

Speculation and Risk Sharing with New Financial Assets

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Abstract

While the traditional view of financial innovation emphasizes the risk sharing role of new financial assets, belief disagreements about these assets naturally lead to speculation, which represents a powerful economic force in the opposite direction. This paper investigates the effect of financial innovation on portfolio risks in an economy when both the risk sharing and the speculation forces are present. Financial assets provide hedging services but they are also subject to speculation because traders do not necessarily agree about their payoffs. I define *the average variance* of traders' net worths as a measure of portfolio risks for this economy, and I decompose it into two components: *the uninsurable variance*, defined as the average variance that would obtain if there were no belief disagreements, and *the speculative variance*, defined as the residual variance that results from speculative trades based on belief disagreements. Financial innovation always decreases the uninsurable variance because new assets increase the possibilities for risk sharing. My main result shows that financial innovation also always increases the speculative variance. This is true even if traders completely agree about the payoffs of new assets. The intuition behind this result is *the hedge-more/bet-more effect*: Traders use new assets to hedge their bets on existing assets, which in turn enables them to place larger bets and take on greater risks. My second set of results concern endogenous financial innovation. When belief disagreements are sufficiently large, a profit seeking market maker introduces assets that *maximize* the average variance among all possible choices, completely disregarding the risk sharing motive for financial innovation. My third set of results concern a calibration of the model in the context of the GDP indexed assets proposed by Athanasoulis and Shiller (2001). When traders have reasonable amounts of belief disagreements, these assets would have the unintended consequence of increasing their consumption risks.

JEL Classification: G1, G11, D53, D61.

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1 Introduction

According to the traditional view of financial innovation, new financial assets facilitate the diversification and the sharing of risks.¹ However, this view does not take into account that new assets are often associated with much uncertainty, especially because they do not have a long track record. Belief disagreements come as a natural by-product of this uncertainty and change the implications of risk taking in these markets. In particular, market participants' disagreements about how to value new assets naturally lead to speculation, which represents a powerful economic force that tends to increase risks.

An example is offered by the recent crisis. Assets backed by pools of subprime mortgages (e.g., subprime CDOs) became highly popular in the run-up to the crisis. One role of these assets is to allocate the risks to market participants who are best able to bear them. The safer tranches are held by investors that are looking for safety (or liquidity), while the riskier tranches are held by financial institutions who are willing to hold these risks at some price. While these assets (and their CDSs) should have served a stabilizing role in theory, they became a major trigger of the crisis in practice, when a fraction of financial institutions realized losses from their positions. Importantly, the same set of assets also generated considerable profits for some market participants,² which suggests that at least some of the trades on these assets were speculative. What becomes of the risk sharing role of new assets when market participants use them to speculate on their different views?

To address this question, this paper analyzes the effect of financial innovation on portfolio risks in a model that features both the risk sharing and the speculation motives for trade. Traders with income risks take positions in a set of financial assets, which enables them to share and diversify some of their background risks. However, traders have belief disagreements about asset payoffs, which induces them to take also speculative positions on assets. I assume traders have mean-variance preferences over net worth. In this setting, a natural measure of portfolio risk for a trader is the variance of her net worth (calculated according to her own beliefs). I define *the average variance* as an average of this risk measure across all traders. I further decompose the average variance into two components: *the uninsurable variance*, defined as the variance that would obtain if there were no belief disagreements, and *the speculative variance*, defined as the residual amount of variance that results from speculative trades based on belief disagreements. I model financial innovation as an expansion of the set of assets available for trade. My main result characterizes the effect of financial innovation on each component of the average variance. In line with the traditional view, financial innovation always decreases the uninsurable variance because new assets increase the possibilities for risk sharing. Theorem 1 shows that financial innovation also always increases the speculative variance. Moreover, there

¹Cochrane (2001) summarizes this view as follows: "Better risk sharing is much of the force behind financial innovation. Many successful new securities can be understood as devices to more widely share risks."

²Lewis (2010) provides a detailed description of investors that took a short position on housing related assets in the run-up to the recent crisis.

exist economies in which this increase in the speculative variance is sufficiently strong that financial innovation increases the average variance (by an arbitrary amount).

My analysis identifies two distinct channels by which financial innovation increases the speculative variance. First, new assets lead to new disagreements because they are associated with new uncertainties. Second, and perhaps more importantly, new assets also amplify speculation on existing disagreements. To illustrate the second channel, Theorem 1 shows that new assets increase the speculative variance even if traders completely agree about their pay-offs. This result is somewhat surprising because traders use new assets to hedge some of the speculative risks they have been undertaking from their bets on existing assets. In view of this direct hedging effect, one could expect new assets (on which there is complete agreement) to reduce the speculative variance. This view does not take into account a powerful amplification mechanism, *the hedge-more/bet-more* effect.

To illustrate this effect, consider the following example. Suppose two traders have differing views about the Swiss Franc, which is highly correlated with the Euro. The optimist believes the Franc will appreciate while the pessimist believes it will depreciate. Traders do not disagree about the Euro, perhaps because they disagree about the prospects of the Swiss economy but not about the Euro zone. First suppose traders can only take positions on the Franc and not the Euro. In this case, traders' positions in the Franc will be determined by a standard risk-return trade-off. Traders may not take too large positions on the Franc especially because the Franc is affected by multiple sources of risks, e.g., the shocks that affect the Swiss economy as well as the shocks to the Euro zone. To bet on their belief differences, traders must bear all of these risks, which might make them reluctant to take large positions. Suppose instead the Euro is also introduced for trade. In this case, traders will complement their positions in the Franc by taking the opposite positions in the Euro. This is because the complementary positions enable traders to hedge the risks that also affect the Euro, which they don't disagree about, and to take purer bets on the Franc. With purer bets, traders bear less risk for each unit position on the Franc, which in turn enables them to take larger positions. Put differently, when traders are able to hedge more, they are induced to bet more. Theorem 1 shows that the hedge-more/bet-more effect is sufficiently strong that the introduction of the Euro in this example (and more generally, a new asset) increases the speculative variance.

Theorem 1 takes the new assets as exogenous and analyzes their impact on portfolio risks. In practice, new financial assets are endogenously introduced by economic agents with profit incentives. A sizeable literature emphasizes risk sharing as a major driving force in endogenous financial innovation [see, for example, Allen and Gale (1994) or Duffie and Rahi (1995)]. A natural question is to what extent the risk sharing motive for financial innovation is robust to the presence of belief disagreements. I address this question by introducing a profit seeking market maker that innovates new assets for which it subsequently serves as the intermediary. The market maker's expected profits are proportional to traders' perceived surplus from trading new assets. Thus, traders' speculative trading motive, as well as their risk sharing motive,

creates innovation incentives for the market maker. In particular, the optimal asset design (characterized in Theorem 2) depends on the size and the nature of belief disagreements, in addition to the risk sharing possibilities. When traders have common beliefs, the market maker innovates assets that minimize the average variance, as in Demange and Laroque (1995) and Athanasoulis and Shiller (2000). In contrast to these traditional results, Theorem 3 also characterizes the polar opposite case: When traders' belief disagreements are sufficiently large, the market maker innovates assets which *maximize* the average variance among all possible choices, completely disregarding the risk sharing motive for financial innovation.

A natural question is how large belief disagreements must be to make these results practically relevant. To address this question, I consider a calibration of the model in the context of the national income markets proposed by Shiller (1993), and analyzed in detail by Athanasoulis and Shiller (2001). Assets whose payoffs are linked to (various combinations of) national incomes could in principle facilitate the sharing of income risks among different countries. Athanasoulis and Shiller (2001) characterize the optimal design of such assets. They also calibrate their model for G7 countries, and argue that the innovation of a couple of these assets would lead to large welfare gains in view of the reduction in individuals' consumption risks. I consider the effect of belief disagreements on their results about consumption risks. Using exactly their data and calibration, I find that reasonable amounts of belief disagreements about the GDP growth rates of G7 countries (much smaller than implied by the Survey of Professional Forecasters) imply that these new assets would actually increase average consumption risks. Intuitively, per-capita income risks in developed countries is small relative to their per-capita incomes. Moreover, income risks are correlated across developed countries. Thus, even if these risks are perfectly diversified, the reduction in the standard deviation of consumption amounts to a relatively small fraction of income. In contrast, with a typical calibration for the relative risk aversion parameter, $\theta^{relative} = 3$, traders are willing to risk a greater fraction of their incomes in their pursuit for speculative gains. Consequently, a small amount of belief disagreements is sufficient to ensure that the increase in the speculative variance dominates the relatively small decrease in uninsurable variance.

The rest of the paper is organized as follows. The next subsection discusses the related literature. Section 2 introduces the basic environment. This section also uses simple examples to illustrate the two channels by which new assets increase traders' portfolio risks. Section 3 characterizes the equilibrium and decomposes traders' portfolio risks into the uninsurable and the speculative variance components. Section 4 presents the main result, which characterizes the effect of financial innovation on these two components. Section 5 analyzes endogenous financial innovation. Section 6 discusses the positive and the normative implications of the results. Section 7 presents the calibration exercise and Section 8 concludes. Appendix A contains the results and proofs omitted from the main text.

1.1 Related Literature

As the above discussion clarifies, my paper belongs to a sizeable literature on financial innovation and security design [see also Van Horne (1985), Miller (1986), Ross (1988), Merton (1989, 1992), Duffie and Jackson (1989), Cuny (1993), Tufano (2003)]. This literature, with the exception of a few recent papers (some of which are discussed below), has not explored the implications of heterogeneous beliefs for security design. For example, in their survey of the literature, Duffie and Rahi (1994) note that “one theme of the literature, going back at least to Working (1953) and evident in the Milgrom and Stokey (1982) no-trade theorem, is that an exchange would rarely find it attractive to introduce a security whose sole justification is the opportunity for speculation.” The results of this paper show that this observation does not apply if traders have *heterogeneous prior beliefs* rather than heterogeneous information. The observation also does not apply if traders have heterogeneous information but security prices do not reveal information fully due to the presence of noise traders. The analogues of my results can be derived for this alternative setting. The important economic ingredient is that traders continue to have some disagreements after observing asset prices. In addition, the quantitative results of this paper suggest that a relatively small amount of belief disagreements of this type is sufficient to ensure that speculation is a significant factor in financial innovation.

My paper is most closely related to the recent work of Brock, Hommes, and Wagener (2009), who also emphasize the destabilizing effects of financial innovation in the presence of belief disagreements. The two papers are complementary in the sense that they use different ingredients, and they focus on different aspects of instability. Brock et al.’s (2009) main ingredient is reinforcement learning: That is, they assume traders choose their beliefs according to a fitness measure, such as past profits made by the belief. They show that, with reinforcement learning, new assets make the steady-state corresponding to the fundamental asset price more likely to be dynamically unstable. In contrast, this paper takes traders’ prior beliefs as given, and establishes a static instability result (increase in speculative variance) regardless of how those beliefs are formed. In particular, my results do not rely on reinforcement learning.

My paper is part of a recent literature which emphasizes that increased trading opportunities might increase portfolio risks when traders have distorted or different beliefs. Weyl (2007) notes that cross-market arbitrage might create additional risks when investors have “mistaken beliefs.” Dieckman (2009) shows that rare-event insurance can increase portfolio risks when traders disagree about the frequency of these events. The contribution of my paper is to systematically characterize the effect of financial innovation on portfolio risks for a general environment with belief disagreements and mean-variance preferences. I show that new assets reduce uninsurable variance, which captures the insights of the traditional literature on financial innovation, but that they also always increase the speculative variance through two distinct channels. I also characterize the endogenous asset design with belief disagreements, and show that it is partly driven by the speculation motive for trade.

My paper also contributes to a literature which analyzes the welfare implications of financial

innovation. A strand of the general equilibrium theory, e.g., Hart (1975) and Elul (1994), has shown that new assets that only partially complete the market may make all agents worse off in view of general equilibrium price effects. Stein (1987) shows that speculation driven by financial innovation can reduce welfare through informational externalities. I abstract away from these channels by focusing on an economy with single good (hence, no relative price effects) and heterogeneous prior beliefs (hence, no information). In particular, the competitive equilibrium in this economy is constrained Pareto efficient if agents' welfare is calculated according to their own subjective beliefs. However, several economists, e.g., Stiglitz (1989), Mongin (1997), Gilboa, Samet, and Schmeidler (2004) and Kreps (2012), have noted that the Pareto criterion might not be appropriate when agents have different beliefs. The key insight is that agents' beliefs are inconsistent with one another, which creates a "collective irrationality" (see Section 6 for details). In recent work, Brunnermeier, Simsek, Xiong (2012) and Gilboa and Schmeidler (2012) propose alternative welfare criteria that could be used in environments with belief disagreements. Applying Brunnermeier, Simsek, Xiong's (2012) criterion in my paper detects financial innovation as inefficient when new assets increase portfolio risks. That said, I do not take a strong normative stance in this paper. Instead, my primary goal is to systematically characterize the positive effects of new assets on portfolio risks.³

My paper is also related to a large literature that analyzes the implications of belief disagreements for asset prices or volume of trade. A very incomplete list includes Miller (1977), Harrison and Kreps (1978), Varian (1985, 1989), Harris and Raviv (1993), Kandel and Pearson (1995), Zapatero (1998), Chen, Hong and Stein (2003), Scheinkman and Xiong (2003), Geanakoplos (2009), Cao (2010), Simsek (2011). The main difference of my paper from this literature is the focus on the effect of belief disagreements on *the riskiness of traders' portfolios*, rather than the riskiness (and the level) of prices or the volume of trade. My model is closest to Lintner (1969), who generalizes the capital asset pricing model to an environment in which beliefs and risk aversion coefficients are heterogeneous. However, Lintner (1969) does not analyze the risk sharing implications of this theory.

