# Welfare Analysis of Currency Regimes with Defaultable Debts 

Aloisio Araujo (EPGE/FGV and IMPA)
Marcia Leon (Banco Central do Brasil)
Rafael Santos (Banco Central do Brasil)
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## Presentation

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## 1. MOTIVATION

Use the self-fulfilling debt crisis model of Cole-Kehoe to evaluate financial aspects of currency regimes:

- Dollarization
- Common Currency
- Local Currency

The optimal currency regime depends on:

- Correlation of External Shocks (Refinancing Risks) among countries of a monetary union
- Risk of Political Inflation


## 2 - The Cole-Kehoe Model <br> Review of Economic Studies(2000)

It has two parts:
a) a dynamic, stochastic general equilibrium model, with probability $\pi$ of a self-fulfilling debt crisis occurring;
b) a simulation exercise to obtain the debt-crisis zone and the welfare levels for an economy under a possible speculative attack on its public debt.

## 2 - The Cole-Kehoe Model

- One good: $f\left(k_{t}\right)$;
- Three participants:
(i) national consumers;
(ii) international bankers; and
(iii) the government.
- One sunspot $\zeta_{t}$ : bankers' confidence that government will not default; i.i.d., uniform $[0,1]$ and $\mathrm{P}\left[\zeta_{t} \leq \pi\right]=\pi$
- $\zeta_{t}$ also indicates the refinancing risk faced by indebted economies.
- Foreign-currency debt, $B_{t}$ : in the hands of int'I bankers; probability $\pi$ of no rollover in the crisis zone; if there is default, it is full. $\left(Z_{t}=0\right)$. No default $\left(Z_{t}=1\right)$.


## 2 - The Cole-Kehoe Model

(i) Consumer's problem ${ }_{\infty}$

$$
\max _{c_{t}, k_{t+1}} E \sum_{t=0}^{\infty} \beta^{t}\left[\varrho c_{t}+v\left(g_{t}\right)\right]
$$

s.t.

$$
\begin{gathered}
c_{t}+k_{t+1}-k_{t} \leq(1-\theta)\left[a_{t} f\left(k_{t}\right)-\delta k_{t}\right] \\
k_{0}>0
\end{gathered}
$$

$a_{t}$ - productivity factor
If the government has defaulted, then $a_{t}=\alpha, 0<\alpha<1$.
Otherwise, $a_{t}=1$.

## 2 - The Cole-Kehoe Model

(ii) International bankers' problem

$$
\max _{x_{t}, b_{t+1}} E \sum_{t=0}^{\infty} \beta^{t} x_{t}
$$

s.t.

$$
x_{t}+q_{t}^{*} b_{t+1} \leq \bar{x}+z_{t} b_{t}
$$

$$
b_{0}>0
$$

$q^{*}{ }_{t}$ - price, at $t$, of one-period government bond that pays one good, if there is no default.

## 2 - The Cole-Kehoe Model

(iii) Government

Benevolent and with no commitment.

Decision variables: $B_{t+1}, z_{t}, g_{t}$
Budget constraint

$$
g_{t}+z_{t} B_{t} \leq \theta\left[a_{t} f\left(k_{t}\right)-\delta k_{t}\right]+q_{t}^{*} B_{t+1}
$$

Strategic behavior since foresees $q_{t}^{*}, c_{t}, k_{t+1}, g_{t}, z_{t}, a_{t}$

## 2 - The Cole-Kehoe Model

- Timing of actions within a period
a) $\zeta$ is realized and state $S=\left(K, B, a_{-1}, \zeta\right)$
b) government, given $q^{*}=q^{*}\left(s, B^{\prime}\right)$, chooses $B^{\prime}$
c) bankers decide whether to purchase $B^{\prime}$
d) government chooses $Z$ and $g$
e) consumers, given $a(S, Z)$, choose $C$ and $k^{\prime}$


## 2 - The Cole-Kehoe Model

- An Equilibrium
a) Characterization of consumers and bankers behavior

Consumers: $k^{\prime}$ takes three values: $k^{n}>k^{\pi}>k^{d}$ depending on $E\left[a^{\prime}\right]$
$k^{n}, E\left[a^{\prime}\right]=1 ; k^{\pi}, E\left[a^{\prime}\right]=1-\pi+\pi \alpha ; k^{d}, E[a]=\alpha$
Bankers: $\quad q^{*}$ takes three values: $\beta, \beta(1-\pi), 0$ depending on $E\left[z^{\prime}\right]$ since $q^{*}=\beta E\left[z^{\prime}\right]$
$\beta, E\left[z^{\prime}\right]=1 ; \beta(1-\pi), E\left[z^{\prime}\right]=1-\pi ; 0, E\left[z^{\prime}\right]=0$

