Theoretical **RE**search in **N**euroeconomic **D**ecision-making
(www.neuroeconomictheory.org)

Resource Allocation in the Brain

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Neuroeconomic Theory

Use evidence from neuroscience to revisit economic theories of decision-making

- Examples of neuroscience evidence include:

  Existence of multiple brain systems, interactions between systems, physiological constraints, etc.

- Revisiting theories of decision-making includes:

  Building models of bounded rationality based not on inspiration or casual observation but on the physiological constraints of our brain

  → derive behavioral biases from brain limitations

The brain is, so it should be modeled as, a multi-system organization

This paper (1)

Build a model of constrained optimal behavior in multi task decision-making based on evidence from neuroscience:

i. Different brain systems are responsible for different tasks. Neurons in a system respond exclusively to features of that particular task.

ii. The brain allocates resources (oxygen, glucose) to systems. Resources are transformed into energy that make neurons fire (fMRI measure blood oxygenation, PET measure changes in blood flows, etc.).

iii. More complex tasks necessitate more resources. Performance suffers if resources needs are not filled.

iv. Resources are scarce: "biological mechanisms place an upper bound on the amount of cortical tissue that can be activated at any given time".

This paper (2)

v. Central Executive System (**CES**) coordinates the allocation of resources:
   - Active when two tasks are performed simultaneously.
   - Not active if only one task, if two sequential tasks, or if two tasks but subject instructed to focus on only one.

vi. Asymmetric information in the brain: neuronal connectivity is very limited → information carried by neurotransmitters reaches some systems but not others.

The model

- Three systems (0, 1, 2) perform three types of tasks:
  - System 0 (**S_0**) controls motor skill functions. Needs θ_0 are known
  - Systems 1 and 2 (**S_1** and **S_2**) control higher order cognitive functions (mental rotation, auditory comprehension, face recognition) Needs θ_1 and θ_2 depend on task complexity and are privately known
  - Distributions $F_1(\theta_1)$ and $F_2(\theta_2)$ satisfy Increasing Hazard Rate (IHR)
- Another system, Central Executive System (**CES**), allocates resources $\{x_0, x_1, x_2\}$ to **S_0, S_1, S_2**
- Performance of system l is $U_l(x_l, \theta_l) = -\frac{1}{\beta_l}(x_l - \theta_l)^2$ with $l \in \{0,1,2\}$.

  Systems are tuned to respond only to their task
- **CES** maximizes (weighted) sum of performances of systems: $U_0+U_1+U_2$
- Resources are scarce and bounded at k: $x_0 + x_1 + x_2 \leq k$
- Each system requires a minimum of resources to operate: $x_l \geq 0$

The model

CES
central executive system

$$x_0 + x_1 + x_2 < k$$

$$U_0 + U_1 + U_2$$

x_0 x_1 x_2

S_0
(lifting)

S_1
(rotation)

S_2
(spelling)

$$U_0(x_0,\theta_0) = -\frac{1}{\beta_0}(x_0 - \theta_0)^2$$

$$U_1(x_1,\theta_1) = -\frac{1}{\beta_1}(x_1 - \theta_1)^2$$

$$U_2(x_2,\theta_2) = -\frac{1}{\beta_2}(x_2 - \theta_2)^2$$

Motor function

θ_0 "public"

Cognitive functions 1 and 2

θ_1 and θ_2 "private"

Benchmark case: full information

$$\max_{x_0, x_1, x_2} \quad \frac{1}{\beta_0} U_0\left(x_0(\theta_1, \theta_2), \theta_0\right) + \frac{1}{\beta_1} U_1\left(x_1(\theta_1, \theta_2), \theta_1\right) + \frac{1}{\beta_2} U_2\left(x_2(\theta_1, \theta_2), \theta_2\right)$$

$$\text{s.t.} \quad x_0(\theta_1, \theta_2) + x_1(\theta_1, \theta_2) + x_2(\theta_1, \theta_2) \le k \qquad \text{(R)}$$

$$x_0(\theta_1, \theta_2) \ge 0,\ x_1(\theta_1, \theta_2) \ge 0,\ x_2(\theta_1, \theta_2) \ge 0 \qquad \text{(F)}$$