2 Basic Environment and Main Channels

Consider an economy with two dates, $\{0, 1\}$, and a single consumption good, which will be referred to as a dollar. There are a finite number of traders denoted by $i \in I = \{1, 2, \dots, |I|\}$. Each trader is endowed with e dollars at date 0, which is constant. Trader i is also endowed

³From the normative point of view, my paper is also related to a recent literature that analyzes the role of financial innovation in generating financial instability. Gennaioli, Shleifer and Vishny (2010) investigate the neglected risks associated with new financial assets. Rajan (2005) and Calomiris (2008) emphasize the effect of financial innovation on agency problems. My paper identifies the increase in speculation (and speculative variance) as an additional channel by which financial innovation decreases financial stability. Posner and Weyl (2012) push the normative implications of speculation further by calling for a regulatory authority, along the lines of the FDA, which approves financial products based on whether they will increase or decrease the portfolio risks. The potentially destabilizing role of speculation is also discussed in Stiglitz (1989), Summers and Summers (1991), and Stout (1995).

with w_i dollars at date 1, which is a random variable that captures the trader's background risks. Traders only consume at date 1, and they can transfer resources to date 1 by investing in one of two ways. They can invest in cash which yields one dollar for each dollar invested. Alternatively, they can invest in risky assets denoted by $j \in J = \{1, \dots, |J|\}$. Asset j is in fixed supply, normalized to zero, and it pays a^j dollars at date 1, which is a random variable. Assets' payoffs and prices are respectively denoted by $|J| \times 1$ column vectors $\mathbf{a} = (a^1, \dots, a^{|J|})'$.

The uncertainty in this economy is captured by an $|m| \times 1$ random vector, $\mathbf{v} = (v_1, \dots, v_m)'$. Traders have potentially heterogeneous prior beliefs about \mathbf{v} . They agree about the variance of \mathbf{v} , which simplifies the analysis, but they might disagree about the mean of \mathbf{v} . In particular: **Assumption (A1)**. Trader i 's belief for the random variable \mathbf{v} is given by the Normal distribution, $N(\boldsymbol{\mu}_i^{\mathbf{v}}, \Lambda^{\mathbf{v}})$, where $\boldsymbol{\mu}_i^{\mathbf{v}} \in \mathbb{R}^m$ is the mean vector and $\Lambda^{\mathbf{v}}$ is an $m \times m$ covariance matrix with full row rank.⁴

Traders' date 1 endowment can be written in terms of \mathbf{v} as:

$$w_i = (\mathbf{W}_i)' \mathbf{v},$$

for an $|m| \times 1$ vector $\mathbf{W}_i \in \mathbb{R}^m$. Asset j 's payoff can also be written in terms of \mathbf{v} as:

$$a^j = (\mathbf{A}^j)' \mathbf{v},$$

for an $|m| \times 1$ vector $\mathbf{A}^j \in \mathbb{R}^m$. The vectors, $\{\mathbf{A}^j\}_j$, are linearly independent, which ensures that the assets are not redundant.

At date 0, the asset is traded in a competitive market at price p^j . Trader i 's position in asset j is denoted by $x_i^j \in \mathbb{R}$. Given the $|J| \times 1$ price vector $\mathbf{p} = (p^1, \dots, p^{|J|})'$, each trader chooses a $|J| \times 1$ asset portfolio, $\mathbf{x}_i = (x_i^1, \dots, x_i^{|J|})'$, and invests the rest of her budget, $e - \mathbf{x}_i' \mathbf{p} \in \mathbb{R}$, in cash.⁵ With these investment decisions, her net worth at date 1 is given by:

$$n_i = e - \mathbf{x}_i' \mathbf{p} + w_i + \mathbf{x}_i' \mathbf{a}. \quad (1)$$

Trader i maximizes subjective expected utility over net worth at date 1. Her utility function takes the CARA form. Since the asset payoffs and endowment shocks are jointly Normally

⁴Note that I have not specified the empirical (or realized) distribution for \mathbf{v} . This distribution does not matter for much of the analysis in this paper. In particular, the mean of the empirical distribution does not play a role in any of the results. This is because the main goal of this paper is to characterize traders' portfolio risks, for which it is not necessary to take a position on who is right on average. In addition, the variance of the empirical distribution plays only a limited role. This is because traders' portfolio risks could be defined by using their *perceived* variance, $\Lambda^{\mathbf{v}}$, without reference to the empirical variance. This is the approach that will be taken in the main text. Appendix A.1 generalizes the main results to the case in which portfolio risks are defined with the empirical variance.

⁵Note that traders are allowed to take unrestricted negative positions in risky assets or cash, that is, both short selling and leverage are allowed. Similarly, the asset payoffs can take negative values because the environment is frictionless. In particular, there is no limited liability and repayment is enforced by contracts.

distributed, the trader's optimization reduces to the usual mean-variance problem:⁶

$$\max_{\mathbf{x}_i} E_i [n_i] - \frac{\theta_i}{2} \text{var}_i [n_i]. \quad (2)$$

Here, θ_i denotes the trader's absolute risk aversion coefficient, while $E_i [\cdot]$ and $\text{var}_i [\cdot]$ respectively denote the mean and the variance of the trader's portfolio according to her beliefs.

The equilibrium in this economy is a collection of asset prices, \mathbf{p} , and portfolios, $(\mathbf{x}_1, \dots, \mathbf{x}_{|I|})$, such that each trader i chooses her portfolio to solve problem (2) and prices clear asset markets, that is,

$$\sum_i x_i^j = 0 \text{ for each } j \in J.$$

I will capture financial innovation in this economy as an expansion of the set of traded assets, J . Before I turn to the general characterization, I use a simple example to illustrate the effects of financial innovation on portfolio risks.

2.1 An illustrative example

Suppose there are two traders with the same coefficient of risk aversion, i.e., $I = \{1, 2\}$ and $\theta_1 = \theta_2 \equiv \theta$. The fundamental risk is captured by 2 uncorrelated random variables, v_1, v_2 . Traders' date 1 endowments are perfectly negatively correlated:

$$w_1 = v \text{ and } w_2 = -v, \text{ where } v = v_1 + \alpha v_2.$$

In particular, traders' endowments are affected by both both sources of fundamental risk, with the weight, α , capturing the relative importance of the second risk.

As a benchmark suppose traders have common beliefs about v_1 and v_2 given by $N(0, 1)$. In this benchmark, first consider the case in which there are no assets, i.e., $J = \emptyset$. In this case, there is no trade and traders' net worths are given by:

$$n_1 = e + v \text{ and } n_2 = e - v. \quad (3)$$

Traders' net worths are risky because they are unable to hedge their endowment risks. Next suppose a new asset is introduced with payoff

$$a^1 = v = v_1 + \alpha v_2.$$

⁶The only role of the CARA preferences and the Normality assumption is to generate the mean-variance optimization in (2). In particular, the results of this paper apply as long as traders' portfolio choice can be reduced to the form in (2) over net worth. An important special case is the continuous-time model in which traders have time-separable expected utility preferences (which are not necessarily CARA), and asset returns and background risks follow diffusion processes. In this case, the optimization problem of a trader at any date can be reduced to the form in (2) (see Ingersoll, 1987). The only caveat is that the reduced form coefficient of absolute risk aversion, θ_i , is endogenous since it depends on the trader's value function. Thus, in the continuous trading environment, the results of this paper apply at a trading date conditional on traders' coefficients of absolute risk aversion, $\{\theta_i\}_i$.

In this case, traders' equilibrium portfolios are given by:

$$x_1^1 = -1 \text{ and } x_2^1 = 1$$

(and the equilibrium price is $p^1 = 0$). Traders' net worths are constant and given by

$$n_1 = n_2 = e.$$

Thus, the benchmark analysis shows that, with common beliefs, financial innovation enables traders to hedge and diversify their idiosyncratic risks.

Next suppose traders have heterogeneous prior beliefs for some of the fundamental risk in this economy. In particular, traders have common beliefs for v_2 given by the distribution, $N(0, 1)$. They also know that v_1 and v_2 are uncorrelated. However, they disagree about the distribution of v_1 . Trader 1's prior belief for v_1 is given by $N(\varepsilon, 1)$ while trader 2's belief is given by $N(-\varepsilon, 1)$. The parameter ε captures the level of belief disagreements. I next use this specification to illustrate the two channels by which new assets increase portfolio risks.

Channel 1: New assets generate new disagreements

Consider the case in which asset 1 is available for trade. Since traders disagree about the mean of v_1 , they also disagree about the mean of the asset payoff, $a^1 = v_1 + \alpha v_2$. In this case, it is easy to check that the asset price is $p^1 = 0$ (by symmetry) and that traders' portfolios are:

$$\begin{aligned} x_1^1 &= -1 + x_1^S \text{ and } x_2^1 = 1 + x_2^S, \\ \text{where } x_1^S &= \frac{\varepsilon}{\theta} \frac{1}{1 + \alpha^2} \text{ and } x_2^S = -\frac{\varepsilon}{\theta} \frac{1}{1 + \alpha^2}. \end{aligned} \quad (4)$$

Note that traders' positions deviate from the optimal risk sharing benchmark in view of their belief disagreements. I define the difference as the traders' *speculative portfolios* and denote it by $\{x_i^S\}_i$. Traders' net worths can also be calculated as:

$$n_1 = e + \frac{\varepsilon}{\theta} \frac{v_1 + \alpha v_2}{1 + \alpha^2} \text{ and } n_2 = e - \frac{\varepsilon}{\theta} \frac{v_1 + \alpha v_2}{1 + \alpha^2}. \quad (5)$$

If $\varepsilon > \theta(1 + \alpha^2)$, then traders' net worths are riskier than the case in which no new asset is introduced [cf. Eq. (3)]. Intuitively, trader 1 is so optimistic about the asset's payoff that she takes a positive net position, despite the fact that her endowment covaries positively with the asset payoff. Consequently, the new asset increases the riskiness of her net worth. Hence, when traders' disagreements about the asset payoff are sufficiently large, financial innovation increases traders' portfolio risks.

Channel 2: New assets amplify existing disagreements

Next consider the introduction of a second asset with payoff:

$$a^2 = v_2.$$

Note that traders do not disagree on the payoff of this new asset. Nonetheless, this asset also increases traders' portfolio risks through a second channel: By amplifying traders' speculation on existing disagreements. To see this, first consider traders' equilibrium portfolios in this case which can be calculated as:

$$\begin{bmatrix} x_1^1 \\ x_1^2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \begin{bmatrix} x_1^{1,S} \equiv \frac{\varepsilon}{\theta} \\ x_1^{2,S} \equiv -\alpha \frac{\varepsilon}{\theta} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x_2^1 \\ x_2^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} x_2^{1,S} \equiv -\frac{\varepsilon}{\theta} \\ x_2^{2,S} \equiv \alpha \frac{\varepsilon}{\theta} \end{bmatrix}, \quad (6)$$

As before, traders' portfolios feature speculative positions, $\left\{ \begin{bmatrix} x_i^{1,S} \\ x_i^{2,S} \end{bmatrix} \right\}_i$, which represent the deviations from the optimal risk sharing benchmark. Given these positions, traders' net worths are given by:

$$n_1 = e + \frac{\varepsilon}{\theta} v_1 \quad \text{and} \quad n_2 = e - \frac{\varepsilon}{\theta} v_1. \quad (7)$$

Note that the magnitude of traders' speculative positions on asset 1 is greater than the earlier setting in which asset 2 was not available [cf. Eqs. (6) and (4)]. Importantly, traders' net worths are also riskier [cf. Eqs. (7) and (5)]. Put differently, the innovation of asset 2, about which traders do not disagree, enables traders to take greater speculative positions on asset 1 and increases their portfolio risks.

The intuition for this result is related to an important economic force: the *hedge-more/bet-more* effect. When only asset 1 is available, traders' speculative positions and portfolio risks are decreasing in α [cf. Eqs. (4) and (5)]. Intuitively, asset 1 provides the traders with only an *impure bet* because its payoff also depends on the risk, v_2 , on which traders do not disagree. To take speculative positions, traders must also hold these additional risks, which makes them reluctant to bet. When asset 2 is also available, traders complement their speculative positions in asset 1 by taking the opposite positions in asset 2 [Eq. (6)]. This enables them to take a purer bet on the risk, v_1 . When traders are able to take purer bets, they also take larger bets, which in turn leads to greater portfolio risks [Eq. (5)].

