Biology and the Arguments of Utility

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"Biological Basis of Economic Preferences and Behavior"

Introduction

- Study why our utility functions have the *arguments* they do
- Why do we care about *anything* other than offspring?

(Implication: we sacrifice offspring for other goods)

- We show that the *marginal utility* of a given action does not depend only on its *fitness value*

Note: “Innate/primary” vs. “conditioned/secondary” arguments

Observations:

1. Innate arguments are numerous – e.g., Linden, 2001

food, prestige, body temperature, view, sex, effort

General Problem

Principal: Natural selection (maximizes fitness)

Agent: Individual (maximizes utility)

- Principal selects *utility function* of Agent
- Agent selects expected-utility-maximizing action, which randomly determines fitness
- Fitness-maximizing action depends on unknown state of nature ¾

What is the expected-fitness-maximizing utility function when each player has only partial information about ¾?

Related Literature

- *Biology and Economics*: Becker (76), Robson (01a,b), Samuelson (04), Samuelson-Swinkels (06), Rayo-Becker (07), Robson-Samuelson (07), Robson and Szentes (08), Netzer (09), Herold -Netzer (11), Robson-Szentes-Iantchev (11), Ely-Lleras-Muney (12), Alger-Weilbull (12)

- *Optimal Delegation*: Holmstrom (84), Aghion -Tirole (97), Dessein (02), Alonso-Matouschek (08), Armstrong-Vickers (10)

- *Monotone Comparative Statics*: Milgrom-Shannon (94), Athey (02)

- Key differences vis-à-vis standard principal-agent model:

(1) All actions are contractable

Model

- $x \in R^{N}$: Agent's action
- ¾ ∈ R: state; s, t ∈ R: Principal and Agent's signals of ¾

¾~f(¾|s,t); ¾~g(¾|t)

(f and g increasing in s,t according to FOSD)

- y ∈ R: fitness

y = φ(x,¾)

(φ(x,¾) strictly concave in x; $\varphi_{x_i\text{¾}}$(x,¾), $\varphi_{x_i x_k}$(x,¾) > 0)

- x^{FB}(s,t) = $\text{argmax}_x \int \varphi$(x,¾)f(¾|s,t)d¾
- x_i^{FB}(s,t) increasing in s and t (for all i)

Timing

1. Principal privately observes $s \in R$, selects utility function

$$U: (x,y,s) \to R$$

2. State ¾ $\in R$ is drawn
3. Agent observes U, observes t, solves

$$\max_x E[U(x,y,s) \mid t]$$

4. Fitness y is realized

How can the principal implement $x^{FB}(s,t)$ for all s,t?

Case 1: Principal communicates s

Suppose the Agent knows (s,t)

Remark 1: The utility function

$$U(x,y,s) \equiv y$$

is then optimal for all s.

Proof: Agent maximizes $\int \varphi$(x,¾)f(¾|s,t)d¾, which by definition delivers x^{FB}(s,t).

Case 2: U can depend on t (as well as s)

Remark 2: The utility function

$$U(x,y,s,t) = U(x,s,t) = \begin{cases} 1 & \text{if } x = x^{FB}(s,t) \\ 0 & \text{otherwise} \end{cases}$$

is then optimal for all s.

Proof: Agent is an “automaton” who automatically chooses $x^{FB}(s,t)$.