Benchmark case: full information

$$\max_{x_0,x_1,x_2} \quad \frac{1}{\beta_0}U_0\left(x_0\left(\theta_1,\theta_2\right),\theta_0\right)+\frac{1}{\beta_1}U_1\left(x_1\left(\theta_1,\theta_2\right),\theta_1\right)+\frac{1}{\beta_2}U_2\left(x_2\left(\theta_1,\theta_2\right),\theta_2\right)$$

$$\text{s.t. } x_0\left(\theta_1,\theta_2\right)+x_1\left(\theta_1,\theta_2\right)+x_2\left(\theta_1,\theta_2\right)\le k \qquad \text{(R)}$$

$$x_0\left(\theta_1,\theta_2\right)\ge 0,\ x_1\left(\theta_1,\theta_2\right)\ge 0,\ x_2\left(\theta_1,\theta_2\right)\ge 0 \qquad \text{(F)}$$

Solution under full information (assuming (R) binds and (F) does not):

$$x_l^F=\theta_l-\frac{\beta_l}{\sum\beta}\left(\sum\theta-k\right)$$

Distribute k according to needs (θ_0, θ_1, θ_2) weighed by importance (β_0, β_1, β_2)

$$U_l^F\left(x_l^F;\theta_l\right)=-\frac{\beta_l}{\left(\sum\beta\right)^2}\left(\sum\theta-k\right)^2$$

Utility of a system depends on total needs (sum of θ_l) and relative importance (β_l) but not on how needs are distributed among the systems (θ_1 v. θ_2)

Roadmap

1. Normative approach: optimal allocation given private information if **CES** could use any conceivable communication mechanism
   - General properties
   - Comparative statics
2. Positive approach: can this allocation be implemented using a physiologically plausible mechanism?
3. Applications
   - Task inertia
   - Task separation

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The optimization problem

Optimal allocation of resources when needs of systems 1 and 2 are unknown and **CES** can use any mechanism. Using the revelation principle:

$$\max_{x_0, x_1, x_2} \iint \frac{1}{\beta_0} U_0\left(x_0(\theta_1, \theta_2), \theta_0\right) + \frac{1}{\beta_1} U_1\left(x_1(\theta_1, \theta_2), \theta_1\right) + \frac{1}{\beta_2} U_2\left(x_2(\theta_1, \theta_2), \theta_2\right) dF_1(\theta_1) dF_2(\theta_2)$$

$$\text{s.t. } U_i\left(x_i\left(\theta_i, \theta_j\right), \theta_i\right) \geq U_i\left(x_i\left(\tilde{\theta}_i, \theta_j\right), \theta_i\right) \quad \forall i, j \in \{1,2\}, \forall \theta_i, \tilde{\theta}_i, \theta_j \quad \text{(IC)}$$

$$x_0(\theta_1, \theta_2) + x_1(\theta_1, \theta_2) + x_2(\theta_1, \theta_2) \leq k \quad \forall \theta_1, \theta_2 \quad \text{(R)}$$

$$x_0(\theta_1, \theta_2) \geq 0,\ x_1(\theta_1, \theta_2) \geq 0,\ x_2(\theta_1, \theta_2) \geq 0 \quad \forall \theta_1, \theta_2 \quad \text{(F)}$$

The solution

- Optimal mechanism **M**:
  - resource cap $\bar{x}_1(\theta_2)$ for $\mathbf{S_1}$
  - resource cap $\bar{x}_2(\theta_1)$ for $\mathbf{S_2}$
- Equilibrium allocation under **M**:
  - $x_1^*(\theta_1,\theta_2) \;=\; \min\{\, \theta_1,\; \bar{x}_1(\theta_2) \,\}$
  - $x_2^*(\theta_1,\theta_2) \;=\; \min\{\, \theta_2,\; \bar{x}_2(\theta_1) \,\}$
  - $x_0^*(\theta_1,\theta_2) \;=\; k - x_1(\theta_1,\theta_2) - x_2(\theta_1,\theta_2)$
- What are the optimal caps $\bar{x}_1(\theta_2)$ and $\bar{x}_2(\theta_1)$?