3 Equilibrium and the Decomposition of Average Variance

This section characterizes the equilibrium, and decomposes traders' portfolio risks into two components which respectively correspond to traders' risk sharing and speculative motives for trade. The main result in the next section characterizes the effect of financial innovation on the two components of portfolio risks.

Given assumption (A1), trader i believes the asset payoffs are Normally distributed,

$N(\boldsymbol{\mu}_i, \Lambda)$, with

$$\boldsymbol{\mu}_i \equiv \mathbf{A}' \boldsymbol{\mu}_i^y \text{ and } \Lambda \equiv \mathbf{A}' \Lambda^y \mathbf{A},$$

where $\boldsymbol{\mu}_i$ is a $|J| \times 1$ vector and Λ is a $|J| \times |J|$ matrix. In addition, trader i believes that her endowment is Normally distributed, and that the covariance of her endowment with the asset payoffs is given by:

$$\boldsymbol{\lambda}_i = \mathbf{A}' \Lambda^y \mathbf{W}_i,$$

where $\boldsymbol{\lambda}_i$ is a $|J| \times 1$ matrix. Given these beliefs, traders' portfolio demand [cf. problem (2)] can be solved in closed form. Aggregating traders' demands and using market clearing, asset prices are given by:

$$\mathbf{p} = \frac{1}{|I|} \sum_{i \in I} \left(\frac{\bar{\theta}}{\theta_i} \boldsymbol{\mu}_i - \bar{\theta} \boldsymbol{\lambda}_i \right), \quad (8)$$

where $\bar{\theta} \equiv (\sum_{i \in I} \theta^{-1} / |I|)^{-1}$ is the Harmonic mean of traders' absolute risk aversion coefficients. Intuitively, an asset commands a higher price if traders are on average optimistic about its payoff, or if it on average covaries negatively with traders' endowments.

Using the price expression (8), a trader's equilibrium portfolio can also be solved as:

$$\begin{aligned} \mathbf{x}_i &= \mathbf{x}_i^R + \mathbf{x}_i^S, \quad \text{where} \\ \mathbf{x}_i^R &= -\Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \quad \text{and} \quad \mathbf{x}_i^S = \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}. \end{aligned} \quad (9)$$

Here, the expression

$$\tilde{\boldsymbol{\lambda}}_i = \boldsymbol{\lambda}_i - \frac{\bar{\theta}}{\theta_i} \frac{1}{|I|} \sum_{\bar{i} \in I} \boldsymbol{\lambda}_{\bar{i}} \quad (10)$$

denotes the relative covariance of the trader's endowment, and

$$\tilde{\boldsymbol{\mu}}_i = \boldsymbol{\mu}_i - \frac{1}{|I|} \sum_{\bar{i} \in I} \frac{\bar{\theta}}{\theta_{\bar{i}}} \boldsymbol{\mu}_{\bar{i}} \quad (11)$$

denotes her relative optimism. Note that the trader's portfolio decomposes into two components. The first component, \mathbf{x}_i^R , is the portfolio that would obtain if there were no belief belief disagreements (i.e., if $\tilde{\boldsymbol{\mu}}_i = 0$ for each i). Hence, I refer to \mathbf{x}_i^R as the trader's *risk sharing portfolio*. The optimal risk sharing portfolio is determined by traders' endowment risks and their risk tolerances. The second component, \mathbf{x}_i^S , captures traders' deviations from this benchmark in view of their belief disagreements, $\tilde{\boldsymbol{\mu}}_i$. Hence, I refer to \mathbf{x}_i^S as the *speculative portfolio* of trader i .

Eqs. (8) – (11) complete the characterization of equilibrium in this economy. The main goal of this paper is to analyze the effect of financial innovation on portfolio risks. Given the mean-variance framework, a natural measure of portfolio risk for a trader i is the variance of

her net worth, $var_i(n_i)$. I consider an average of this measure across all traders, *the average variance*, defined as follows:

$$\Omega = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} var_i(n_i) = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} (\mathbf{W}'_i \Lambda^v \mathbf{W}_i + 2\mathbf{x}'_i \boldsymbol{\lambda}_i + \mathbf{x}'_i \Lambda \mathbf{x}_i). \quad (12)$$

A couple comments about this definition are in order. First, the portfolio risk of a trader is calculated according to traders' (common) belief for the variance Λ^v . Appendix A.1 generalizes the main results to the case in which portfolio risks are defined with the empirical variance (that is, the variance that would be reflected ex-post in the data). Second, note that traders that are relatively more risk averse are given a greater weight in the average, Ω .

I use Ω as my main measure of average portfolio risks for two reasons. First, Section 6 shows that Ω is a natural measure of welfare in this economy (although it is not equivalent to welfare according to the Pareto criterion). The second justification is provided by the following lemma.

Lemma 1. *The risk sharing portfolios, $\{\mathbf{x}_i^R\}_i$, minimize the average variance, Ω , among all feasible portfolios:*

$$\min_{\{\mathbf{x}_i \in \mathbb{R}^{|J|}\}_i} \Omega \quad s.t. \quad \sum_i \mathbf{x}_i = 0. \quad (13)$$

When there are no belief disagreements, i.e., $\tilde{\boldsymbol{\mu}}_i^y = 0$ for each i , then the complete portfolios and the risk sharing portfolios coincide, i.e., $\mathbf{x}_i = \mathbf{x}_i^R$ for each i . Thus, Lemma 1 shows that Ω is the measure of risks that would be minimized in equilibrium absent belief disagreements. Thus, it is natural to take Ω as the measure of average risks, and to characterize the extent to which it deviates from the minimum benchmark in (13) when traders have belief disagreements.

To this end, I let Ω^R denote the minimum value of problem (13) and refer to it as the *uninsurable variance*. I also define $\Omega^S = \Omega - \Omega^R$ and refer to it as the *speculative variance*. This provides a decomposition of the average variance into two components:

$$\Omega = \Omega^R + \Omega^S.$$

The main result in the next section concerns the effect of financial innovation on Ω^R and Ω^S . The next lemma characterizes the two components of average variance in terms of the exogenous parameters of the model. The forms of Ω^R and Ω^S are intuitive. Eq. (14) illustrates that the uninsurable variance is lower when the assets provide better risk sharing opportunities, captured by larger $\tilde{\boldsymbol{\lambda}}_i$. Similarly, Eq. (15) illustrates that the speculative variance is greater when the assets feature greater belief disagreements, captured by larger $\tilde{\boldsymbol{\mu}}_i$.

Lemma 2. *The uninsurable variance is given by:*

$$\Omega^R = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} (\mathbf{W}'_i \Lambda^v \mathbf{W}_i - \tilde{\boldsymbol{\lambda}}'_i \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i), \quad (14)$$

and the speculative variance is given by:

$$\Omega^S \equiv \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} \left(\frac{\tilde{\boldsymbol{\mu}}_i'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} \right). \quad (15)$$

4 Financial Innovation and Portfolio Risks

I model financial innovation as an expansion of the set of traded assets. For this purpose, it is useful to define the notation, $z(\hat{J})$, to refer to the equilibrium variable z when only a subset $\hat{J} \subset J$ of the assets in J are traded. I next present the main result.

Theorem 1 (Financial Innovation and Portfolio Risks). *Suppose J consists of a set of old assets, J_O , and a set of new assets, J_N (formally, $J = J_O \cup J_N$ where J_O and J_N are disjoint sets).*

(i) *Financial innovation always reduces the uninsurable variance, that is:*

$$\Omega^R(J_O \cup J_N) \leq \Omega^R(J_O).$$

(ii) *Financial innovation always increases the speculative variance, that is:*

$$\Omega^S(J_O \cup J_N) \geq \Omega^S(J_O).$$

The first part of this theorem is a corollary of Lemma 1, and it shows that financial innovation always provides some risk sharing benefits. This part formalizes the traditional view of financial innovation in the context of this model. On the other hand, the second part of the theorem identifies a second force which always operates in the opposite direction. In particular, when there are belief disagreements, financial innovation also always increases the speculative variance. Hence, the net effect of financial innovation on average variance is ambiguous, and it depends on the relative strength of the two forces. Furthermore, it is easy to see that there exist economies in which the increase in the speculative variance is sufficiently large that financial innovation increases the average variance (by an arbitrary amount).

Most of the literature on financial innovation considers the special case without belief disagreements. Theorem 1 shows that the common-beliefs assumption is restrictive, as it shuts down an important economic force by which financial innovation has a positive effect on portfolio risks. It is also worth emphasizing the generality of Theorem 1. The result applies for all sets of existing and new assets, J_O and J_N , with no restrictions on the joint distribution of asset payoffs or traders' beliefs for \mathbf{v} [except for the relatively mild Assumption (A1)]. For example, Theorem 1 shows that financial innovation increases the speculative variance even if there are no belief disagreements about new assets (as in Example 1).

The rest of this section provides a sketch proof and a complementary intuition for the second part of Theorem 1. The proof proceeds in three steps. First, the form of \mathbf{x}_i^S in Eq.

(9) implies that the speculative portfolio, \mathbf{x}_i^S , solves the following version of the mean-variance problem:

$$\max_{\mathbf{x}_i \in \mathbb{R}^J} (\tilde{\boldsymbol{\mu}}_i)' \mathbf{x}_i - \frac{\theta_i}{2} \mathbf{x}_i' \Lambda \mathbf{x}_i. \quad (16)$$

Moreover, the speculative variance, Ω^S , is found by averaging the variance costs for each trader at the solution to this problem:

$$\Omega^S = \frac{1}{|I|} \sum_i \frac{\theta_i}{\theta} (\mathbf{x}_i^S)' \Lambda \mathbf{x}_i^S. \quad (17)$$

Intuitively, problem (16) is the traders' mean-variance problem in a hypothetical economy that is identical except that traders have no background risks (i.e., $\mathbf{W}_i = 0$ for all $i \in I$), so that the only motive for trade is speculation. The solution to this problem gives the speculative portfolio in the actual economy, and also determines the speculative variance as captured by (17).

Second, financial innovation relaxes the constraint set of problem (16). That is, when the asset set is $\hat{J} = J^O \cup J^N$, traders are able to make all the speculative trades they could make when the asset set is $\hat{J} = J^O$, and some more. Put differently, new assets expand the “betting possibilities frontier” for traders. This observation reinforces the insights of Example 1. In particular, financial innovation increases the betting possibilities frontier through two distinct channels: By creating new disagreements, and by enabling traders to take purer bets on existing disagreements.

Third, when the constraint set of problem (16) is more relaxed, each trader obtains a greater certainty-equivalent payoff from betting. Moreover, since the problem is a quadratic optimization, expected payoffs at the optimum are proportional to the expected variance of the payoffs, that is: $(\tilde{\boldsymbol{\mu}}_i)' \mathbf{x}_i^s = 2 \frac{\theta_i}{2} (\mathbf{x}_i^s)' \Lambda \mathbf{x}_i^s$. Consequently, a relaxed constraint set also increases the expected variance, $\frac{\theta_i}{2} (\mathbf{x}_i^s)' \Lambda \mathbf{x}_i^s$. Intuitively, at the optimal speculative portfolio, higher expected returns go hand-in-hand with higher risks. These steps establish that financial innovation increases the speculative variance of each trader. It follows that financial innovation also increases the average speculative variance in Eq. (17), completing the proof (see Appendix A.3 for an alternative proof based on matrix algebra).

A complementary intuition for Theorem 1 can be provided by characterizing traders' speculative risks in terms of the Sharpe ratios of their speculative portfolios. To this end, consider the hypothetical economy in the above proof in which there are no background risks. Define the *speculative Sharpe ratio* of a trader as the Sharpe ratio of her portfolio in this hypothetical

economy.⁷ Using Eqs. (8) – (11), this ratio can be calculated as:

$$Sharpe_i^S = \frac{(\mathbf{x}_i^S)'(\boldsymbol{\mu}_i - \mathbf{p})}{\sqrt{(\mathbf{x}_i^S)' \Lambda \mathbf{x}_i^S}} = \sqrt{\tilde{\boldsymbol{\mu}}_i' \Lambda^{-1} \tilde{\boldsymbol{\mu}}_i}. \quad (18)$$

Next consider the trader's *portfolio return* given by n_i/e (where recall that e is the trader's initial net worth). The standard deviation of this return can also be calculated as:

$$\sigma_i^S \equiv \frac{1}{e} \sqrt{(\mathbf{x}_i^S)' \Lambda \mathbf{x}_i^S} = \frac{1}{\theta_i e} \sqrt{\tilde{\boldsymbol{\mu}}_i' \Lambda^{-1} \tilde{\boldsymbol{\mu}}_i}. \quad (19)$$

Note that the ratio, $\theta_i e$, provides a measure of trader i 's coefficient of relative risk aversion. Thus, combining Eqs. (18) and (19) gives the familiar result that the standard deviation of the portfolio return is equal to the Sharpe ratio of the optimal portfolio divided by the coefficient of relative risk aversion (see Campbell and Viceira, 2002). This textbook result applies also in this model when there are no background risks.

