Marriage with Labor Supply*

Nicolas Jacquemet[†] Jean-Marc Robin[‡]

January 2012

Abstract

We propose a search-matching model of the marriage market that extends Shimer and Smith (2000) to allow for labor supply. We characterize the steady-state equilibrium when exogenous divorce is the only source of risk. The estimated matching probabilities that can be derived from the steady-state flow conditions are strongly increasing in both male and female wages. We estimate that the share of marriage surplus appropriated by the man increases with his wage and that the share appropriated by the woman decreases with her wage. We find that leisure is an inferior good for men and a normal good for women.

Keywords: Marriage search model, Collective labor supply, Structural estimation.

JEL classification: C78, D83, J12, J22.

---

*We benefited from comments by Elizabeth Caucutt, Pierre-André Chiappori, Bernard Fortin, Guy Lacroix, Costas Meghir, Rob Shimer, as well as conference participants at Congrès annuel de la Société Canadienne de Sciences Economiques, Montebello, 2008, Annual CIRPEE Workshop, Mont-Orford), 2008, INED Workshop « Recent advances in the economics of the household », Paris, 2010, ANR-Cemmap workshop « Matching and sorting : theory and applications », Sciences-Po, Paris, 2010, ICREA-MOVE Conference on Family Economics, Autonoma Barcelona, 2011, Conference "Economic analysis using linked employer and employee data: bringing together theory and empirics", Porto University, 2011, SED Annual Meeting, Gent, 2011, and seminar participants at various seminars. We gratefully acknowledge financial support from ANR-GINDHILA. Robin gratefully acknowledges financial support from the Economic and Social Research Council through the ESRC Centre for Microdata Methods and Practice grant RES-589-28-0001, and from the European Research Council (ERC) grant ERC-2010-AdG-269693-WASP.

[†]Paris School of Economics and University Paris 1 Panthéon–Sorbonne ; Nicolas.Jacquemet@univ-paris1.fr; Centre d'Economie de la Sorbonne, MSE, 106 Bd. de l'hôpital, 75013 Paris, France.

[‡]Sciences Po, Paris and University College of London; jeanmarc.robin@sciences-po.fr; Sciences-Po, Department of Economics, 28 rue des St Peres, 75007 Paris, France.

1 Introduction

One of the key issues in understanding how tax policies affect labor supply is the intra-household allocation of time and consumption. This is in particular the case of welfare benefits, such as the Working Family Tax Credit program in the UK and the Earned Income Tax Credit in the US, aimed at providing work incentives and a safety net against poverty at the same time. The models used to address these issues typically take the household as a unit with unitary preferences (e.g. Eissa and Hoynes, 2004). The Collective models of the family (Chiappori, 1988, 1992) offer a solution for improvement by modeling intrahousehold resource allocation, but the interest of this framework for policy evaluation is in turn limited by its inability to predict the impact of welfare policies on the sharing rule. Yet, the factors influencing the sharing rule, such as sex ratios or rules about divorce, are now well understood.[1] They are distribution factors conditioning the process of match formation and implicitly determining the threats that each household member can summon in the strategic negotiation for sharing resources. The ultimate model to evaluate welfare policies for the family must therefore be an equilibrium model of both match formation and intra-household resource allocation. The present paper offers one attempt at constructing such a model.

The literature on the family can be broadly classified into three main strands. A first series of papers address the issue of intra-household resource allocation for a given population of families, without worrying about their formation. The oldest papers used bargaining models.[2] Becker's (1981) theory of the benevolent dictator is a special theory of efficient resource sharing that does not use Nash bargaining. Chiappori's Collective model is another one, in many ways the least restrictive of all.[3] Then, in the last ten years a large variety of search-matching models of marriage markets have been proposed to explain certain stylized facts or trends such as declining marriage rates or increasing female college graduation rates, and to analyze various policies affecting the family.[4] At the same time, the general theory of search and matching in marriage markets (existence of equilibrium, multiplicity, conditions of positive assortative mating, PAM, etc.) was further developed,[5] but generated almost no empirical microeconomic applications. Wong (2003) stands like one rare exception.

[1]See Grossbard-Shechtman (1984), Brien (1997), Lundberg, Pollak, and Wales (1997), Chiappori, Fortin, and Lacroix (2002), DelBoca and Flinn (2005), Amuedo-Dorantes and Grossbard (2007), Seitz (2009).

[2]See Manser and Brown, 1980, McElroy and Horney, 1981, Lundberg and Pollak, 1993, 1996).