## 2 - The Cole-Kehoe Model

b) Definition: Crisis Zone with probability $\pi$

Debt interval that a crisis can occur with probability $\pi$.
For one-period gov't bonds and $S=\left(k^{\pi}, B, 1, \zeta\right)$ :

$$
\left(\bar{b}\left(k^{n}\right), \bar{B}\left(k^{\pi}, \pi\right)\right]
$$

c) Government choices:
$B^{\prime} \leq \bar{b}\left(k^{n}\right)$ - no crisis zone
$\bar{b}\left(k^{n}\right)<B^{\prime} \leq \bar{B}\left(k^{\pi}, \pi\right)$ - crisis zone
$B^{\prime}>\bar{B}\left(k^{\pi}, \pi\right)$ - full default only zone

## 3 - Local-currency debt model

Araujo and Leon (RBE, 2002)

- Public debt denominated in two currencies: foreign, $B_{t}$, and local, $D_{t}$
- A full default on $B_{t}$ may be avoided through a partial default on debt denominated in local currency, $D_{t}$
- $D_{t}$ only in the hands of national investors; credit rollover always.
- Government decision variable to partial default, $\mathbf{v}$.

No partial default, local bond pays one good $(v=1)$.
Otherwise, it pays less than one good, $(v=\phi), \phi<1$.

## 3 - Local-currency debt model

- Cost of partial default: productivity falls to $\alpha^{\phi}>\alpha$ If $z=0$ (full default on $B_{t}$ ), then $a=\alpha$ forever
If $v=\phi$ (partial default on $D_{t}$ ), then $a=\alpha^{\phi}$ forever
- Intense speculative attack:

If $\zeta_{t}<\pi^{\mathrm{d}}$, then $\mathrm{z}=0$ and full default on $B_{t}$

- Moderate speculative attack:

If $\pi^{\mathrm{d}}<\zeta_{\mathrm{t}}<\pi^{\mathrm{up}}$, then $\mathrm{z}=1$ and a fraction $\varphi$ of $B_{t}$ is renewed and there is partial default on $D_{t}$ to avoid a full default on $B_{t}$.

## 3 - Local-currency debt model

- Political Inflation

$$
\text { If } \pi^{\text {up }}<\zeta_{t}<\pi^{\text {up } \psi} \text {, then } z=1 \text { and total } B_{t} \text { is }
$$

renewed, but there is partial default on $D_{t}$.

- Risk of political inflation, $\pi^{\mathrm{p}}$

$$
\pi^{\mathrm{p}}=\pi^{\mathrm{up} \psi}-\pi^{\mathrm{up}}
$$

- Partial default revenues:
$\rightarrow$ to avoid full default on $B_{t}$; or
$\rightarrow$ for political purposes (risk of political inflation)


## 3 - Local-currency debt model

An equilibrium is analogous to the original C-K

- Consumers' new budget constraint:

$$
c_{t}+k_{t+1}-k_{t}+q_{t} d_{t+1} \leq(1-\theta)\left[a_{t} f\left(k_{t}\right)-\delta k_{t}\right]+v_{t} d_{t}
$$

besides $C_{t}$ and $k_{t+1}$ also chooses $d_{t+1}$

- Government new budget constraint:

$$
g_{t}+z_{t} B_{t}+v_{t} D_{t} \leq \theta\left[a_{t} f\left(k_{t}\right)-\delta k_{t}\right]+q_{t}^{*} B_{t+1}+q_{t} D_{t+1}
$$

besides $B_{t+1}, z_{t}$ and $g_{t}$ also chooses $D_{t+1}$ and $v_{t}$

## 4. Common-currency debt model

- I countries in a monetary union and a central government
- Each country $i$ issues debt in common currency, $D_{t}^{i}$
- Possibility of a partial default on common-currency debt, which depends on decision process.
- Partial-default decision: Member-countries vote: $v^{i}$; and Union decision: $\mathbf{v}^{\mathbf{u}}$


## 4. Common-currency debt model

- Two decision processes are considered:

1) The right of veto: $v^{u}=\phi \Leftrightarrow v^{i}=\phi$, for all i
2) Political influence over the union's central bank:

Each member implements its decision with probability $p w^{i}$ and $\Sigma p w^{i}=1$.