Theorem 1 can then be understood from the lenses of Eqs. (18) and (19). Financial innovation increases the traders' speculative Sharpe ratios by expanding the betting possibilities frontier through the two distinct channels emphasized before. Once traders are able to obtain higher Sharpe ratios, they also undertake greater speculative risks, providing a complementary intuition for the main result.

5 Endogenous Financial Innovation

The analysis so far has taken the set of new assets as exogenous. In practice, many financial products are introduced endogenously by economic agents with profit incentives. A large literature emphasizes risk sharing as a major driving force for endogenous financial innovation [see, for example, Allen and Gale (1994), Duffie and Rahi (1995), Demange and Laroque (1995), Athanasoulis and Shiller (2000, 2001)].⁸ A natural question, in view of the results in the earlier sections, is to what extent the risk sharing motive for financial innovation is robust to the presence of belief disagreements. To address this question, this section endogenizes the asset design by introducing a profit seeking market maker and obtains two main results. First, the optimal asset design depends on the size and the nature of traders' belief disagreements, in addition to the possibilities for risk sharing. Second, when traders' belief disagreements are sufficiently large, the market maker designs assets that *maximize* traders' average portfolio risks among all possible choices, completely disregarding the risk sharing motive for financial

⁷Recall that the Sharpe ratio of a portfolio is defined as the expected portfolio return in excess of the risk-free rate (which is normalized to 0 in this model) divided by the standard deviation of the portfolio return.

⁸Risk sharing is one of several drivers of financial innovation emphasized by the previous literature. Other factors include mitigating agency frictions, reducing asymmetric information, minimizing transaction costs, and sidestepping taxes and regulation (see Tufano, 2004, for a recent survey). These other factors, while clearly important, are left out of the analysis in this paper to focus on the effect of belief disagreements on the risk sharing motive for innovation.

innovation.

The main feature of the model in this section is that the assets, J , are introduced by a market maker. The market maker is constrained to choose $|J| \leq m$ assets, but is otherwise free to choose the asset design, \mathbf{A} . Here, recall that the matrix, $\mathbf{A} = [\mathbf{A}^1, \mathbf{A}^2, \dots, \mathbf{A}^{|J|}]$, captures the asset payoffs which are given by $a^j = (\mathbf{A}^j)' \mathbf{v}$ for each j . Thus, the market maker's choice of \mathbf{A} affects the belief disagreements and the relative covariances according to [cf. Eqs. (11) and (10)]:

$$\tilde{\boldsymbol{\mu}}_i(\mathbf{A}) = \mathbf{A}' \tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} \text{ and } \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) = \mathbf{A}' \Lambda^{\mathbf{v}} \tilde{\mathbf{W}}_i,$$

where the deviation terms are defined as:

$$\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} = \boldsymbol{\mu}_i^{\mathbf{v}} - \frac{1}{|I|} \sum_{\bar{i} \in I} \frac{\bar{\theta}}{\theta_{\bar{i}}} \boldsymbol{\mu}_{\bar{i}}^{\mathbf{v}} \text{ and } \tilde{\mathbf{W}}_i = \mathbf{W}_i - \frac{\bar{\theta}}{\theta_i} \frac{1}{|I|} \sum_{\bar{i} \in I} \mathbf{W}_{\bar{i}}.$$

Once the market maker chooses the asset design, \mathbf{A} , the assets are traded in a competitive market similar to the previous sections. The market maker intermediates these trades which enables it to extract some of the surplus from traders. To keep the analysis simple, suppose the market maker can extract the full surplus.⁹ In particular, the market maker sets a fixed membership fee, π_i , for each trader i and makes a take it or leave it offer. If trader i accepts the offer, then she can trade the available assets in the competitive market. Otherwise, trader i is out of the market, and her net worth is given by her endowment, $e + \mathbf{W}'_i \mathbf{v}$.

The equilibrium of this economy can be characterized backwards. First consider the competitive equilibrium after the market maker has chosen \mathbf{A} and traders decided whether or not to participate in the market. Assume that all traders have accepted the offer, which will be the case in equilibrium. In view of the mean-variance framework, traders' portfolio choices are independent of the fixed fees they have paid. In particular, the equilibrium is characterized as in the earlier sections.

Next consider the fixed fees the market maker charges for a given choice of \mathbf{A} . If trader i rejects the offer, she receives the certainty equivalent payoff from her endowment. Otherwise, she receives the certainty equivalent payoff from her equilibrium portfolio net of the fixed fee, $\pi_i(\mathbf{A})$. The market maker sets $\pi_i(\mathbf{A})$ so that the trader is just indifferent to accept the offer. Straightforward calculations (relegated to Appendix A.4) show that the market maker's expected total profits are given by:

$$\sum_{i \in I} \pi_i(\mathbf{A}) = \sum_i \frac{\theta_i}{2} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right). \quad (20)$$

This expression reflects the two motives for trade in this economy. Traders are willing to pay

⁹The results below remain unchanged under the less extreme (reduced form) assumption that the market maker extracts a constant fraction, $\zeta \in (0, 1]$, of the surplus regardless of the choice of \mathbf{A} .

to trade assets that facilitate better risk sharing [i.e., larger $\tilde{\lambda}_i(\mathbf{A})$], or to trade assets that generate greater belief disagreements [i.e., larger $\tilde{\mu}_i(\mathbf{A})$].

The market maker chooses an asset design, \mathbf{A} , that maximizes the expected profits in (20). Note that many choices of \mathbf{A} represent the same trading opportunities over the space of the underlying risks, \mathbf{v} (and thus, also generate the same profits). Thus, suppose without loss of generality that the market maker's choice is subject to the following normalizations:

$$\Lambda = \mathbf{A}'\Lambda^{\mathbf{v}}\mathbf{A} = \mathbf{I}_{|J|}, \quad \text{and} \quad \left((\Lambda^{\mathbf{v}})^{1/2} \mathbf{A} \right)_1^j \geq 0 \text{ for each } j \in J. \quad (21)$$

Here, $(\Lambda^{\mathbf{v}})^{1/2}$ denotes the unique positive definite square root of the matrix, $\Lambda^{\mathbf{v}}$. The first condition in (21) normalizes the variance of assets to be the identity matrix, $\mathbf{I}_{|J|}$. This condition determines the column vectors of the matrix for normalized asset payoffs, $(\Lambda^{\mathbf{v}})^{1/2} \mathbf{A}$, up to a sign. The second condition resolves the remaining indeterminacy by adopting a sign convention for these column vectors.

Theorem 2 (Optimal Asset Design). *Suppose the matrix*

$$\frac{1}{|I|} \sum_i \frac{\theta_i}{\theta} \left((\Lambda^{\mathbf{v}})^{-1/2} \frac{\tilde{\mu}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2} \tilde{\mathbf{W}}_i \right) \left((\Lambda^{\mathbf{v}})^{-1/2} \frac{\tilde{\mu}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2} \tilde{\mathbf{W}}_i \right)' \quad (22)$$

is non-singular. Then, an asset design is optimal if and only if the columns of the matrix for normalized asset payoffs, $(\Lambda^{\mathbf{v}})^{1/2} \mathbf{A}$, correspond to the eigenvectors corresponding to the $|J|$ largest eigenvalues of the matrix in (22). If the eigenvalues are distinct, then the asset design is uniquely determined by this condition along with the normalizations in (21). Otherwise, the asset design is determined up to a choice of the $|J|$ largest eigenvalues.

This result generalizes the results in Demange and Laroque (1995) and Athanassoulis and Shiller (2000) to the case with belief disagreements, $\tilde{\mu}_i^{\mathbf{v}} \neq \mathbf{0}$. Importantly, the expressions (20) and (22) show that financial innovation is partly driven by the size and the nature of traders' belief disagreements. The size of the belief disagreements, $\left\| (\Lambda^{\mathbf{v}})^{-1/2} \tilde{\mu}_i^{\mathbf{v}} \right\|$, (along with the risk aversion coefficients, θ_i) determine to what extent endogenous innovation is driven by the speculation motive for trade as opposed to risk sharing. Assuming that this term is significant, the nature of the belief disagreements, $\frac{(\Lambda^{\mathbf{v}})^{-1/2} \tilde{\mu}_i^{\mathbf{v}}}{\left\| (\Lambda^{\mathbf{v}})^{-1/2} \tilde{\mu}_i^{\mathbf{v}} \right\|}$, bias the choice of assets towards those that maximize the opportunities for speculation.

The next result characterizes the optimal asset design further in two extreme cases: when traders have common beliefs, and when their belief disagreements are very large.

Theorem 3 (Optimal Asset Design and Portfolio Risks). *Consider a collection of economies which are identical except for beliefs given by $\mu_{i,K}^{\mathbf{v}} = K\mu_i^{\mathbf{v}}$ for all i , where $K \geq 0$ is a parameter that scales belief disagreements. Suppose the matrix in (22) is non-singular with distinct eigenvalues for each K .¹⁰ Let $\Omega_K(\cdot)$ denote the average variance and \mathbf{A}_K denote the*

¹⁰The assumption of distinct eigenvalues can be relaxed at the expense of additional notation.

optimal asset design (characterized in Theorem 2) for each K .

(i) With no belief disagreements, i.e., $K = 0$, the market maker innovates assets that minimize the average variance:

$$\mathbf{A}_0 \in \arg \min_{\hat{\mathbf{A}}} \Omega_0 \left(\hat{\mathbf{A}} \right) \text{ subject to (21).}$$

For the next part, suppose there exists at least two traders with different beliefs, i.e., $\boldsymbol{\mu}_i^{\mathbf{y}} \neq \boldsymbol{\mu}_{\bar{i}}^{\mathbf{y}}$ for some $i, \bar{i} \in I$. Let $\Omega_K(\emptyset)$ denote the average variance without any assets.

(ii) As $K \rightarrow \infty$, the market maker innovates assets that maximize the average variance. In particular, the limits $\lim_{K \rightarrow \infty} \mathbf{A}_K$ and $\lim_{K \rightarrow \infty} \frac{1}{K^2} \Omega_K \left(\hat{\mathbf{A}} \right)$ exist and are finite, and they satisfy:¹¹

$$\lim_{K \rightarrow \infty} \mathbf{A}_K \in \arg \max_{\hat{\mathbf{A}}} \left(\lim_{K \rightarrow \infty} \frac{1}{K^2} \Omega_K \left(\hat{\mathbf{A}} \right) \right) \text{ subject to (21).} \quad (23)$$

Without belief disagreements, the market maker innovates assets that minimize average portfolio risks in this economy, as illustrated by the first part of the theorem. The second part provides a sharp contrast to this traditional view. When traders' belief disagreements are large, the market maker innovates assets that *maximize* average portfolio risks, completely disregarding the risk sharing motive for innovation. Thus, belief disagreements change the nature of financial innovation as well as its impact on portfolio risks. This result complements the main result, Theorem 1, by identifying sufficient conditions under which assets that increase portfolio risks endogenously emerge in this economy.

6 Discussion and Welfare Implications

While Theorems 1 and 3 show that financial innovation may increase portfolio risks, they do not reach any welfare conclusions. In fact, it is easy to see that financial innovation in this economy results in a Pareto improvement if traders' welfare is calculated according to their own beliefs. This is because each trader perceives a large expected return from her speculative positions in new assets, which justifies the additional risks that she is taking. This observation naturally raises concerns about the significance of Theorems 1 and 3. This section preempts these concerns, and clarifies the positive and the normative content of these results.

Even absent any normative considerations, Theorems 1 and 3 are important because of their positive implications. A large literature in financial innovation has argued that financial assets improve welfare *because they facilitate risk sharing* (see, for example, Allen and Gale, 1994, or Duffie and Rahi, 1995). Theorem 1 shows that, while new assets might improve welfare, they might do so for quite different reasons than emphasized in the previous literature. In particular, when belief disagreements are sufficiently large, traders' welfare gains do not come from a

¹¹The scale factor, $\frac{1}{K^2}$, in (23) ensures that the limit $\lim_{K \rightarrow \infty} \frac{1}{K^2} \Omega_K \left(\hat{\mathbf{A}} \right)$ is finite, so that the optimization problem is well defined.

decrease in their risks, but from an increase in their perceived expected returns. Importantly, this analysis also generates a new testable implication: The introduction of new assets will increase traders' average portfolio risks as long as traders' belief disagreements are sufficiently large. Theorem 3 generates a further testable implication that profit seeking market makers might introduce new assets that will increase risks, as opposed to assets that will facilitate risk sharing. Among other things, these results might provide an explanation for why most of the macro futures markets proposed by Shiller (1993) have not been adopted in practice, despite the fact that they are in principle very useful for risk sharing purposes.

