau, g, b'_\theta(b_o), b, \underline{\omega}) = 0$ in (57) at b. It again follows from Theorem 4.10 of Stokey, Lucas and Prescott (1989) that $V_\theta(b)$ is differentiable at b_θ^{**} with derivative $V'_\theta(b_\theta^{**}) = \varphi'_\theta(b_\theta^{**}) = -(1+\rho)[1+\lambda_\theta^o(b_\theta^{**})] = -[1+\lambda_\theta^o(b_\theta^{**})]/\beta$. The derivative is the same as above, with the difference that at b_θ^{**} we have $\mu_\theta^o(b_o) = 0$. So (55) is equal to (60) at b_θ^{**}. ■

We can now show:

Lemma A.7. $b^q \in [b_H^q, b_L^q]$.

Proof. From the definition of b^q, we know that if V_H and V_L are differentiable at b^q it must be the case that

$$q = -\beta E V'_\theta(b^q) \tag{61}$$

Assume first that $b^q < b^q_H$. Then, by Lemma A.6, (61) would imply $q = 1$, a contradiction. Assume next that $b^q > b^q_L$. This would imply that for each cost state θ, we have that $\tau_\theta(b^q) > \tau^q_\theta$. Using the first order conditions for τ^q_θ and the expressions in Lemma A.6, we can show that this implies $\beta V'_\theta(b^q) < -q$. This implies: $-\beta E V'_\theta(b^q) > q$: again a contradiction. We conclude that $b^q \in [b^q_H, b^q_L]$ as claimed. ■

We can use this result to establish the assertion in the proposition that $b^q > r^o_H/\rho$. Since $r^q_H > r^o_H$, we have from Lemma A.7 that

$$b^q \geq b^q_H = \frac{r^q_H + b^q}{1+\rho} > \frac{r^o_H + b^q}{1+\rho}$$

Multiplying this inequality through by $1+\rho$ yields the result.

6.5.2 Proof of countercyclical behavior

We begin with the following useful result.

Lemma A.8. *For all $b \in [b^q_H, \overline{b}]$ it is the case that $\lambda_H(b) > \lambda_L(b)$, where $\lambda_\theta(b)$ is the Lagrange multiplier on the budget constraint for the problem*

$$\max \left\{ \begin{array}{c} b' - b(1+\rho) + x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A n_e}\right)^2}{2} + \gamma \ln g + \beta E V_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0,\ g + \frac{x_\theta(\tau)}{A} \leq n_w \ \&\ b' \in [\underline{b}, \overline{b}] \end{array} \right\}.$$

Proof. Let $b \in [b^q_H, \overline{b}]$. Suppose, contrary to the claim, that $\lambda_L(b) \geq \lambda_H(b)$. Then, by the concavity of the value function, we know that $b'_L(b) \geq b'_H(b)$. Following the same basic steps as in the proof of Lemma A.4, we can show that:

$$R_L(\tau_L(b), \underline{\omega}) - \underline{\omega} g_L(b) > R_H(\tau_H(b), \underline{\omega}) - \underline{\omega} g_H(b). \tag{62}$$

Since $b \geq b^q_H$, moreover, we know that $\lambda_H(b) > 0$ and hence that the budget constraint is binding in the high cost state. Thus, if (62) holds, we have

$$R_L(\tau_L(b), \underline{\omega}) - \underline{\omega} g_L(b) + b'_L(b) - (1+\rho)b > R_H(\tau_H(b), \underline{\omega}) - \underline{\omega} g_H(b) + b'_H(b) - (1+\rho)b = 0.$$

This implies $\lambda_L(b) = 0$. So we have $\lambda_L(b) < \lambda_H(b)$, a contradiction. ■

We now prove:

Lemma A.9. *In equilibrium: (i) $b'_H(b) > b$ for all $b \in [\underline{b}, \overline{b})$, and, (ii) $b'_L(b) > b$ for all $b \in [\underline{b}, b^q)$ and $b'_L(b) < b$ for all $b \in (b^q, \overline{b}]$.*

Proof (i) We need to show that $b'_H(b) > b$ for all $b \in [\underline{b}, \overline{b})$. Let $b \in [\underline{b}, \overline{b})$. Suppose first that $b \leq b^q_H$. Then, we have that $b'_H(b) = b^q \geq b^q_H > b$. Suppose next that $b > b^q_H$. We know that $b'_H(b) > b^q$ and that $b'_H(b)$ satisfies the first order condition:

$$1 + \lambda_H(b) = -\beta EV'_\theta(b'_H(b))$$

where $\lambda_H(b)$ is the Lagrangian multiplier on the budget constraint on the maximization problem (50). We also know from the proof of Lemma A.6 that

$$-\beta V'_\theta(b) = \begin{cases} 1 + \lambda_\theta(b) & \text{if } b > b^q_\theta \\ 1 & \text{if } b < b^q_\theta \end{cases} \tag{63}$$