- Correlation of external shocks, $\rho$

The external shock (refinancing risk), $\zeta^{i}$, of each country $i$ correlates with the one from the other countries.

## 5. Computed Model Results

- Numerical Findings follow from the welfare analysis of alternative currency regimes, depending on the risk of political inflation, $\pi^{\mathrm{p}}$, and the correlation of external shocks (refinancing risks), $\rho$.
- A country (country A) has to decide either to maintain its local-currency regime, or to join a common-currency regime with a partner country (country B), or to dollarize by adopting the currency of a third country.
- Country $B$ is assumed to have all parameters equal to those of country $A$, except for a possible change in the risk of political inflation.


## 5. Computed Model Results

- Numerical Finding 1

The bigger the risk of political inflation, the larger the region where dollarization maximizes welfare. (See Figure 2)

- Numerical Finding 2

The larger the correlation of external shocks $\rho$, the larger the region where common-currency maximizes welfare. (See Figure 2)

## 5. Computed Model Results

- Numerical Finding 3

As $\pi^{\mathrm{pB}}$ decreases the range for $\rho$ in which the commoncurrency regime is optimal increases over the Dollar region and decreases over the Local-Currency region. (Compare Figures 2 and 3 )

Note: In Figure 2, the risk of political inflation of country B, $\pi^{\mathrm{pB}}$, is 0.7 and, in Figure 3, is zero.

## 5. Computed Model Results

- Numerical Finding 4

For high levels of the risk of political inflation in country A, $\pi^{\mathrm{pA}}$, the region where dollarization is preferred increases as $\mathrm{p}^{\mathrm{wA}}$ increases.
(See Figure 4)

Optimal Monetary Arrangement ( $\mathrm{n}=2$ )
Decision process: Right of Veto
Risk of political inflation in the other country (B): 0.7 and 0

Figure 2


Optimal Monetary Arrangement ( $\mathrm{n}=2$ )
Political Weight in the decision process: $0,0.4$ and 0.8 Risk of political inflation in the other country (B): 0.7
Figure 4


External Shocks Correlation ( $\rho$ )

## 6. Conclusions

- Choices of currency regimes considering financial aspects:

Low risk of political inflation
and low external correlation $\Rightarrow$ Local-currency regime
High risk of political inflation
and high external correlation $\Rightarrow$ Common-currency regime
High risk of political inflation and low correlation $\quad \Rightarrow$ Dollarization

THANK YOU FOR YOUR ATTENTION

## 5. Computed Model Results

Benchmark: the Brazilian economy (1998/2001)

| Length (Years) | Model | Brazil (98-01) |
| :--- | :--- | :--- |
| Average Maturity | 1 | $\in[.4 ; 2.2]$ |
| Average Duration | 1 | $\in[.2 ; .9]$ |
| Variables Relative to GDP | Model | Brazil (98-01) |
| External Debt | 45 | $\in[31 ; 45]$ |
| External Public Debt | 45 | $\in[9 ; 24]$ |
| Local Currency Public Debt | 30 | $\in[27 ; 31]$ |
| Capital Outflow | 4 | - |
| Investment | 16 | $\in[20 ; 22]$ |
| Private consumption | 60 | $\in[61 ; 62]$ |
| Public Expenditure | 20 | 19 |


| Parameters | Model |
| :--- | :--- |
| $\beta$ | 0.95 |
| $\theta$ | 0.30 |
| $v(\mathrm{~g})$ | $\ln (\mathrm{g})$ |
| $\mathrm{f}(\mathrm{k})=\mathrm{k}^{\lambda}$ | $\mathrm{k}^{0.4}$ |
| $\delta$ | 0.05 |
| $\alpha$ | 0.95 |
| $\alpha^{\phi}$ | 0.998 |
| $\varphi$ | 0.62 |
| $\phi$ | 0.85 |
| $\pi^{\mathrm{d}}$ | 0.04 |
| $\pi^{\mathrm{i}}$ | 0.04 |
| $\pi^{\mathrm{p}}$ | $\in[0 ; 0.9]$ |
| $\rho$ | $\in[-0.3 ; 1]$ |

$\rho$ is the correlation between moderate attacks, conditional to the no occurrence of an intense one.