The solution

<u>Sketch of proof</u>

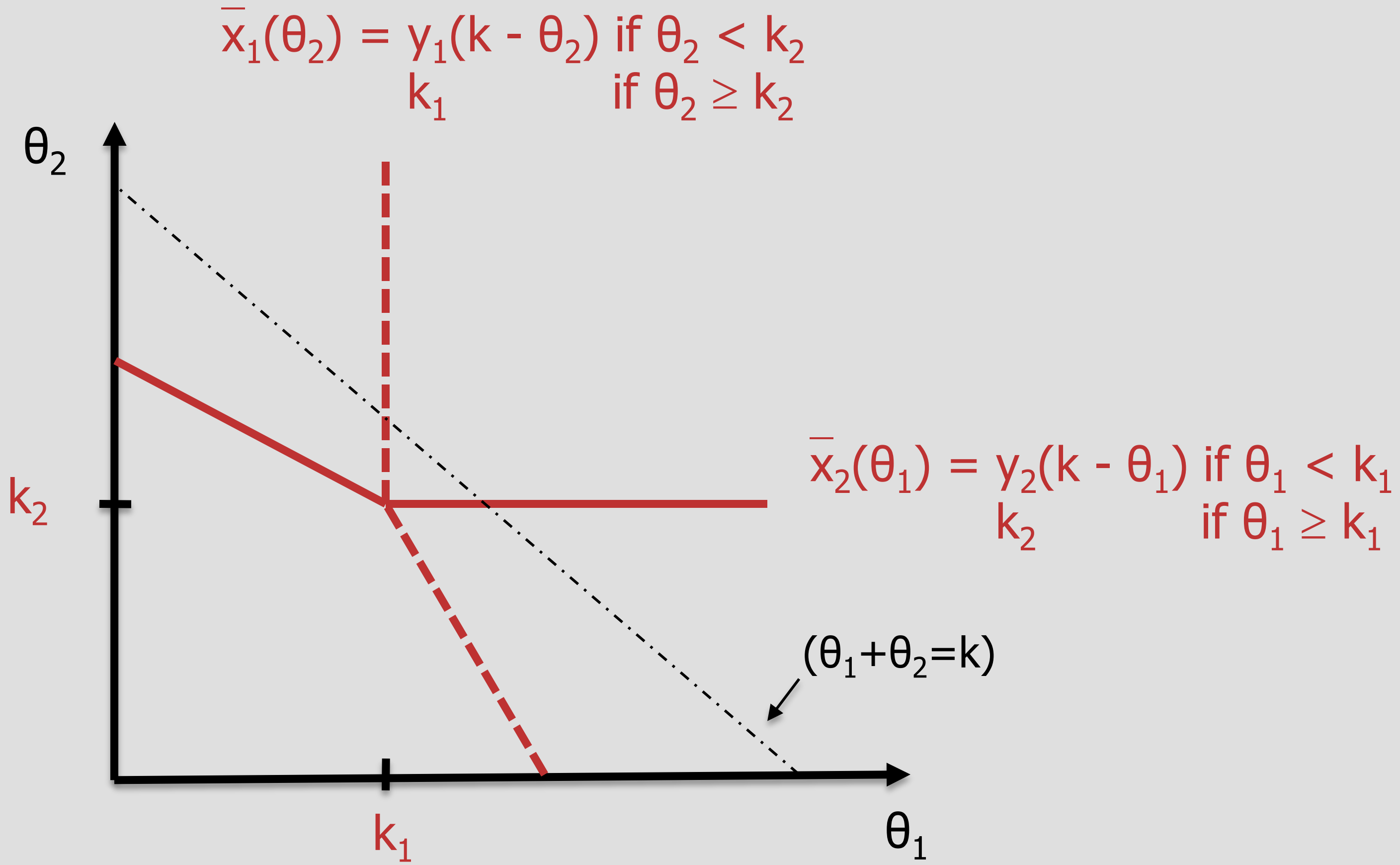
1. Derive optimal allocation with only 2 systems (1 with private info.)

2. Use it to derive "priority mechanism":
   - **P_1** : optimal mechanism under the requirement that **S_1** always obtains the resources it requests
   - **P_2** : optimal mechanism under the requirement that **S_2** always obtains the resources it requests.

3. Compare **P_1** and **P_2** . Show that optimum is a hybrid of both:
   it behaves like **P_1** for certain (θ_1,θ_2) and like **P_2** for some other (θ_1,θ_2).