Theorems 1 and 3 might also have a normative content depending on the interpretation of belief disagreements. There are two distinct ways in which traders might come to have different (prior) beliefs. On the one hand, these differences might reflect traders' different subjective prior beliefs as in Savage (1954). Under this interpretation, traders' beliefs are convenient representations of their preferences under uncertainty. The fact that they have different beliefs is simply a reflection of their differing personal experiences (and possibly also their limited past opportunities to learn). Consequently, the Pareto criterion is arguably the right criterion despite the fact that traders have different beliefs. On the other hand, belief differences might also emerge from overconfidence (which is commonly observed in experimental studies) or other psychological biases. Under this second interpretation, traders' belief disagreements represent *belief distortions*, and the Pareto criterion is arguably no longer appropriate. While all traders expect to receive a high perceived return, at most one of these expectations can be correct. Put differently, traders' beliefs under the second interpretation represent a "collective irrationality."¹² The welfare criterion should ideally correct for this irrationality. However, there is a practical problem because the planner might not know the nature of traders' belief distortions. In particular, it is not clear which trader's belief the planner should use to evaluate welfare.

It is perhaps fortunate that speculation is inefficient *regardless of whose belief one uses to evaluate welfare* (as long as that same belief is consistently applied to calculate each trader's welfare). In recent work, Brunnermeier, Simsek, and Xiong (2012) propose an alternative welfare criterion which is specifically designed to capture this aspect of speculation. They say that an allocation is *belief-neutral Pareto inefficient* if it is Pareto inefficient according to any convex combination of agents' beliefs. To apply this criterion in this model, let subscript $\mathbf{h} = (h_1, \dots, h_{|I|})$, with $h_i \geq 0$ and $\sum_i h_i = 1$, denote an arbitrary convex combination of agents' beliefs. In particular, the distribution of \mathbf{v} according to belief \mathbf{h} is given by $N(\boldsymbol{\mu}_{\mathbf{h}}^{\mathbf{v}}, \Lambda^{\mathbf{v}})$, where $\boldsymbol{\mu}_{\mathbf{h}}^{\mathbf{v}} = \sum_i h_i \boldsymbol{\mu}_i^{\mathbf{v}}$. Consider the sum of traders' certainty equivalent net worths under belief \mathbf{h} ,

¹²The second interpretation, and the associated collective irrationality, is also supported by a large body of empirical evidence which shows that excessive trading significantly worsens portfolio performance of non-institutional investors, e.g., Odean (1999), Barber and Odean (2000), Barber, Lee, Liu, and Odean (2009). To give one example, Barber and Odean (2000) analyze a large sample of brokerage accounts held by households in the US, and show that households that trade the most frequently underperform the market return by more than 5% per year.

given by:

$$N_{\mathbf{h}} = \sum_{i \in I} \left(E_{\mathbf{h}} [n_i] - \frac{\theta_i}{2} \text{var}_{\mathbf{h}} (n_i) \right).$$

Note that this expression is a measure of welfare under belief \mathbf{h} , in the sense that any allocation $\tilde{\mathbf{x}}$ that yields a higher $N_{\mathbf{h}}$ than another allocation \mathbf{x} can also be made to Pareto dominate allocation \mathbf{x} (under belief \mathbf{h}) after combining it with appropriate ex-ante transfers. Using the expressions (1) and (12), along with the market clearing condition $\sum_{i \in I} \mathbf{x}_i = 0$, this welfare measure can further be simplified to:

$$N_{\mathbf{h}} = E_{\mathbf{h}} \left[\sum_{i \in I} e + w_i \right] - \frac{\bar{\theta}}{2} \Omega.$$

In particular, the welfare measure for this economy has two components: An expected endowment component which does not depend on traders' portfolios, and the average variance component which depends on the portfolios but which is independent of belief \mathbf{h} [cf. Eq. (12)]. Consequently, regardless of the belief \mathbf{h} , an allocation $\tilde{\mathbf{x}}$ yields a higher welfare than \mathbf{x} if and only if it yields a smaller average variance. Intuitively, the portfolio allocations in this economy do not generate expected net worth since they simply redistribute wealth across traders. Hence, the portfolios affect the welfare measure, $N_{\mathbf{h}}$, only through their effect on traders' portfolio risks.

It follows that the average variance, Ω , emerges as a belief-neutral measure of welfare for this economy. In particular, financial innovation is belief-neutral inefficient whenever the introduction of new assets increases the average variance. Despite this observation, I do not take a strong normative stance in this paper, and emphasize the positive implications of Theorems 1 and 3. This is because there are some important ingredients missing from this model which might change the welfare arithmetic. Most importantly, traders' beliefs in this model contain no information about asset prices. When traders have some information, even pure speculation can be beneficial by increasing the informational efficiency of prices. I leave a more complete analysis of welfare for future work.

7 A Quantitative Exploration

The results in the previous sections have theoretically established that belief disagreements, when they are sufficiently large, change the nature of financial innovation as well as its effect on portfolio risks. A natural question is how large belief disagreements should be to make these results practically relevant. To address this question, this section considers a calibration of the model in the context of the national income markets analyzed by Athanasoulis and Shiller (AS, 2001). AS argue that assets whose payoffs are linked to (various combinations of) national incomes would lead to large welfare gains *because they would enable individuals to internationally diversify their consumption risks*. I first replicate AS's empirical results by

mapping their model and calibration to this framework. I then show that, with reasonable amounts of belief disagreements, the new assets proposed by AS would have the unintended consequence of *increasing* individuals' consumption risks.

Replicating Athanasoulis and Shiller (2001) results in this framework

To replicate the AS results, consider a setting in which the underlying risks correspond to the income shocks of G7 countries. More specifically, let $c \in C = \{1, \dots, |C|\}$ denote a G7 country, and suppose (as AS does) that the yearly income per-capita of country c is given by:

$$y(c) = y_{past}(c) + \alpha_{past}(c) + v(c).$$

Here, $y_{past}(c)$ denotes last year's income per-capita, $\alpha_{past}(c)$ denotes the predetermined change in income per-capita, and $v(c)$ denotes a zero-mean random variable which captures the yearly shock to income per-capita. Let $\mathbf{v} = \{v(c)\}_{c \in C}$ denote the $|C| \times 1$ vector which captures the underlying uncertainty. Suppose \mathbf{v} is Normally distributed with the $|C| \times |C|$ variance matrix, $\mathbf{\Lambda}^{\mathbf{v}}$.

A key challenge for empirical analysis is to estimate $\mathbf{\Lambda}^{\mathbf{v}}$. AS specify a spatial correlation model for $\mathbf{\Lambda}^{\mathbf{v}}$, which they estimate using data from the Penn World Table over the years 1950-1992. I use exactly the same specification to reconstruct their estimate for $\mathbf{\Lambda}^{\mathbf{v}}$, which facilitates the comparison of results. However, as it will become clear, the results are robust to reasonable variations in $\mathbf{\Lambda}^{\mathbf{v}}$. To interpret the numerical results below, it is useful to note that the standard deviation of $v(c)$ is estimated to be the same for each country and given by $\sigma^{v(c)} = \$364$ in 1985 dollars. The average per-capita income of G7 countries in 1992 is $\bar{y} = \$14784$ (in 1985 dollars), which implies that the standard deviation of yearly income growth in a G7 country is about 2.46%.

In this setting, the introduction of assets linked to country income shocks, \mathbf{v} , can facilitate international risk sharing. To see this, let $I(c)$ denote the set of individuals in country c that will actually trade the new assets, which I take to be proportional to the population of country c .¹³ To focus on international risk sharing, suppose (as AS do) that traders in the same country experience the same income shocks: That is, the income shock of trader $i(c)$ is given by $(\mathbf{W}_c)' \mathbf{v}$, where \mathbf{W}_c is a $|C| \times 1$ vector that has 1 in its c^{th} entry and 0 everywhere else. To simplify the analysis, suppose (as AS do) also that traders have the same risk aversion coefficients, $\theta_{i(c)} \equiv \theta$ for each trader $i(c)$. Finally, suppose that new assets $j \in J = \{1, \dots, |J|\}$ are linear combinations of the countries' income shocks, $a^j = (\mathbf{A}^j)' \mathbf{v}$ for each $j \in J$. AS characterize the optimal design of these assets that maximizes a social welfare function. The top panel of Figure 1 illustrates the optimal design when only two assets can be introduced. The most

¹³AS take $I(c)$ to be equal to the population of country c . This assumption is unreasonable since it is well documented that a large fraction of households do not participate in risky financial markets at all (see Campbell, 2006, for evidence from the US). I make the less stringent assumption that $I(c)$ is proportional to the population of the country. This is without loss of generality since the level of participation does not play an important role for the results in this section.

	$A^1_c (\times 10^{-4})$	$A^2_c (\times 10^{-4})$	$x^1_{i(c)}$	$x^2_{i(c)}$	$A^1_c / \text{Pop}_c (\times 10^{-9})$	$A^2_c / \text{Pop}_c (\times 10^{-9})$
Canada	1.66	-0.14	-58.58	3.54	6.05	-0.53
the US	37.22	-11.50	-141.24	30.32	14.60	-4.51
Japan	-20.41	-29.77	159.26	161.45	-16.46	-24.01
France	-4.23	10.17	71.28	-119.19	-7.37	17.73
Germany	-5.43	11.41	80.64	-117.84	-8.33	17.53
Italy	-4.80	9.97	80.34	-115.90	-8.30	17.24
the UK	-4.02	9.87	67.24	-114.69	-6.95	17.06

	Risk exposure of individuals in the US before innovation	With two assets	With complete markets
Beta_Canada	0.00	-0.02	0.04
Beta_US	1.00	0.44	0.40
Beta_Japan	0.00	0.20	0.19
Beta_France	0.00	0.09	0.09
Beta_Germany	0.00	0.11	0.10
Beta_Italy	0.00	0.10	0.09
Beta_UK	0.00	0.09	0.09
Std of cons. (1985 dollars)	363.99	315.41	314.36

Figure 1: The top table illustrates the asset design and the equilibrium portfolios for the benchmark without belief disagreements (with two assets). The last two columns display the asset design normalized by the country populations for comparison with Table 1 of AS. The bottom table shows the effect of financial innovation on consumption risks in the US. The columns display the slope coefficients in the following regression, $Consumption = \alpha + \sum_{c=1}^7 \beta_c v_c$, which has a perfect fit in the model.

important asset to create resembles an income swap between the US and Japan. Intuitively, this asset enables risk sharing between the traders in the US and Japan. The model picks the US and Japan because these are large countries whose income shocks are relatively less correlated (since they are geographically far), which increases the benefits from risk sharing. The second most important asset also resembles an income swap, this time between Japan and the core EU region, for a similar reason.

When there are no belief disagreements, my analysis in the earlier sections replicates the results in AS.¹⁴ In particular, traders' risky asset portfolios are also identical in both settings. The top table of Figure 1 illustrates these equilibrium portfolios (cf. Table 1 of AS). Note that much of the trade in the first asset is among the traders in Japan and the US who take the opposite positions to diversify their income risks. Moreover, traders' consumption risks are also identical in both settings. The second panel of Figure 1 illustrates these risks for a sample country, the US. Before financial innovation, the consumption of traders in the US has an exposure of one to the US income shock, v_{US} , and an exposure of zero to the income shocks of other countries. The new assets enable the traders to reduce their exposure to the

¹⁴This might seem surprising since AS consider a dynamic model, whereas this paper considers a static model. However, in view of CARA preferences, the dynamic and the static models are equivalent in several aspects. In particular, the equilibrium portfolios of risky assets are identical in both settings. Moreover, the variance of an individual's consumption in the dynamic model is the same as the variance of her net worth in the static model. It follows that the average variance, Ω , accurately describes the average consumption risks in the AS model.

US income shock by taking on some exposure to the income shocks of other countries (in particular, Japan). Consequently, traders are able to diversify and reduce their consumption risks. With two assets, the standard deviation of their consumption declines from about \$364 (2.46% of average income) to \$315.4 (2.13% of average income). Introducing additional assets reduces risks further but there are diminishing returns.

Effect of belief disagreements on consumption risks

I next consider the robustness of the AS results to the presence of belief disagreements. Modeling belief disagreements represents two additional empirical challenges. The first challenge is to calibrate traders' belief disagreements. In line with assumption (A1), I assume that traders know the variance matrix, $\mathbf{\Lambda}^v$, but they disagree about the mean of income shocks, μ^v . To keep the analysis simple, I assume that a trader's belief for the mean of a country's income shock is an i.i.d. draw from a Normal random variable,

$$\mu_i^v(c) \sim N\left(0, (\sigma^\mu)^2\right). \quad (24)$$

The assumption that a trader has uncorrelated beliefs for various countries is admittedly arbitrary. However, the qualitative conclusions in this section are robust to variations of the structure of a trader's beliefs, as long as there are disagreements across traders.