Suppose that $b'_H(b) \leq b$. Then if $b \geq b^q_L$, we have that

$$1 + \lambda_H(b) = -\beta EV'_\theta(b'_H(b)) \leq -\beta EV'_\theta(b) = \alpha(1 + \lambda_H(b)) + (1 - \alpha)(1 + \lambda_L(b)) < 1 + \lambda_H(b)$$

since $\lambda_H(b) > \lambda_L(b)$ for all $b \geq b^q_H$ by Lemma A.8. If $b < b^q_L$, we have that

$$1 + \lambda_H(b) = -\beta EV'_\theta(b'_H(b)) \leq -\beta EV'_\theta(b) = \alpha(1 + \lambda_H(b)) + (1 - \alpha) < 1 + \lambda_H(b).$$

(ii) We first show that $b'_L(b) > b$ for all $b \in [\underline{b}, b^q)$. Let $b \in [\underline{b}, b^q)$. Then since $b^q < b^q_L$, we know that $b'_L(b) = b^q > b$. We next show that $b'_L(b) < b$ for all $b \in (b^q, \overline{b}]$. Let $b \in (b^q, \overline{b}]$. Suppose first that $b \leq b^q_L$. Then we know that $b'_L(b) = b^q < b$. Now suppose that $b > b^q_L$. We know that $b'_L(b)$ satisfies the first order condition:

$$1 + \lambda_L(b) = -\beta EV'_\theta(b'_L(b)).$$

Suppose that $b'_L(b) \geq b$. Then since $b > b^q_L$ we have that

$$1 + \lambda_L(b) = -\beta EV'_\theta(b'_L(b)) \geq -\beta EV'_\theta(b) = \alpha(1 + \lambda_H(b)) + (1 - \alpha)(1 + \lambda_L(b)) > 1 + \lambda_L(b),$$

where the last step relies on (63) and the fact that by Lemma A.8 $\lambda_H(b) > \lambda_L(b)$. This is a contradiction. ■

6.5.3 The stable distribution

Let $\psi_t(b)$ denote the distribution function of the current level of debt at the beginning of period t. The distribution function $\psi_0(b)$ is exogenous and is determined by the economy's initial level of debt b_0. The transition function implied by the equilibrium is given by

$$H(b,b') = \begin{cases} \Pr\{\theta' \text{ s.t. } b'_{\theta'}(b) \leq b'\} & \text{if } \exists\, \theta' \ \ s.t.\ b'_{\theta'}(b) \leq b' \\ 0 & otherwise \end{cases},$$

for any $b' \in [b^q, \overline{b}]$. $H(b,b')$ is the probability that in the next period the initial level of debt will be less than or equal to $b' \in [b^q, \overline{b}]$ if the current level of debt is b. Using this notation, the distribution of debt at the beginning of any period $t \geq 1$ is defined inductively by $\psi_t(b) = \int_z H(z,b) d\psi_{t-1}(z)$. The sequence of distributions $\langle \psi_t(b) \rangle$ converges to the distribution $\psi(b)$ if we have that $\lim_{t\to\infty} \psi_t(b) = \psi(b)$ for all $b' \in [b^q, \overline{b}]$. Moreover, $\psi^*(b)$ is an invariant distribution if

$$\psi^*(b) = \int_z H(z,b) d\psi^*(z).$$

We now establish that any sequence of equilibrium debt distributions $\langle \psi_t(b) \rangle$ converges to a unique invariant distribution $\psi^*(b)$.

It is easy to prove that the transition function $H(b,b')$ has the Feller Property and that it is monotonic in b (see Ch. 8.1 in Stokey, Lucas and Prescott (1989) for definitions). Define the function $H^m(b,b')$ inductively by $H^0(b,b') = H(b,b')$ and $H^m(b,b') = \int_z H(z,b') dH^{m-1}(b,z)$. By Theorem 12.12 in Stokey, Lucas and Prescott (1989), therefore, the result follows if the following "mixing condition" is satisfied:

Mixing Condition: *There exists an* $\epsilon > 0$ *and* $m \geq 0$*, such that* $H^m(\overline{b}, b^q) \geq \epsilon$ *and* $H^m(\underline{b}, b^q) \leq 1 - \epsilon$*.*

We proceed in two steps.