[3]See Chiappori and Donni (2009) for a recent survey of non-unitary models of the household. Chiappori's seminal contributions generated a long list of papers building on the model of the family as a Pareto equilibrium. We can only cite a few of them: Browning, Bourguignon, Chiappori, and Lechene (1994), Fortin and Lacroix (1997), Browning and Chiappori (1998), Chiappori, Fortin, and Lacroix (2002), Mazzocco (2004, 2007), Blundell, Chiappori, Magnac, and Meghir (2007), etc. Note also that the assumption of efficient allocations within the family has been disputed, in particular, by DelBoca and Flinn (2005, 2006, 2009).

[4]See e.g. Aiyagari, Greenwood, and Guner (2000), Greenwood, Guner, Knowles, Greenwood, Guner, and Knowles (2000), Caucutt, Guner, and Knowles (2002), Gould and Paserman (2003), Fernandez, Guner, and Knowles (2005), Chiappori and Weiss (2006, 2007), Chiappori and Oreffice (2008)

[5]See e.g. Sattinger (1995), Lu and McAfee (1996), Burdett and Coles (1997), Shimer and Smith (2000), Sattinger (2003), Eeckhout (1999), Eeckhout and Kircher (2010b,a).

Recently, the perfect-information assignment framework of Shapley and Shubik (1971) was revived in order to integrate the collective model within a matching framework.[6] In this paper, we aim at a similar target. However, instead of assuming a frictionless environment, we assume that single individuals randomly search for a partner and that they can date only one person at a time. We design a search-matching model of the marriage market with labor supply by building on the seminal works of Sattinger (1995), Lu and McAfee (1996) and Shimer and Smith (2000). The equilibrium model of this paper thus defines the outside option as the value of remaining single, which is equal to the instantaneous utility of the wage plus the option value of an eventual future marriage. Couples are formed when enough public goods are produced in the association. The surplus is divided via Nash bargaining.

There are many reason that explain our preference for search models. First, casual experience seems to suggest that it takes time and trial to find the right partner in life. Second, sequential search naturally yields mismatch. Third, it is easy with a search model to deal with continuous individual characteristics and difficult in a perfect-assignment model. Fourth and finally, forward looking behavior and risk are natural ingredients of search-matching models.

Despite a rather complicated structure the model remains tractable thanks to the steady state assumption. Although an important application of the matching framework is understanding long-term demographic changes such as increasing divorce and remarriage rates, we assume that these changes are slow enough for a steady state to hold at least approximately at all times. In other words, flows can vary over time in a trended or cyclical way, but net flows must remain small compared with gross flows. We show that steady-state flow conditions deliver important identifying restrictions on matching probabilities, and indirectly on the relationship between transfers—or the sharing rule defined as the share of male transfers in total transfers—and wages.

The steady-state flow conditions relate the matching probability to wage distributions in a simple way. Despite a small correlation between spouses' wages (25%), we find that the matching probability increases in both wages exponentially. So its shape is rather flat for most wages but very steep when both wages cross the median. We take this result as a strong indication of positive assortative matching, albeit unconventional.

Under the steady-state restriction, a lot of information can be drawn from cross-section data. Using SIPP data we observe that married men earn more than anybody else, and that the distribution of wages is about the same for married women, single women and single men. The model's estimates let us conclude from these facts that a man who is more productive in the labor market is also more attractive in the marriage market both by the higher income that he brings to the family and also by the larger public good value that he contributes to. Women contribute to family income and marriage surplus in the same way, but female wage is a less efficient input of household production than male wage. This makes a high wage a relatively less attractive trait

[6] See Choo and Siow (2006), Choo, Seitz, and Siow (2008b,a), Chiappori, Iyigun, and Weiss (2008), Siow (2009), Chiappori, Iyigun, and Weiss (2009), Chiappori, Salanie, and Weiss (2010).

in a woman than in a man. Consequently, high-wage men are over-represented among married couples and they are in a favorable position to bargain a higher share of the rent.

Hours worked do not bring any additional information on matching but allow to identify preferences for consumption and leisure. Married men, at all wages, work more that anybody else and married women less. We show that intrahousehold transfers cannot explain this fact. High-wage men take a bigger share of the rent and high-wage women a smaller share. If men, receiving bigger transfers, work more, it has to be that leisure is an inferior good for them. For women, the opposite is true: more non labor income implies more specialization into household production.

These results should not be taken at face value. The model is overly simplistic. First there are many dimensions of individual heterogeneity that matter in marriage decisions. Second, the reasons that make marriage attractive are certainly more sophisticated and diverse than household production, the way we model it in this paper. Children, for example, are certainly an important component of the marriage externality, which deserves to be properly modeled. Third and lastly, but the list could be longer, individuals and couples are subject to shocks that we rule out completely. Future versions of the model will see to that. This paper's main achievement is to show that search-matching models, with realistic distributions of individual heterogeneity, are more than theoretical objects of interest, and that they can be used efficaciously in empirical work.

The layout of the paper is as follows. First we construct the model. Second we study identification. Third we estimate the model non parametrically. The last section concludes. An appendix details the numerical techniques used to perform the nonparametric analyses.