The specification in (24) has the benefit of capturing disagreements with a single parameter. In particular, let $\delta^v = \frac{\sigma^\mu}{\sigma^v}$ denote the cross-sectional standard deviation of traders' beliefs for income shocks relative the standard deviation of the same shock. Note that δ^v represents a measure of belief disagreements which is independent of linear transformations of $v(c)$. AS results correspond to the case in which $\delta^v = 0$. I next show that, with the specification in (24), $\delta^v = 0.02$ is sufficient to overturn their results about consumption risks. Disagreements of this size do not seem unreasonable. According to the Philadelphia Fed's *Survey of Professional Forecasters* (SPF) on macroeconomic forecasts in the US, the interquartile range of forecasts of US yearly GDP growth is on average given by 0.70% between the first quarter of 1992 and the third quarter of 2011. Over the same period, the historical standard deviation of the US yearly GDP growth is 2.08%. This suggests $\delta^v = 0.25$, which is an order of magnitude larger than the parameter that I consider, $\delta^v = 0.02$.¹⁵

The second challenge for empirical analysis with belief disagreements is to calibrate the risk aversion coefficient. This coefficient plays an important role because it affects the size of traders' speculative portfolios, but not their risk sharing portfolios [cf. Eq. (9)]. As a benchmark, I follow AS and calibrate the relative risk aversion coefficient as $\theta^{relative} = 3$, "as representing a consensus by many who work in this literature as a reasonable value to assume." This implies

¹⁵A caveat is order at this point. Note that the traders' belief disagreements in the model are non-informational, i.e., they agree to disagree. However, the δ^v observed in the SPF might reflect to some extent forecasters' private information, which creates an upward bias in the calibration. While it is difficult to adjust δ^v for this bias, the fact that the unadjusted δ^v is an order of magnitude larger than necessary suggests that the results would continue to hold for reasonable amounts of private information.

an absolute risk aversion coefficient $\theta = \frac{\theta^{relative}}{\bar{y}} = 0.00020$. Using a larger calibration for $\theta^{relative}$ makes the results less stark but it does not overturn the qualitative conclusions in this section.¹⁶

To illustrate the equilibrium with belief disagreements, I start by considering the portfolio allocations for a few traders whose beliefs are specified exactly and who are representative of a larger class of traders with similar beliefs. Let a *moderate* be someone whose belief for the income shocks of all countries is the same as the mean belief. In contrast, define a *c'-optimist* as someone whose belief for the income shock of country *c'* is exactly one standard deviation above the mean belief, and whose belief for all other income shocks is equal to the mean belief. The top table of Figure 2 shows the portfolio allocations for moderates and *c'*-optimists when there are two new assets. A moderate holds exactly the risk sharing portfolio in her respective country, which is illustrated in the third and the fourth columns. Thus, her portfolio is unaffected by the presence of belief disagreements. In contrast, a *c'*-optimist who lives in country *c* combines the risk sharing portfolio for country *c* with her speculative portfolio, which is illustrated in the last two columns. Note that, for a US-optimist and a Japan-optimist, the speculative portfolio is comparable in magnitude to (and often larger than) the risk sharing portfolios. Consequently, the complete portfolio of a US-optimist or a Japan-optimist is significantly influenced by the speculation motive for trade.

The bottom table in Figure 2 shows the consumption risks for moderates and US-optimists who live in the US. Moderates continue to diversify their risks in this case, as illustrated by the second column. However, the third column illustrates that a US-optimist does not diversify her risks. The risk sharing considerations would require this individual to take a short position in the first asset. However, her optimism about the US induces her to take a long position. When there are two assets, the two forces almost perfectly balance for this individual (cf. the top table), who remains exposed to the US income shock (cf. the bottom table). The last column illustrates the case with complete markets, in which case the speculation motive for trade dominates for a US-optimist. In this case, this individual has a greater exposure to the US income shock, and consequently greater consumption risks, than before financial innovation.

This analysis illustrates that, with belief disagreements, financial innovation has a different qualitative effect on the income risks of moderates and optimists. Guided by the earlier analysis, I assess the overall effect by considering $\sqrt{\Omega}$, which provides a quadratic average of the standard deviation of consumption over traders in G7 countries. The left table in Figure 3 shows that financial innovation increases this average for each country, $\sqrt{\Omega_c}$. The right panel of Figure 3 plots the world average standard deviation of consumption normalized by average income, $\sqrt{\Omega}/\bar{y}$. In contrast with the case without belief disagreements (the dashed line), financial innovation increases $\sqrt{\Omega}/\bar{y}$ (the solid line).

¹⁶For an extreme case, suppose $\theta^{relative} = 50$ which is the level of the relative risk aversion parameter that is necessary to rationalize the equity premium puzzle in a CRRA environment, but which is also considered to be implausibly high (Campbell, 2000). In this case, the AS results are overturned as long as belief disagreements are as high as implied by the SPF data, $\delta^v = 0.25$.

	$A^1_c (\times 10^{-4})$	$A^2_c (\times 10^{-4})$	$x^{R,1}_{i(c)}$	$x^{R,2}_{i(c)}$	$x^{S,1}_{c\text{-optimist}}$	$x^{S,2}_{c\text{-optimist}}$
Canada	1.66	-0.14	-58.58	3.54	5.96	-0.52
the US	37.22	-11.50	-141.24	30.32	133.53	-41.25
Japan	-20.41	-29.77	159.26	161.45	-73.22	-106.81
France	-4.23	10.17	71.28	-119.19	-15.16	36.49
Germany	-5.43	11.41	80.64	-117.84	-19.47	40.94
Italy	-4.80	9.97	80.34	-115.90	-17.22	35.75
the UK	-4.02	9.87	67.24	-114.69	-14.42	35.40

	Risk exposure of individuals in the US before innovation	With two assets, for a moderate	With two assets, for a US-optimist	With complete markets, for a US-optimist
Beta_Canada	0.00	-0.02	-0.00	-0.57
Beta_US	1.00	0.44	0.98	1.38
Beta_Japan	0.00	0.20	0.05	0.12
Beta_France	0.00	0.09	-0.01	0.03
Beta_Germany	0.00	0.11	-0.01	0.06
Beta_Italy	0.00	0.10	-0.01	0.05
Beta_UK	0.00	0.09	-0.01	0.01
Std cons. (1985 \$s)	363.99	315.41	360.86	402.94

Figure 2: The top table illustrates the equilibrium portfolios with belief disagreements ($\delta^v = 0.02$) and with two assets. The third and the fourth columns illustrate the risk sharing portfolios in country c . These are also the complete portfolios of *moderates*, who have the mean belief for each country. The last two columns illustrate the speculative portfolios of *c'-optimists*, whose beliefs for country c' are one standard deviation above the mean and whose beliefs for all other countries are equal to the mean. The bottom table illustrates the consumption risks in the US for moderates and US-optimists (see Figure 1 for a detailed explanation).

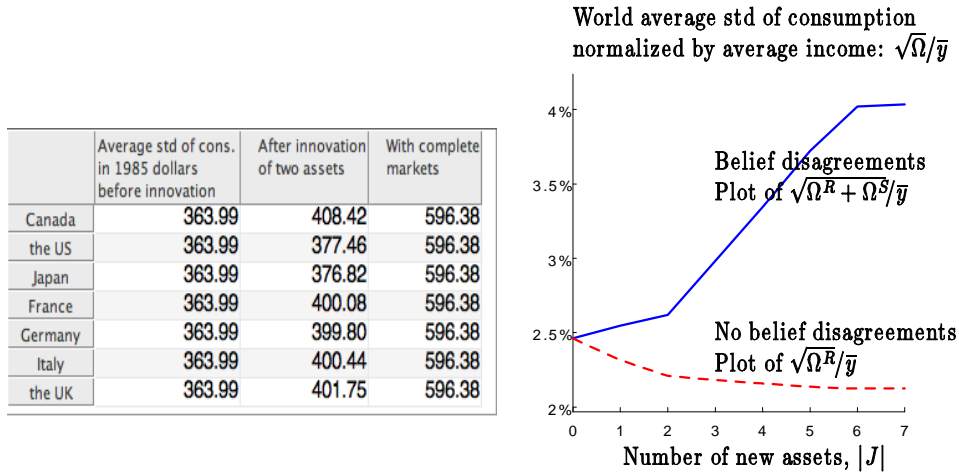


Figure 3: The panels illustrate the effect of financial innovation on average consumption risks in each country (left) and in all G7 countries (right) with belief disagreements parameterized by $\delta^v = 0.02$.

Intuition for the quantitative results

As illustrated by Figure 3, the main result of this section is equivalent to saying that the decrease in the uninsurable portfolio risks, $\sqrt{\Omega^R(\emptyset) - \Omega^R(J)}/\bar{y}$, is quantitatively smaller than the increase in the speculative portfolio risks, $\sqrt{\Omega^S(J) - \Omega^S(\emptyset)}/\bar{y} = \sqrt{\Omega^S(J)}/\bar{y}$. To understand the intuition for this result, it is useful to analyze the determinants of each term in turn.

The first term, $\sqrt{\Omega^R(\emptyset) - \Omega^R(J)}/\bar{y}$, is the same as the reduction in portfolio risks in the AS benchmark without belief disagreements. Figure 3 illustrates that $\sqrt{\Omega^R(J)}/\bar{y}$ decreases from 2.46% to 2.21% for the case with two assets, and to 2.13% for the complete markets case. This implies:

$$\frac{\sqrt{\Omega^R(\emptyset) - \Omega^R(J)}}{\bar{y}} = \begin{cases} 1.08\%, & \text{with two assets,} \\ 1.24\%, & \text{with complete markets,} \end{cases} \quad (25)$$

which is small in magnitude. Intuitively, the income risks in developed countries are small relative to their average incomes. Moreover, income risks are correlated across developed countries. Thus, even if these risks are perfectly diversified, the reduction in the standard deviation of normalized income risks, 2.46% to 2.13%, is small in magnitude. Consequently, the reduction in the uninsurable portfolio risks is also small.

Thus, the AS results are overturned as long as the second term, $\sqrt{\Omega^S(J)}/\bar{y}$, is greater than 1.24%. To understand when this is the case, recall from Eqs. (18) – (19) that the standard deviation of the speculative portfolio of an individual i (normalized by average income) can be written in terms of her speculative Sharpe ratio:

$$\sigma_i^S = \frac{1}{\theta^{relative}} \sqrt{\tilde{\mu}_i \Lambda^{-1} \tilde{\mu}_i} = \frac{1}{\theta^{relative}} Sharpe_i^S. \quad (26)$$

Here, recall that $Sharpe_i^S$ is the Sharpe ratio that trader i would perceive in a hypothetical scenario in which there are no background risks. In this scenario, a textbook result of mean-variance analysis applies and characterizes the standard deviation of the speculative portfolio return as in (26). Recall also that the term, $\sqrt{\Omega^S(J)}/\bar{y}$, is a (quadratic) average of the expression in (26) over all traders. Thus, given $\theta^{relative} = 3$ and the threshold 1.24%, it suffices to have that traders' speculative Sharpe ratios “on average” exceed 3.72%.

Speculative Sharpe ratios at this order of magnitude do not seem unreasonable. To see this, it is useful to characterize the speculative Sharpe ratio for a particular individual, the US-optimist. To this end, first consider a benchmark case in which there is only one asset whose payoff is equal to v_{US} . In this case, the Sharpe ratio of a US-optimist has a simple expression:

$$Sharpe_i^S = \frac{\tilde{\mu}_{US\text{-optimist}}^{v(US)}}{\sigma^{v(US)}} = \frac{\sigma^{\mu(US)}}{\sigma^{v(US)}} = \delta^v.$$

Here, the first equality uses Eq. (26), the second equality uses the definition of a US-optimist,

and the last equality uses the definition of δ^v . Intuitively, the expected excess payoff perceived by the US-optimist is equal to one cross-sectional standard deviation, $\sigma^{\mu(US)}$, while the risk of the payoff is equal to the standard deviation, $\sigma^{v(US)}$. Thus, the speculative Sharpe ratio, $\frac{\sigma^{\mu(US)}}{\sigma^{v(US)}}$, in this case is exactly equal to the parameter, $\delta^v = 2\%$. When there are more assets, the speculative Sharpe ratio is typically even greater than this expression in view of the hedge-more/bet-more effect. With complete markets, the US-optimist is able to obtain a Sharpe ratio of 3.80%. This is greater than 3.72%, which is the threshold required for speculative risks to dominate the reduction in uninsurable risks (on average). This observation provides an intuition for the earlier result illustrated in Figure 2: For a US-optimist living in the US, complete markets generate higher consumption risks than before financial innovation.

The speculative Sharpe ratio exceeds the required threshold, 3.72%, not just for the US-optimist but also many other similarly optimistic (or pessimistic) traders: Those whose beliefs for the US income shock are more than one standard deviation above mean, those who are sufficiently pessimistic about the US income shock, those who are sufficiently optimistic or pessimistic about other countries' income shocks, and so on. Consequently, $\delta^v = 2\%$ is sufficient to ensure that the speculative Sharpe ratios exceed 3.72% on average, providing the intuition for the quantitative results.