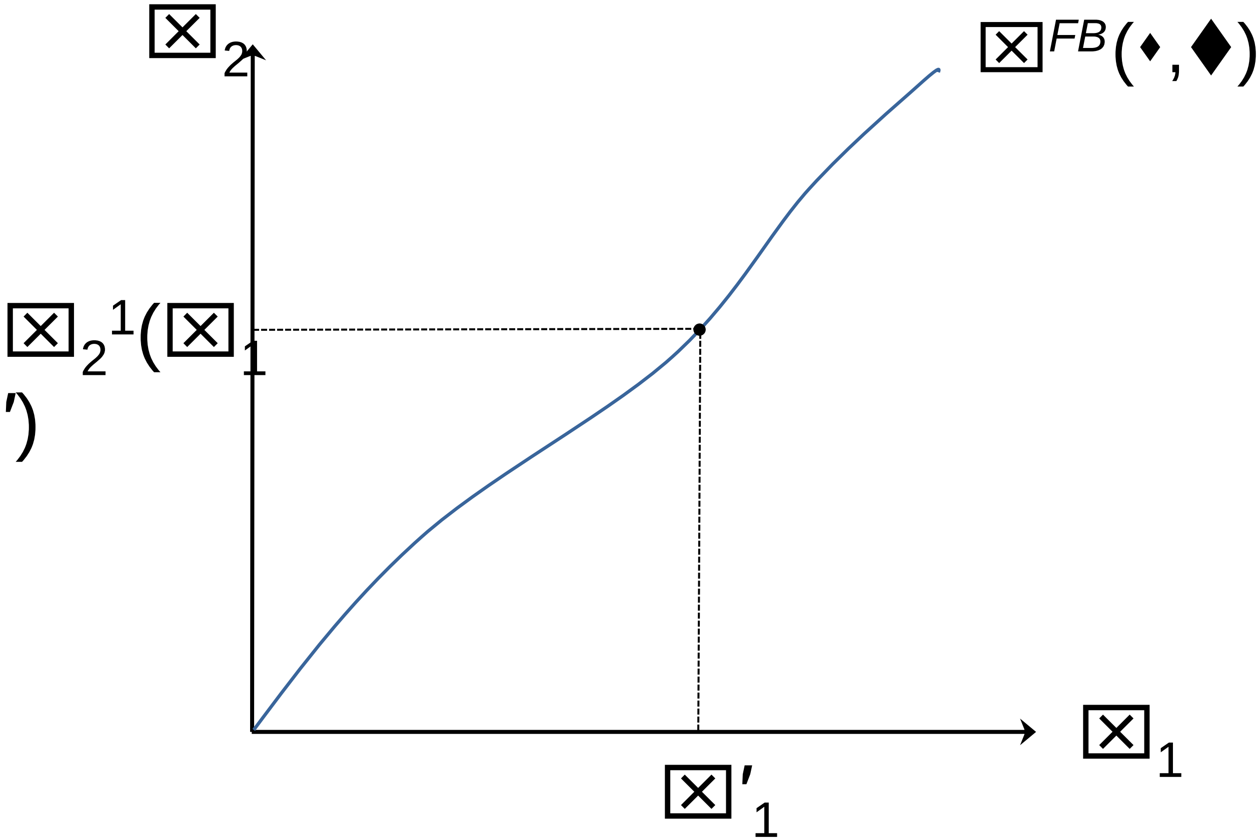
Case 3: U cannot depend on t and agent does not know s

Definition 1: Let

- $x^i(x_i,s): R \times R \rightarrow R^N$ and
- $t^i(x_i,s): R \times R \rightarrow R$ such that:

A. $x^i(x_i,s) = x_{i,}$

B. $x^i(x_i,s) = x^{FB}(s, t^i(x_i,s))$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Theorem 1: An optimal utility function is

$$U(x,y,s) = \varphi\,(x,\text{¾}) + \alpha(x,s),$$

where $\alpha(x,s) = \sum_{i} \alpha^i(x_i,s)$, and

$$\alpha^i(x_i,s) = -\int^{xi}\int \varphi_{x_i}(x^i(z,s))g(\text{¾}|t^i(z,s))d\text{¾}dz.$$

Illustration of Theorem

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Suppose N = 1. Agent's objective:

$$E[U|t] = \int \varphi(x,¾)g(¾|t)d¾ + \alpha(x,s)$$

FOC for x:

$$\alpha_x(x,s) = -\int \varphi_x(x,¾)g(¾|t)d¾$$

Since $x^{FB}(s, t)$ is monotonic, we can obtain

$t(x,s)$ such that $t(x^{FB}(s, t),s) = t$ for all s,t,

and so the R.H.S. above is a function of (x,s).

Illustration of Theorem

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$$\alpha_x(x,s) = -\int \varphi_x(x,¾)g(¾|t(x,s))d¾$$

SOC for x:

$$d^2U/dx^2 = -\int \varphi_x(x,¾)g_t(¾|t(x,s))d¾ < 0,$$

which follows from single-crossing and FOSD.

