A Search Theory of the Peacock's Tail

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Literature

1. Costly Signaling

2. Social Assets

   Postlewaite and Mailath (2006)

Model

- males differ in a binary attribute $\{a, d\}$
- females differ in endowment $E \sim U[0, 1]$
- attribute is genetic, endowment is not

matching market

- there are two markets for the males M_a and M_d
- females decide which market to enter
- match as many as possible in each market

reproduction

- c-male and E-female reproduce $q\left(c,E\right)$ offspring.
- half of the offspring is male
- death after reproduction or if unmatched

Assumptions

A1. $q(a,E) > q(d,E)$ for all $E \in [0,1)$.

A2. $q(d,E)/q(a,E)$ is increasing in E.

A3. $q(a,E) < E(q(d,E') : E' \geq E)$ for all $E \in [0,1)$.

A4. $1/[\partial \lg q(a,E)/\partial E] - 1/\partial \lg q(d,E)/\partial E \leq 1/2$ for all $E \in [0,1)$.

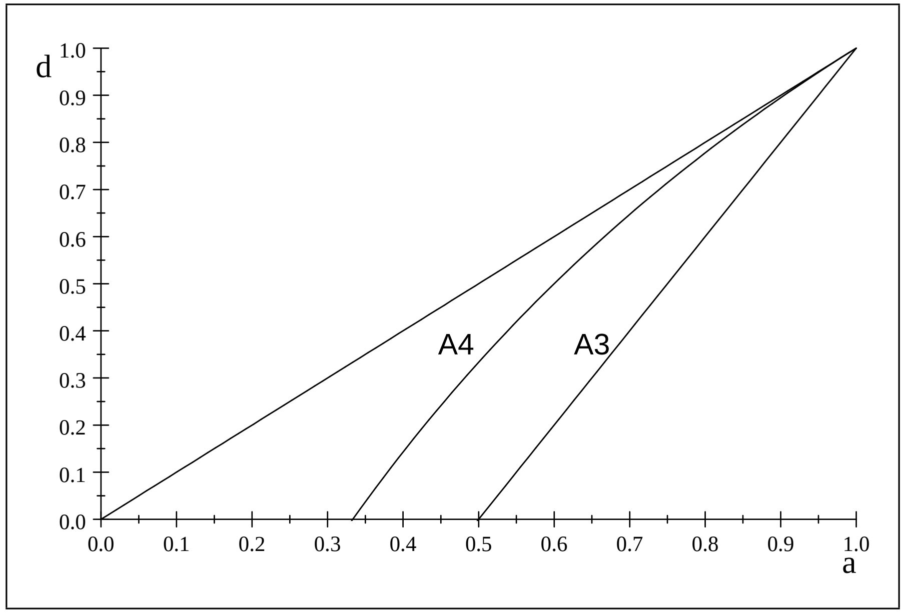
Example

$$q\left(c,E\right)=c+\left(1-c\right)E\ \left(c\in\{a,d\}\right)$$

$a>d\Rightarrow$A1, A2

$d>2a-1\Rightarrow$A3

$d>\left(3a-1\right)/\left(a+1\right)\Rightarrow$A4



d
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
a
A4
A3

State Space

(μ, S)

μ : fraction of $d-$males

S : population strategies of females

Assume that females want to maximize the expected number of offspring

A2 $\Rightarrow$ if an E-female enters the d-market then E' $(> E)$ also enters d-market

restrict attention to cutoff strategies: E^*

μ : fraction of $d-$males

Equilibrium

(μ, E^*) is an equilibrium if

(1) E^* is a best-response to (μ, E^*) and

(2) μ is constant over time

Proposition

The only equilibria are $(0, 1)$ and $(1, 0)$.

WHTS: no interior equilibrium

If (μ, E^*) is an interior equilibrium

(i) a and d males have the same reproductive values

(ii) E^*-female is indifferent between the two markets

Constant μ

claim.