Effect of belief disagreements on the asset design

The analysis in this section took the assets in AS as exogenous. When $\delta^v = 0$, these are identical to the endogenous asset design characterized in Section 5. In particular, without belief disagreements, a profit seeking market maker chooses the same set of assets as the social planner in AS. However, assuming $\delta^v = 0.02$ also dramatically changes the endogenous asset design. In this case, new assets are directed towards maximizing the opportunities for speculation rather than risk sharing. The exact asset design depends on the specification of the structure of traders' beliefs in (24), which is admittedly arbitrary. This result nonetheless highlights that, with reasonable amounts of belief disagreements, financial innovation is determined by the nature of traders' belief disagreements as opposed to the nature of their risks.

8 Conclusion

This paper analyzed the effect of financial innovation on portfolio risks in a standard mean-variance setting in which both the speculation and risk sharing forces are present. In this framework, I have defined the average variance of traders' net worths as a natural measure of portfolio risks. I have also decomposed the average variance into two components: the uninsurable variance, defined as the variance that would obtain if there were no belief disagreements, and the speculative variance, defined as the residual amount of variance that results from speculative trades based on belief disagreements. My main result characterized the effect of financial innovation on both components of the average variance. Financial innovation always reduces

the uninsurable variance through the traditional channels of diversification and the efficient transfer of risks. However, financial innovation also *always* increases the speculative variance, through two distinct economic channels. First, new assets generate new disagreements. Second, new assets amplify traders' speculation on existing disagreements. The increase in speculative variance may be sufficiently large that financial innovation may lead to a net increase in average variance. A calibration of the model revealed that this is in fact the case for the GDP indexed assets proposed by Athanasoulis and Shiller (2001), as long as traders have reasonable amounts of belief disagreements for the GDP growth rates of G7 countries.

I have also analyzed endogenous financial innovation by considering a profit seeking market maker who introduces new assets for which it subsequently serves as the intermediary. The market maker's profits are proportional to traders' perceived surplus from trading the new assets. Consequently, traders' speculative motive for trade as well as their risk sharing motive for trade creates innovation incentives for the market maker. When belief disagreements are sufficiently large, the market maker innovates assets which *maximize* the average variance among all possible choices, completely disregarding the risk sharing motive for financial innovation.

A number of avenues for future research are opened by this paper. The first open question concerns the policy implications of the results. The equilibrium allocation is Pareto efficient despite the fact that trade in new securities may increase traders' portfolio risks. However, as discussed in Section 6, the notion of Pareto efficiency with heterogeneous beliefs is somewhat unsatisfactory. In recent work, Brunnermeier, Simsek, Xiong (2012) propose an alternative welfare criterion which detects financial innovation in this economy as inefficient whenever new assets increase portfolio risks. Despite this observation, I do not take a strong normative stance in this paper (and emphasize the positive implications of the results). This is because there are some important ingredients missing from the model, e.g., informational efficiency of prices, which might change the welfare arithmetic. I leave a more complete analysis of welfare for future work.¹⁷

A second avenue of new research concerns the evolution of belief disagreements. This paper analyzed financial innovation in a model in which traders' beliefs are exogenously fixed. In a companion paper, I consider financial innovation in a model in which traders' beliefs evolve over time. The novel feature of this dynamic setting is that traders learn from past observations of asset payoffs. Under appropriate assumptions, traders' belief disagreements on a given set of new assets disappear in the long run. Thus, in these environments, there is a tension between the short run and the long run effects of new assets on portfolio risks. I leave the resolution of this tension for future research.

¹⁷There is a second reason to analyze the policy implications further. While this paper illustrates the results in a standard mean-variance framework without externalities, the main mechanisms apply also in richer environments that may feature externalities. For example, if the traders are financial intermediaries that do not fully internalize the social costs of their losses (or bankruptcies), then an increase in speculation may lead to an inefficiency even according to the usual Pareto criterion.

A Appendices: Omitted Results and Proofs

A.1 Financial Innovation and Empirical Portfolio Risks

The analysis in the text described the effect of financial innovation on traders' portfolio risks calculated according to their own beliefs [cf. Eq. (12)]. This appendix shows that, under a slight strengthening of assumption (A1), the main result, Theorem 1, continues to hold after replacing the perceived variances in its statement with empirical variances (that is, the risks that would be reflected ex-post in the data). To this end, suppose that the underlying risks, \mathbf{v} , have an empirical (or ex-post realized) distribution denoted by $N(\boldsymbol{\mu}_{emp}^{\mathbf{v}}, \Lambda_{emp}^{\mathbf{v}})$. Suppose also that traders' beliefs are related to the empirical distribution. In particular, traders know the empirical variance, $\Lambda_{emp}^{\mathbf{v}}$, but they do not know the empirical mean, $\boldsymbol{\mu}_{emp}^{\mathbf{v}}$. Trader i has a prior belief for the empirical mean parameter, $\boldsymbol{\mu}_{emp}^{\mathbf{v}}$, given by the Normal distribution, $N(\boldsymbol{\mu}_i^{\mathbf{v}}, \Lambda_i^{\mathbf{v}})$. The following assumption about beliefs simplifies the analysis:

Assumption (A1^S). For each i , $\Lambda_i^{\mathbf{v}} = \tau^{-1}\Lambda_{emp}^{\mathbf{v}}$ for some constant $\tau > 0$.

This assumption implies assumption (A1), because the trader's marginal belief for \mathbf{v} can be written as:

$$N(\boldsymbol{\mu}_i^{\mathbf{v}}, \Lambda^{\mathbf{v}}), \text{ where } \Lambda^{\mathbf{v}} = \Lambda_{emp}^{\mathbf{v}} + \Lambda_i^{\mathbf{v}} = \left(1 + \frac{1}{\tau}\right) \Lambda_{emp}^{\mathbf{v}}.$$

The stronger assumption (A1^S) is useful because it ensures that there is a linear wedge between traders' (common) perceived variance, $\Lambda^{\mathbf{v}}$, and the empirical variance, $\Lambda_{emp}^{\mathbf{v}}$. The size of the wedge is controlled by the parameter, τ , which captures the precision of traders' beliefs. Intuitively, traders' perceived uncertainty for \mathbf{v} is greater than the empirical uncertainty of \mathbf{v} because they face additional parameter uncertainty.

Note that the empirical and perceived variances coincide when $\tau = \infty$, i.e., when traders' beliefs are very precise. Thus, in this special case, all of the results of earlier sections apply for the empirical variance as well as the perceived variance of net worths. When $\tau < \infty$, it is not immediately clear that the analogues of earlier results would hold for the empirical variance. The rest of this section shows that this is the case. The following result establishes that, assuming $\tau = \infty$ is without loss of generality as long as the risk aversion coefficients, $\{\theta_i\}_i$, are appropriately adjusted. To state the result, define the *empirical average variance* as:

$$\Omega_{emp} = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} \text{var}_{emp}(n_i). \quad (\text{A.1})$$

Define also the *empirical uninsurable variance*, Ω_{emp}^R as the solution to problem (13) with Ω in the optimization replaced by Ω_{emp} . Finally, define the *empirical speculative variance* as the residual, $\Omega_{emp}^S = \Omega_{emp} - \Omega_{emp}^R$.

Lemma 3. Consider an economy $\mathcal{E}(J)$ that satisfies assumption (A1^S). Consider an alternative economy, $\mathcal{E}_{emp}(J)$, which is identical to $\mathcal{E}(J)$ except for two aspects: (i) traders' precision parameter is adjusted according to $\tau_{emp} = \infty$, and (ii) traders' risk aversion coefficients are

also adjusted according to:

$$\theta_{emp,i} = \left(1 + \frac{1}{\tau}\right) \theta_i \text{ for each } i. \quad (\text{A.2})$$

Then,

- (i) The equilibrium, $(\mathbf{p}, (\mathbf{x}_1, \dots, \mathbf{x}_{|I|}))$, is the same in economies $\mathcal{E}(J)$ and $\mathcal{E}_{emp}(J)$.
- (ii) The empirical average variance and its components, $\Omega_{emp}, \Omega_{emp}^R, \Omega_{emp}^S$, are the same in economies $\mathcal{E}(J)$ and $\mathcal{E}_{emp}(J)$.

The first part follows from Eqs. (8) – (9). Intuitively, traders in economy $\mathcal{E}(J)$ are reluctant to take risky positions not only because they are risk averse but also because they face additional parameter uncertainty. This makes them effectively more risk averse, as captured by the adjustment in (A.2). In fact, $\theta_{emp,i}$ could be viewed as trader i 's effective coefficient of risk aversion. The second part of the lemma follows from the fact that the differences in the two economies do not concern the empirical distributions (or their averages). Since the equilibrium allocations are identical in the two economies, so are the empirical variances.

Lemma 3 is useful because in the alternative economy, $\mathcal{E}_{emp}(J)$, the empirical and the perceived variances coincide. Thus, applying the earlier results for this economy characterizes the effect of financial innovation on the empirical variances in the original economy, $\mathcal{E}(J)$. In particular, the next result follows as a corollary of Lemma 3 and Theorem 1. A similar argument implies that the analogues of the results in Section 5 hold for empirical variances.

Theorem 4 (Financial Innovation and Empirical Portfolio Risks). *Consider an economy $\mathcal{E}(J)$ that satisfies assumption $(A1^S)$, and let J_O and J_N respectively denote the set of old and new assets. Financial innovation always decreases the empirical uninsurable variance and increases the empirical speculative variance, that is:*

$$\Omega_{emp}^R(J_O \cup J_N) \leq \Omega_{emp}^R(J_O) \text{ and } \Omega_{emp}^S(J_O \cup J_N) \geq \Omega_{emp}^S(J_O).$$

A.2 Omitted proofs for Section 3

Proof of Lemma 1. Recall that the objective function for Problem (13) is given by

$$\Omega = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} (\mathbf{W}'_i \Lambda^\nu \mathbf{W}_i + \mathbf{x}'_i \Lambda \mathbf{x}_i + 2\mathbf{x}'_i \boldsymbol{\lambda}_i). \quad (\text{A.3})$$

The first order conditions are given by:

$$\Lambda \mathbf{x}_i + \boldsymbol{\lambda}_i = \gamma \frac{\bar{\theta}}{\theta_i} \text{ for each } i \in I,$$

where $\gamma \in \mathbb{R}^{|J_E|}$ is a vector of Lagrange multipliers. Note that $\mathbf{x}_i^R = -\Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i$ satisfies these first order conditions for the Lagrange multiplier $\gamma = (\sum_{i \in I} \boldsymbol{\lambda}_i) / |I|$. This shows that $\{\mathbf{x}_i^R\}_i$ is the unique solution to Problem (13).

Proof of Lemma 2. Plugging in $\mathbf{x}_i^R = -\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_i$ into the objective function (A.3), the optimal value, Ω^R , is given by:

$$\begin{aligned}\Omega^R &= \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}'_i \Lambda^\nu \mathbf{W}_i + \tilde{\boldsymbol{\lambda}}'_i \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right) - \frac{2}{|I|} \sum_i \tilde{\boldsymbol{\lambda}}'_i \Lambda^{-1} \frac{\theta_i \boldsymbol{\lambda}_i}{\bar{\theta}}, \\ &= \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}'_i \Lambda^\nu \mathbf{W}_i + \tilde{\boldsymbol{\lambda}}'_i \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right) - \frac{2}{|I|} \sum_i \tilde{\boldsymbol{\lambda}}'_i \Lambda^{-1} \frac{\theta_i \tilde{\boldsymbol{\lambda}}_i}{\bar{\theta}} \\ &= \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}'_i \Lambda^\nu \mathbf{W}_i - \tilde{\boldsymbol{\lambda}}'_i \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right).\end{aligned}$$

Here, the second line uses the fact that $\sum_i \tilde{\boldsymbol{\lambda}}_i = 0$ to replace $\frac{\theta_i \boldsymbol{\lambda}_i}{\bar{\theta}}$ with its deviation from average, $\frac{\theta_i \tilde{\boldsymbol{\lambda}}_i}{\bar{\theta}}$. This completes the derivation of Eq. (14).