Step 1. We first show that there exists an $\epsilon > 0$ and $m \geq 0$, such that $H^m(\overline{b}, b^q) \geq \epsilon$. Assume by contradiction that $H^m(\overline{b}, b^q_L) = 0$ for any m. Then the political equilibrium coincides with the planner's solution, since with probability one: $B_\theta(\tau'_\theta(b), g'_\theta(b), b'_\theta(b), b, \underline{\omega}) = 0$. By the argument of Proposition 2, this implies: $\Pr\left(\lim_{n\to\infty} \lambda_{\theta_n}(b_n) = 0\right) = 1$. So $\Pr\left(\lim_{n\to\infty} b'_{\theta_n}(b_n) > \underline{b}\right) = 0$, but then $\Pr\left(\lim_{n\to\infty} b'_{\theta_n}(b_n) > b^q_L\right) = 0$, a contradiction. So there must be an $\varepsilon > 0$ and $m \geq 0$, such that $H^{m-1}(\overline{b}, b^q_L) > \varepsilon$. This implies $H^m(\overline{b}, b^q) \geq (1-\alpha) H^{m-1}(\overline{b}, b^q_L) > 0$.

Step 2. We now show that there exists an $\varepsilon > 0$ and $m \geq 0$, such that $1 - H^m(\underline{b}, b^q) \geq \varepsilon$. With probability $H^{m-1}(\underline{b}, b_L^q)$ the level of debt chosen in period $m-1$ is b_L^q when the initial level of debt is $\underline{b}$. Given this, the probability that the level of debt is larger than b^q in period m is at least $H^{m-1}(\underline{b}, b_L^q)\,[1 - H(b_L^q, b^q)]$. By the previous step and the monotonicity of $H(b, b')$ we have that there is a $\varepsilon > 0$ such that $H^{m-1}(\underline{b}, b_L^q) > \varepsilon$. Since $b_L^q > b_H^q$, we have $b'_H\,(b_L^q) \geq b^q$: it follows that $[1 - H(b_L^q, b^q)]\,H^{m-1}(\underline{b}, b_L^q) \geq \alpha\varepsilon > 0$.

To prove that the stable distribution has full support in $[b^q, \overline{b})$ we now show that for any $b \in [b^q, \overline{b})$, $\psi^*(b) \in (0, 1)$. The fact that $\psi^*(b) > 0$, follows from Step 1 presented above. We therefore only need to show that $\psi^*(b) < 1$. Let $b_0 \in \left[b_H^q, \overline{b}\right]$. Define recursively a sequence b_t such that $b_t = b_H(b_{t-1})$. This sequence is monotonically increasing and bounded, so it converges. Assume that $\lim_{t\to\infty} b_t = b_\infty < \overline{b}$. We have that $b'_H(b_t) \leq b_t + \varepsilon_t$, where $\varepsilon_t > 0$ and $\varepsilon_t \to 0$ as $t \to \infty$. We therefore have:

$$\begin{aligned} 1 + \lambda_H(b_t) &\leq -\beta E V'_\theta(b_t + \varepsilon_t) \\ &= \alpha(1 + \lambda_H(b_t + \varepsilon_t)) + (1 - \alpha)(1 + \lambda_L(b_t + \varepsilon_t)) \\ &< 1 + \lambda_H(b_t + \varepsilon_t) - (1 - \alpha)\Delta^*, \end{aligned} \tag{64}$$

where the last inequality follows from the fact that by Lemma A.8 $\lambda_H(b) > \lambda_L(b)$ for all $b \in [b^q, b_\infty]$, so there is a $\Delta^* > 0$ so that $\lambda_H(b) - \Delta^* > \lambda_L(b)$ in a left neighborhood of b_∞. But since $\lambda_L(b_t)$ is continuous in b, (64) implies $\lambda_H(b_\infty) < \lambda_H(b_\infty) - (1 - \alpha)\Delta^*$, a contradiction.