In any interior equilibrium there are more males than females in the d-market

proof.

$$\mathbb{E}\left[q\left(d,E\right):E\geq E^{*}\right]>q\left(a,E^{*}\right)>\mathbb{E}\left[q\left(a,E\right):E\leq E^{*}\right]$$

a and d grow at the same rate if:

$$\frac{1-E^*}{\mu}\mathbb{E}\left[q\left(d,E\right):E\geq E^*\right]=\mathbb{E}\left[q\left(a,E\right):E\leq E^*\right],$$

or equivalently

$$\int_{E^*}^{1} q\left(d,E\right)dE=\frac{\mu}{E^*}\int_{0}^{E^*} q\left(a,E\right)dE.$$

Define $\mu_1\left(E^*\right)$ by

$$\int_{E^*}^{1} q\left(d,E\right)dE=\frac{\mu_1\left(E^*\right)}{E^*}\int_{0}^{E^*} q\left(a,E\right)dE$$

Observe μ_1 this curve is only defined if $E^*\geq\widetilde{E}$, where $\widetilde{E}$ solves

$$\int_{\widetilde{E}}^{1} q\left(d,E\right)dE=\frac{1}{\widetilde{E}}\int_{0}^{\widetilde{E}} q\left(a,E\right)dE.$$

Best Responses

$E^* \in (0, 1)$ is a best-response iff:

$$q(d, E^*) = \frac{1-\mu}{E^*} q(a, E^*),$$

or equivalently,

$$\frac{q(d, E^*)}{q(a, E^*)} = \frac{1-\mu}{E^*}.$$

Define $\mu_2(E^*)$ as the solution for the following equality:

$$q(d, E^*) = \frac{1-\mu_2(E^*)}{E^*} q(a, E^*).$$

Lemma

(i) $\mu_1(1) = \mu_2(1)$ and

(ii) μ_1 and μ_2 are decreasing.

Lemma

$\nexists E^* \in \left(\widetilde{E}, 1\right) : \mu_1(E^*) = \mu_2(E^*)$.

Corollary

$\nexists$ interior equilibrium

Corollary

$\mu_1(E) > \mu_2(E)$ for all $E \in \left(\widetilde{E}, 1\right)$

Stability

$(\psi, \varphi) : \mathbb{R}_+ \times [0,1]^2 \to [0,1]^2$

If the initial state is $\left(\mu_0, E_0^*\right)$

then $\left(\psi_t\left(\mu_0, E_0^*\right), \varphi_t\left(\mu_0, E_0^*\right)\right)$ is the state at t

Requirements

(1) $\overset{\bullet}{\psi}_t\left(\mu_0, E_0^*\right) > (<)\, 0$ if and only if

$$\frac{1 - E^*}{\mu_t}\mathbb{E}\left[q\left(d, E\right) : E \geq E_t^*\right] > (<)\, \mathbb{E}\left[q\left(a, E\right) : E \leq E_t^*\right].$$

(2) $\overset{\bullet}{\varphi}_t\left(\mu_0, E_0^*\right) > (<)\, 0$ if and only if

$$q\left(d, E_t^*\right) < (>)\, \frac{1 - \mu}{E_t^*} q\left(a, E_t^*\right).$$

definition

(μ, E^*) is a stable equilibrium if

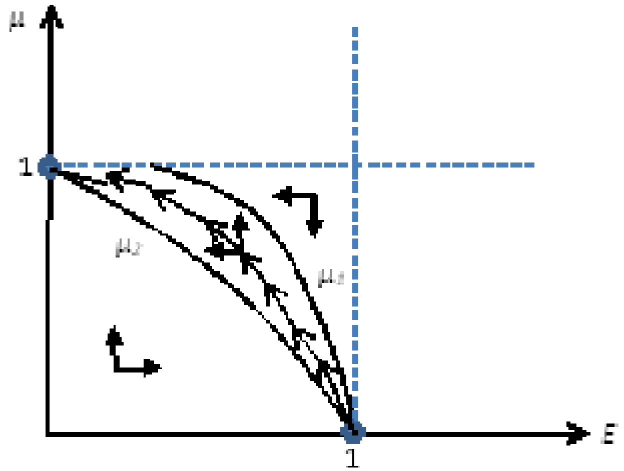
(i) it is an equilibrium, and

(ii) for all $\varepsilon > 0$ there exists an $\varepsilon > 0$, such that if $|\mu_0 - \mu|, \left|E_0^* - E^*\right| < \delta$ then

$$\left|\psi_t\left(\mu_0, E_0^*\right) - \mu\right|, \left|\varphi_t\left(\mu_0, E_0^*\right) - E^*\right| < \varepsilon.$$

Theorem

The state $(1, 0)$ is the unique stable equilibrium.



Phase Diagram

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What if there are many possible attributes?

Economics

- two-sided market
- quality is observable on one side only
- ex-ante investment in quality
- directed search

$\Rightarrow$ unobservable side invests more