Optimal Resource Allocation

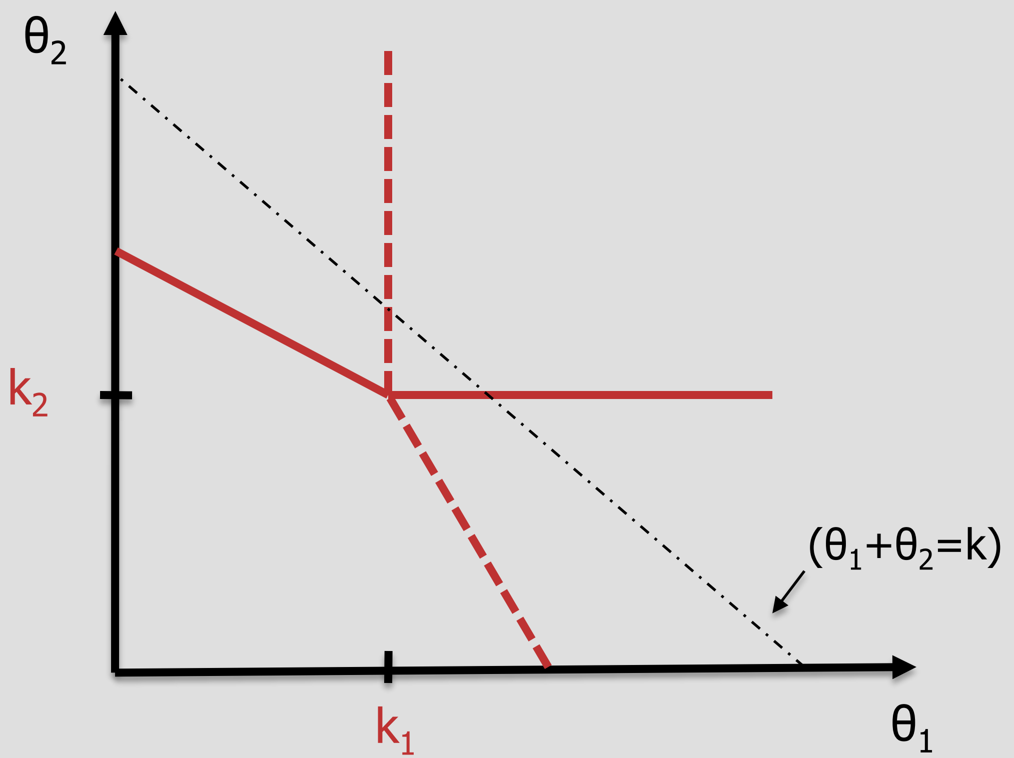
Optimal caps $\bar{x}_1(\theta_2)$ and $\bar{x}_2(\theta_1)$ are first strictly decreasing and then constant in the needs of the other system



Optimal Resource Allocation

Optimal caps $\bar{x}_1(\theta_2)$ and $\bar{x}_2(\theta_1)$ are first strictly decreasing and then constant in the needs of the other system

$$\frac{1}{\beta_1}\left(E\left[\theta_1 \mid \theta_1 > k_1\right] - k_1\right) = \frac{1}{\beta_2}\left(E\left[\theta_2 \mid \theta_2 > k_2\right] - k_2\right) = \frac{1}{\beta_0}\left(\theta_0 - \left(k - k_1 - k_2\right)\right)$$