The argument above implies that for any $b < \overline{b}$, there is a finite T such that starting from any $b_0 \geq b^q$, we have $b_T > b$ with strictly positive probability. This implies that $\psi^*(b) < 1$. ■

6.6 Proof of Proposition 6

Given the discussion in the text, it suffices to establish three properties of the equilibrium revenue requirements. These are (i) that $r_\theta(b)$ is increasing in b for each state θ, (ii) that $r_H(b^q) > r_H^q$, and $r_L(b^q) \leq r_L^q$, and (iii) that $r_H(\overline{b}) > r_H^*$. To see the first property, assume first that $b \leq b_\theta^q$. In this case $b'_\theta(b) = b^q$, so $r_\theta(b) = (1 + \rho)\,b - b'_\theta(b)$ is increasing in b. Assume now that $b > b_\theta^q$. Then we know that $B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \underline{\omega}) = 0$, implying that $b(1 + \rho) - b'_\theta(b) = R_\theta(\tau_\theta(b), \underline{\omega}) - \underline{\omega} g_\theta(b)$. An increase in b implies that $\lambda_\theta(b)$ increases, implying that $R_\theta(\tau_\theta(b), \underline{\omega}) - \underline{\omega} g_\theta(b)$ increases in b. The second property follows from Lemma A.7. For the third property, note that $b'_H(\overline{b}) = \overline{b}$ and so $r_H(\overline{b}) = \rho\overline{b}$. Given the definition of $\overline{b}$, the government budget constraint implies that $(\tau_H(\bar{b}), g_H(\bar{b})) = (1/2, 0)$. Since there is no public sector employment, total employment is therefore

$$\frac{x_H(1/2)}{A} = n_e(A_H - \underline{\omega})/2\xi.$$

This is less than n_w by assumption (see footnote #15). It follows that $r_H(\bar{b}) > r_H^*$. ■

6.7 Proof of the results of Section 4.3

We need to prove that τ_θ^o is smaller than $\tau_\theta^-(r)$ if and only if (26) is satisfied. As in the proof of Proposition 1, let τ_θ^* denote the tax rate at which the slope of the budget line is equal to the slope of the full employment line and let $g_\theta^*(r)$ denote the level of public good that satisfies the budget constraint (32) given this tax rate. Moreover, define r_θ^{**} to be that revenue requirement at which the point $(\tau_\theta^*, g_\theta^*(r))$ is tangent to the full employment line. Then, for all $r \in (r_\theta^o, r_\theta^{**}]$, we know that $\tau_\theta^* \in [\tau_\theta^-(r), \tau_\theta^+(r)]$. Moreover, $\tau_\theta^-(r_\theta^{**}) = \tau_\theta^+(r_\theta^{**}) = \tau_\theta^*$. By definition, we also know that for all $r \in (r_\theta^o, r_\theta^{**}]$, $\tau_\theta^o \notin [\tau_\theta^-(r), \tau_\theta^+(r)]$. Thus, τ_θ^o is smaller than $\tau_\theta^-(r)$ for all $r \in (r_\theta^o, r_\theta^{**}]$ if τ_θ^o is smaller than τ_θ^*, and τ_θ^o is larger than $\tau_\theta^+(r)$ for all $r \in (r_\theta^o, r_\theta^{**}]$ if τ_θ^o is larger than τ_θ^*. Knowing whether τ_θ^o is smaller or larger than τ_θ^* is equivalent to knowing whether g_θ^o is larger or smaller than $g_\theta^*(r_\theta^{**})$. We know g_θ^o from (30), so we just need to compute $g_\theta^*(r_\theta^{**})$. By definition, the tax rate τ_θ^* is such that the slope of the budget line is the same as the slope of the resource constraint. This means that

$$\frac{n_e(A_\theta - \underline{\omega})}{\xi} = \frac{n_e(1 - 2\tau_\theta^*)(A_\theta - \underline{\omega})^2}{\xi\underline{\omega}}, \tag{65}$$

which implies that $\tau_\theta^* = \frac{A_\theta - 2\underline{\omega}}{2(A_\theta - \underline{\omega})}$. We know that the revenue requirement r_θ^{**} is such that the associated budget line is just tangent to the resource constraint at tax rate τ_θ^*. Thus,

$$g_\theta^*(r_\theta^{**}) + \frac{x_\theta(r_\theta^{**})}{A} = n_w. \tag{66}$$

Using (8), this implies that

$$g_\theta^*(r_\theta^{**}) = n_w - \frac{n_e A_\theta}{2\xi}. \tag{67}$$

Thus,

$$g_\theta^o - g_\theta^*(r_\theta^{**}) = \frac{\sqrt{(A_\theta n_e - \xi n_w)^2 + 4\xi n_e \gamma} - \xi n_w}{2\xi} \tag{68}$$

and we have g_θ^o is larger or smaller than $g_\theta^*(r_\theta^{**})$ as γ is larger or smaller than $\frac{A_\theta}{2}\left(n_w - \frac{n_e A_\theta}{2\xi}\right)$ as required. ■