To derive Eq. (15), first consider the expression $|I| \left(\Omega - \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \mathbf{W}'_i \Lambda^\nu \mathbf{W}_i \right)$. Using the definition of the average variance in (12), this expression can be written as:

$$\begin{aligned}\sum_i \frac{\theta_i}{\bar{\theta}} \mathbf{x}'_i \Lambda \mathbf{x}_i + 2 \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \mathbf{x}'_i \boldsymbol{\lambda}_i &= \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right) + 2 \sum_{i \in I} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \frac{\theta_i \boldsymbol{\lambda}_i}{\bar{\theta}} \\ &= \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left[\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right) + 2 \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right] \\ &= \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left[\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} + \tilde{\boldsymbol{\lambda}}_i \right) \right] \\ &= \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \tilde{\boldsymbol{\lambda}}'_i \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i.\end{aligned}$$

Here, the first line substitutes for the portfolio demands from (9); the second line replaces $\frac{\theta_i \boldsymbol{\lambda}_i}{\bar{\theta}}$ with its deviation from average, $\frac{\theta_i \tilde{\boldsymbol{\lambda}}_i}{\bar{\theta}}$ (as in the first part of the proof); and the next two lines follow by simple algebra. Next, using the fact that the last line equals $|I| \left(\Omega - \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \mathbf{W}'_i \Lambda^\nu \mathbf{W}_i \right)$, the average variance can be written as:

$$\Omega = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}'_i \Lambda^\nu \mathbf{W}_i - \tilde{\boldsymbol{\lambda}}'_i \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right) + \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}.$$

Using the definition of Ω^R in (14), it follows that the speculative variance is given by the expression in (15).

A.3 Omitted proofs for Section 4

Proof of Theorem 1. Part (i). By definition, Ω^R , is the optimal value of the minimization problem (13). Financial innovation expands the constraint set of this problem. Thus, it also decreases the optimal value, proving $\Omega^R(J_O \cup J_N) \leq \Omega^R(J_O)$.

Part (ii). The proof is provided in the text. Here, to demonstrate an alternative approach, I provide a second proof using matrix algebra. First note that the definition of Ω^S in (15) implies that

$$\begin{aligned}\Omega^S(J) - \Omega^S(J_O) &= \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\frac{\tilde{\boldsymbol{\mu}}_i'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \frac{\tilde{\boldsymbol{\mu}}_i(J_O)'}{\theta_i} \Lambda(J_O)^{-1} \frac{\tilde{\boldsymbol{\mu}}_i(J_O)}{\theta_i} \right) \\ &= \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i'}{\theta_i} \left(\Lambda^{-1} - \begin{bmatrix} \Lambda(J_O)^{-1} & 0 \\ 0 & 0 \end{bmatrix} \right) \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}.\end{aligned}\quad (\text{A.4})$$

I next claim that the matrix in the parenthesis,

$$\Lambda^{-1} - \begin{bmatrix} \Lambda(J_O)^{-1} & 0 \\ 0 & 0 \end{bmatrix}, \quad (\text{A.5})$$

is positive semidefinite. In view of this claim, Eq. (A.4) implies $\Omega^S(J) \geq \Omega^S(J_O)$, providing an alternative proof for this part.

This claim follows from Lemma 5.16 in Horn and Johnson (2007). This lemma considers a positive definite matrix partitioned into submatrices of arbitrary dimension, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}$. It shows that the matrix A^{-1} is weakly greater than the matrix $\begin{bmatrix} (A_{11})^{-1} & 0 \\ 0 & 0 \end{bmatrix}$ in positive semidefinite order. This in turn implies that the matrix, $A^{-1} - \begin{bmatrix} (A_{11})^{-1} & 0 \\ 0 & 0 \end{bmatrix}$, is positive semidefinite. Invoking this lemma for $A = \Lambda$ and $A_{11} = \Lambda(J_O)$ shows that the matrix in Eq. (A.5) is positive semidefinite, completing the alternative proof.

A.4 Omitted proofs for Section 5

Derivation of the Market Maker's Profit. First note that trader i 's payoff from rejecting the market maker's offer is the certainty equivalent payoff from her endowment:

$$e + \mathbf{W}'_i \boldsymbol{\mu}_i^v - \frac{\theta_i}{2} \mathbf{W}'_i \Lambda^v \mathbf{W}_i. \quad (\text{A.6})$$

Next consider trader i 's certainty equivalent payoff after trading the assets. Using Eq. (9), traders' net worth, n_i , can be written as:

$$n_i = e - \mathbf{x}'_i \mathbf{p} + \left[\mathbf{W}_i + \mathbf{A} \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right) \right]' \mathbf{v}.$$

The certainty equivalent of this expression is given by:

$$\begin{aligned}
e - \mathbf{x}'_i \mathbf{p} + \mathbf{W}'_i \boldsymbol{\mu}_i^{\mathbf{v}} + \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right)' \Lambda^{-1} \boldsymbol{\mu}_i & \tag{A.7} \\
-\frac{\theta_i}{2} \mathbf{W}'_i \Lambda^{\mathbf{v}} \mathbf{W}_i - \frac{\theta_i}{2} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right) - \theta_i \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right) \Lambda^{-1} \boldsymbol{\lambda}_i(\mathbf{A}).
\end{aligned}$$

Since the fixed fee makes the trader indifferent, it is equal to the difference of the expression in (A.7) from the expression in (A.6). That is:

Proof of Theorem 2. To prove the result it is useful to consider the market maker's optimization problem in terms of a linear transformation of assets, $\hat{\mathbf{A}} = (\Lambda^{\mathbf{v}})^{1/2} \mathbf{A}$, where $(\Lambda^{\mathbf{v}})^{1/2}$ is the unique positive definite square root matrix of $\Lambda^{\mathbf{v}}$. Note that choosing $\hat{\mathbf{A}}$ is equivalent to choosing \mathbf{A} . The normalizations in (21) can be written in terms of $\hat{\mathbf{A}}$ as:

$$\hat{\mathbf{A}}' \hat{\mathbf{A}} = \mathbf{I}_{|J|}, \quad \text{and} \quad \tilde{\mathbf{A}}_1^j \geq 0 \text{ for each } j. \tag{A.8}$$

After using the normalization $\Lambda = \mathbf{I}_{|J|}$ and substituting $\hat{\mathbf{A}}$ for \mathbf{A} , the expected profit in (20) can also be written as:

$$\begin{aligned}
\sum_i \pi_i(\hat{\mathbf{A}}) &= \sum_i \frac{\theta_i}{2} \left((\Lambda^{\mathbf{v}})^{-1/2} \frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2} \tilde{\mathbf{W}}_i \right)' \hat{\mathbf{A}} \hat{\mathbf{A}}' \left((\Lambda^{\mathbf{v}})^{-1/2} \frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2} \tilde{\mathbf{W}}_i \right), \\
&= \text{tr} \left(\hat{\mathbf{A}}' \mathbf{M} \hat{\mathbf{A}} \right) = \sum_j \left(\hat{\mathbf{A}}^j \right)' \mathbf{M} \hat{\mathbf{A}}^j, \\
\text{where } \mathbf{M} &= \left((\Lambda^{\mathbf{v}})^{-1/2} \frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2} \tilde{\mathbf{W}}_i \right) \left((\Lambda^{\mathbf{v}})^{-1/2} \frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2} \tilde{\mathbf{W}}_i \right)'. \tag{A.9}
\end{aligned}$$

Here, the second line uses the matrix identity $\text{tr}(XY) = \text{tr}(YX)$ and the linearity of the trace operator, and the last line defines the $m \times m$ matrix, \mathbf{M} . Thus, the market maker's problem reduces to choosing $\hat{\mathbf{A}} = (\Lambda^{\mathbf{v}})^{1/2} \mathbf{A}$ to maximize (A.9) subject to the normalizations in (A.8).

Next note that the first normalization in (21) implies:

$$\left(\hat{\mathbf{A}}^j \right)' \hat{\mathbf{A}}^j = 1 \text{ for each } j. \tag{A.10}$$

Consider the alternative problem of choosing $\hat{\mathbf{A}}$ to maximize the expression in (A.9) subject to the relaxed constraint in (A.10). The first order conditions for this problem are given by

$$\mathbf{M} \hat{\mathbf{A}} = \gamma^j \hat{\mathbf{A}}^j \text{ for each } j,$$

where $\gamma^j \in \mathbb{R}_+$ are Lagrange multipliers. From this expression, it follows that $\left\{ \hat{\mathbf{A}}^j \right\}_j$ correspond to eigenvectors of the matrix, \mathbf{M} , and $\left\{ \gamma^j \right\}_j$ correspond to eigenvalues. Plugging the

first order condition into Eq. (A.9), the expected profit can be written as:

$$\sum_i \pi_i(\hat{\mathbf{A}}) = \sum_j \gamma^j (\hat{\mathbf{A}}^j)' \hat{\mathbf{A}}^j = \sum_j \gamma^j.$$

It follows that the objective value will be maximized if and only if $\{\gamma^j\}_j$ correspond to the $|J|$ largest eigenvalues of the matrix, \mathbf{M} . If the $|J|$ largest eigenvalues are unique, then the optimum vectors, $\hat{\mathbf{A}} = \{\hat{\mathbf{A}}^j\}_j$, are uniquely characterized as the corresponding eigenvectors which have length 1 [cf. Eq. (A.10)] and which satisfy the sign convention in (A.8). If the $|J|$ largest eigenvalues are not unique, then the same argument shows that the vectors, $\{\hat{\mathbf{A}}^j\}_j$, are uniquely determined up to a choice of these eigenvalues.

Finally, consider the original problem of maximizing the expression in (A.9) subject to the stronger condition, $\hat{\mathbf{A}}' \hat{\mathbf{A}} = \mathbf{I}$. Since \mathbf{M} is a symmetric matrix, its eigenvectors are orthogonal. This implies that the solution, $\{\hat{\mathbf{A}}^j\}_j$, to the alternative problem is in the constraint set of the original problem. Since the latter problem has a stronger constraint, it follows that the solutions to the two problems are the same, completing the proof.

Proof of Theorem 3. Part (i). Note that $\tilde{\boldsymbol{\mu}}_{i,0}(\mathbf{A}) = \mathbf{A}' \tilde{\boldsymbol{\mu}}_{i,0}^{\mathbf{v}} = 0$ for any \mathbf{A} . This implies that the expected profit in (20) is given by $\sum_i \frac{\theta_i}{2} \tilde{\boldsymbol{\lambda}}_i(\mathbf{A})' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i(\mathbf{A})$. From Eq. (14), this expression is equal to $c_1 - c_2 \Omega^R(\mathbf{A})$ for some constant c_1 and positive constant c_2 . Thus, maximizing $\sum_i \pi_i(\mathbf{A})$ is equivalent to minimizing $\Omega^R(\mathbf{A})$. Finally, note from Eq. (15), that $\Omega_0^S(\mathbf{A}) = 0$ for any \mathbf{A} . This further implies $\Omega(\mathbf{A}) = \Omega^R(\mathbf{A})$, proving that the market maker innovates assets that minimize $\Omega(\mathbf{A})$.

Part (ii). Consider the following objective function:

$$\frac{1}{K^2} \sum_i \pi_{i,K}(\mathbf{A}), \quad (\text{A.11})$$

which is just a scaling of the expected profit in (20). In particular, maximizing this expression is equivalent to maximizing the expected profit. In view of Theorem 2, the optimal asset design, \mathbf{A}_K , is uniquely determined. This also implies that \mathbf{A}_K is a continuous function. Since \mathbf{A}_K is bounded [from the normalization (21)], it follows that $\lim_{K \rightarrow \infty} \mathbf{A}_K$ exists.

Note also that the limit of the objective function in (A.11) can be calculated as:

$$\begin{aligned} \lim_{K \rightarrow \infty} \frac{1}{K^2} \sum_i \pi_{i,K}(\mathbf{A}) &= \lim_{K \rightarrow \infty} \sum_i \frac{\theta_i}{2} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \frac{\tilde{\boldsymbol{\lambda}}_i(\mathbf{A})}{K} \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \frac{\tilde{\boldsymbol{\lambda}}_i(\mathbf{A})}{K} \right) \\ &= \sum_i \frac{\theta_i}{2} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i}, \end{aligned} \quad (\text{A.12})$$

where the first line uses $\tilde{\boldsymbol{\mu}}_{i,K}(\mathbf{A}) = K \tilde{\boldsymbol{\mu}}_i(\mathbf{A})$ and the second line uses $\tilde{\boldsymbol{\mu}}_i(\mathbf{A}) = \mathbf{A}' \tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}$. In

particular, the objective function remains bounded as $K \rightarrow \infty$. Thus, Berge's Maximum Theorem applies and implies that \mathbf{A}_K is upper hemicontinuous in K over the extended set $\mathbb{R}_+ \cup \{\infty\}$. In particular, $\lim_{K \rightarrow \infty} \mathbf{A}_K$ maximizes the limit objective function in (A.12) subject to the normalization, (21).

Finally, consider the limit of the average variance

$$\lim_{K \rightarrow \infty} \frac{\Omega_K(\hat{\mathbf{A}})}{K^2} = \lim_{K \rightarrow \infty} \left(\frac{\Omega_K^S(\hat{\mathbf{A}})}{K^2} + \frac{\Omega_K^R(\hat{\mathbf{A}})}{K^2} \right) = \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i},$$

where the second equality follows from Eqs. (15) and (14). In view of Eq. (A.12), it follows that $\lim_{K \rightarrow \infty} \mathbf{A}_K$ maximizes $\lim_{K \rightarrow \infty} \frac{1}{K^2} \Omega_K(\hat{\mathbf{A}})$ subject to the normalization, (21), completing the proof.

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