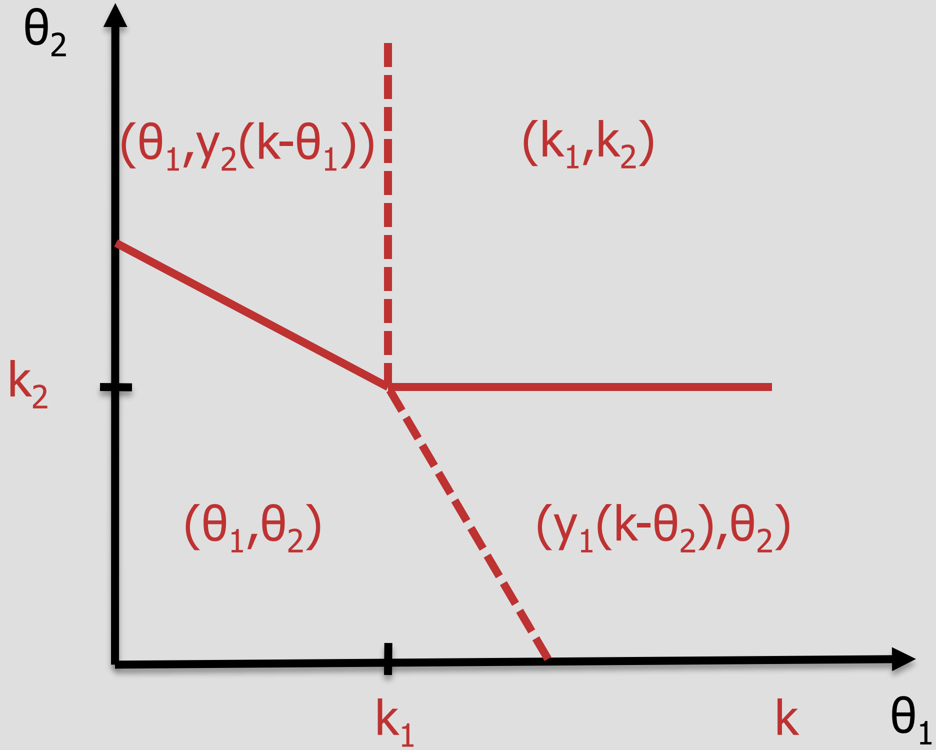


Optimal Resource Allocation

Equilibrium allocation $(x_1^*(\theta_1,\theta_2), x_2^*(\theta_1,\theta_2))$

→ unconstrained for “small” needs and fixed for “large” needs

(with $x_0^*(\theta_1,\theta_2) = k - x_1^*(\theta_1,\theta_2) - x_2^*(\theta_1,\theta_2)$)



Properties

- Equilibrium is unique (under Increasing Hazard Rate)
- k_1 , k_2 , $k - k_1 - k_2$ are guaranteed resources for $\mathbf{S_1}$, $\mathbf{S_2}$, $\mathbf{S_0}$
- Resource monotonicity: if θ_1 ↓, both x_2 ↑ <u>and</u> x_0 ↑
- Comparative statics. Same monotonicity principle:
  - If β_2 ↓ ($\mathbf{S_2}$ more important), then x_2^*↑ , x_1^*↓, x_0^*↓
  - If k ↑ (more resource), then x_0^*↑ x_1^*↑ x_2^* ↑

Implication 1. Let $\beta_1 = \beta_2$. Fix $\theta_1 + \theta_2$ with $\theta_1 > \theta_2$

- Full information: $U_1^F = U_2^F$
- Private information: $U_1^* \leq U_2^*$

  Better performance in easy tasks than in difficult tasks

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Implementation

So far, abstract revelation mechanism: "announce θ'_i, receive $x_i(\theta'_i, \theta'_j)$"

Can **CES** implement the optimal mechanism in a "simple" way and, most importantly, in a way compatible with the physiology of the brain?

- **CES** sends oxygen to $\mathbf{S_1}$, $\mathbf{S_2}$, $\mathbf{S_0}$ at rates k_1 / k, k_2 / k, $(k - k_1 - k_2) / k$.
- Systems deplete oxygen to produce energy. **CES** observes depletion which is a signal that more resources are needed (autoregulation).
- If $\mathbf{S_i}$ stops consumption, oxygen is redirected to $\mathbf{S_j}$ and $\mathbf{S_0}$ at a new rate.
- If both $\mathbf{S_i}$ and $\mathbf{S_j}$ stop consumption, the remaining oxygen is sent to $\mathbf{S_0}$.

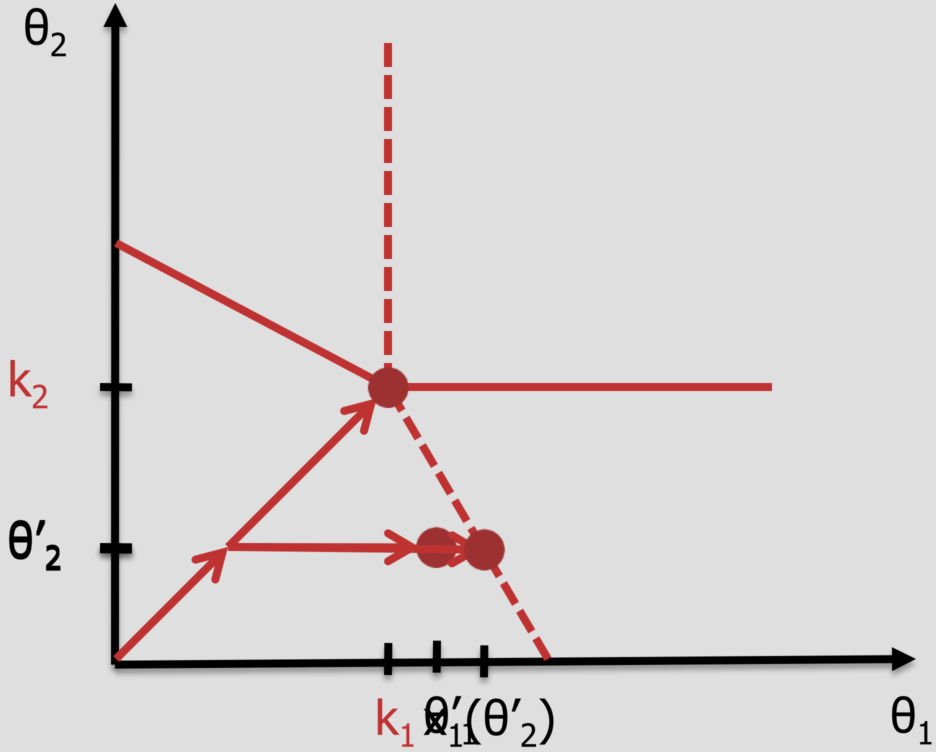
→ $\mathbf{S_i}$ grabs incoming resources up to satiation or up to constraint