Example: Linear φ and E[¾|s,t]

Assume:

A. φ(x,¾) = a(x) + b(x)¾

e.g., x = effort; y = ¾x - C(x)

B. E[¾|s,t] = λs+ (1-λ)t

Optimal Utility:

$$U = \varphi(x,¾) + \frac{\lambda}{1-\lambda}[a(x) + b(x)s]$$

$$\text{FOC: } \lambda s + (1-\lambda)t = -\frac{b'(x^{FB}(s,t))}{a'(x^{FB}(s,t))}$$

Example: Comparative Statics

$$U = \varphi(x, ¾) + \frac{\lambda}{1-\lambda}[a(x) + b(x)s]$$

$$dx/dy|_U = -\alpha_x(x^{FB}(s,t),s,\lambda)$$

$$= -a'(x^{FB}(s,t)) - b'(x^{FB}(s,t))t$$

Remark 2.

- If $t > s$, $dx/dy|_U < 0$,
- If $t < s$, $dx/dy|_U > 0$.

Example: Comparative Statics

Remark 3 (λ).

- If $t > s$, $dx/dy|_U < 0$ and *decreasing* in λ,
- If $t < s$, $dx/dy|_U > 0$ and *increasing* in λ.

Remark 4 (s).

- $dx/dy|_U$ is *increasing* in s.

Example: Effort

x= effort; $y = \frac{3}{4}x - C(x)$

Optimal utility function:

$$U = y - \underbrace{\frac{\lambda}{1-\lambda}[C(x) - sx]}_{\hat{C}(x)}$$

The End

Glimcher
foundations of neuroeconomic analysis, 2010
(value comparison, learning

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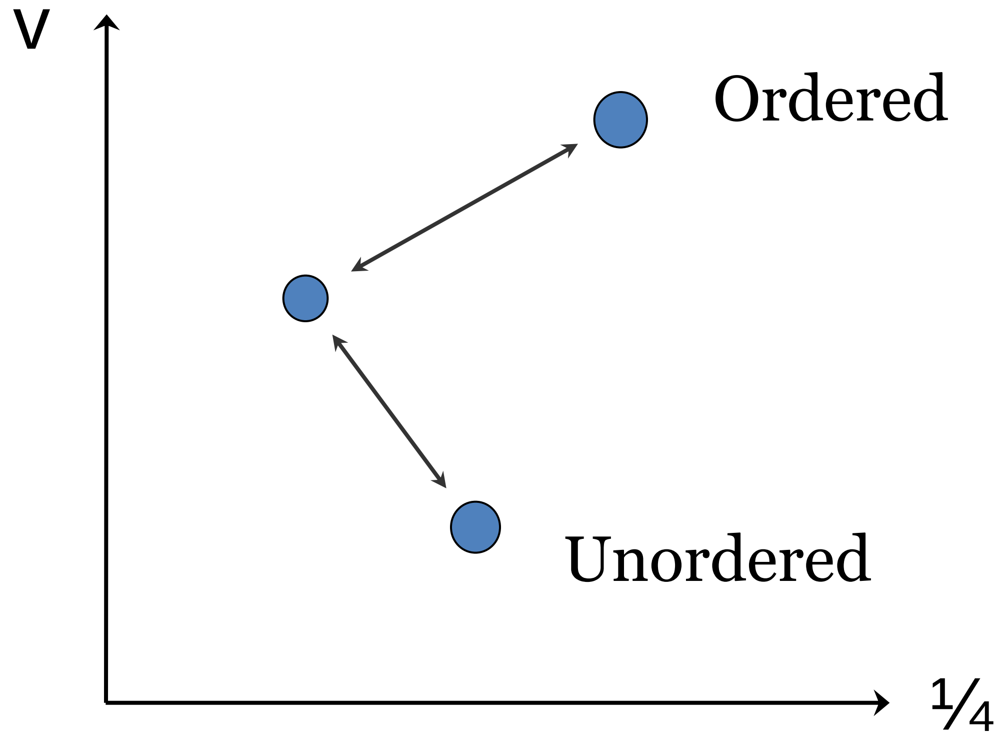

It is (strictly) optimal to separate two (strictly) ordered prospects, and (strictly) optimal to pool two (strictly) unordered prospects.