→ $\mathbf{S_i}$ doesn't need to know needs or even existence of $\mathbf{S_j}$

→ $\mathbf{S_i}$ doesn't need to know its own needs θ_i until they are hit

→ **CES** must be able to redirect resources and change the rates

Implementation



If $\theta_1 > k_1$ and $\theta_2 > k_2$

If $\theta_1 = \theta'_1$ and $\theta_2 = \theta'_2$

If $\theta_1 > x_1(\theta'_2)$ and $\theta_2 = \theta'_2$

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Application 1: task inertia

- **CES** has imperfect knowledge of the distribution of needs
- **CES** gradually learns the distribution through observation of past needs
- How should **CES** adjust current allocation rules based on past needs?
- Learning:
  - Distributions $F_i(\theta_i|s_i)$ depends on unknown but fixed state s_i (is this task usually complex or simple?)
  - Prior belief of state is $p_i(s_i)$
  - Realization of θ_i^t conditionally independent across periods
  - After $\mathbf{S_i}$ reports θ_i^t in period t, **CES** updates belief over s_i
  - Assume $F_i(\theta_i|s_i)$ satisfies MLRP: needs are likely to be high (θ_i high) when task is usually complex (s_i high).

→ <u>Lemma</u>: $G_i(\theta_i^{t+1}|\theta_i^t)$ satisfies MLRP: high θ_i^t implies that s_i is likely to be high which implies that θ_i^{t+1} is also likely to be high

Application 1: task inertia

Assume s_i unknown and compare public info. (θ_1^t,θ_2^t known by **CES** at t) with private info. (θ_1^t,θ_2^t unknown by **CES** at t)

Implication 2. Inertia and path-dependence of the allocation rule.

Under private info. and conditional on present needs, allocation of $\mathbf{S_i}$ is higher if past needs were high:

If $\theta_2^{t-1}\uparrow$, then $x_2^t(\theta_1^t,\theta_2^t)\uparrow$, $x_1^t(\theta_1^t,\theta_2^t)\downarrow$, $x_0^t(\theta_1^t,\theta_2^t)\downarrow$

→ consistent with neuroscience evidence on "task switching cost".

Application 2: task separation

- Is it better to have an integrated system responsible for tasks 1 and 2 or two separate systems each responsible for one task?
- Trade-off:
  - Integrated system allocates more efficiently its resources between tasks 1 and 2
  - Separated systems require less "informational rents": cap of $\mathbf{S_1}$ can depend on announcement of $\mathbf{S_2}$

Implication 3.

- Integration of $\mathbf{S_1}$ and $\mathbf{S_2}$ dominates when motor task is important ("low" β_0)
- Separation of $\mathbf{S_1}$ and $\mathbf{S_2}$ dominates when cognitive tasks are important ("low" β_1 and β_2)

Conclusions

- The brain is a multi-system organization.
- Bounded rationality model based not on inspiration but on physiological constraints of the brain → derive behaviors from brain limitations
- Optimal resource allocation: each system has guaranteed resources (k_0, k_1, k_2). More resources are available only if others are satiated
- Physiologically plausible implementation.
- Resource allocation under capacity constraint and asymmetric information provides an informational rationale for (not a model built to explain):
  - Better performance in easier tasks
  - Task inertia and task switching cost
  - Conditions for integration v. separation of functions
- Model can be straightforwardly applied to standard organization problems:
  - Allocation of resources between research, marketing and production
  - Market split of colluding firms (no transfers!), etc.