2 The Model

The model builds on Shimer and Smith (2000), which we extend to allow for labor supply decisions, non symmetric equilibria, and a general matching function, not necessarily quadratic (Shimer and Smith) or linear (Tröger and Nöldeke, 2009).

2.1 Basic Setup

We consider a marriage market with L_m males and L_f females. The number of married couples is denoted by N and the respective numbers of single males and single females are $U_m = L_m - N$ and $U_f = L_f - N$.

Individuals differ in labor productivity, $x \in [\underline{x}, \overline{x}]$ for males and $y \in [\underline{y}, \overline{y}]$ for females. Let $\nu_m(x)$ and $\nu_f(y)$ denote the density functions of the measures of males of type x and females of type y, with $L_m = \int \nu_m(x)\,\mathrm{d}x$ and $L_f = \int \nu_f(y)\,\mathrm{d}y$. The wage densities for the sub-populations of singles are $u_m(x)$ and $u_f(y)$, with $U_m = \int u_m(x)\,\mathrm{d}x$ and $U_f = \int u_f(y)\,\mathrm{d}y$. The measure density of couples of type (x,y) is $n(x,y)$, with $N=\iint n(x,y)\,\mathrm{d}x\,\mathrm{d}y$ and

$$v_m(x)=\int n(x,y)\,\mathrm{d}y+u_m(x)\,, \tag{1}$$

$$v_f(y)=\int n(x,y)\,\mathrm{d}x+u_f(y). \tag{2}$$

In this paper, labor productivity is the only permanent source of individual heterogeneity. This is also the first dimension to consider as we are interested in modeling family labor supply. However, we shall later introduce a match-specific heterogeneity component.

We assume that only singles search for a partner, ruling out "on-the-marriage" search. The number of meetings per period is a function of the numbers of male and female singles, $M(U_m,U_f)$, and $\lambda_i=\frac{M(U_m,U_f)}{U_i}$ is the instantaneous probability that a searching individual of gender $i=m,f$ meets with a single of the other sex. We also denote $\lambda=\frac{M(U_m,U_f)}{U_mU_f}$.

All datings do not end up in wedlock. We assume that there exists a function $\alpha(x,y)\in[0,1]$ indicating the probability that a match (x,y) be consummated. The matching probability is an equilibrium outcome that will be later derived from fundamentals. The matching set is the support of α. Lastly, matches are exogenously dissolved with instantaneous probability δ.

2.2 Equilibrium Flows

In steady state, flows in and out of the stocks of married couples of each type must exactly balance each other out. This means that, for all (x,y),

$$\delta n(x,y)=u_m(x)\,\lambda_m\frac{u_f(y)}{U_f}\alpha(x,y)=\lambda\,u_m(x)\,u_f(y)\,\alpha(x,y). \tag{3}$$

The left-hand side is the flow of divorces. The right-hand side measures the flow of new (x,y)-marriages. It has three components: a single male of type x, out of the $u_m(x)$ ones, meets a single female with probability λ_m; this woman is of type y with probability $u_f(y)/U_f$; the knot is tied with probability $\alpha(x,y)$.

Integrating equation (3) over y, we obtain the steady-state flow condition for the stock of all married males of type x:

$$\delta\int n(x,y)\,\mathrm{d}y=\lambda u_m(x)\int u_f(y)\alpha(x,y)\,\mathrm{d}y.$$

Using equation (1) to substitute $\int n(x,y)\,\mathrm{d}y$ out of this equation yields

$$\delta\left[v_m(x)-u_m(x)\right]=\lambda u_m(x)\int u_f(y)\alpha(x,y)\,\mathrm{d}y,$$

or, equivalently,

$$u_m(x) = \frac{\delta v_m(x)}{\delta + \lambda \int u_f(y)\alpha(x,y)\,\mathrm{d}y}. \tag{4}$$

By symmetry, the equation defining the equilibrium distribution of wages in the population of single females is

$$u_f(y) = \frac{\delta v_f(y)}{\delta + \lambda \int u_m(x)\alpha(x,y)\,\mathrm{d}x}. \tag{5}$$

2.3 Time Use for Singles

Individuals draw utility from consumption and leisure. Consider a woman with direct utility $V_f(c,\ell)$ for consumption c and leisure ℓ. We first consider the case of a single person. Let p denote the time devoted to home production, say q, that is assumed to be a perfect substitute to purchased consumption. Given productivity y, a female consumer seeks to solve the problem:

$$\max_{c,\ell,p>0} V_f(c,\ell)$$

subject to

$$\begin{aligned} c &= y(T-p-\ell)+q, \\ q &= H_f(p,y), \end{aligned}$$

where T is total time endowment to be split between wage work, leisure and home production, and $H_f(p,y)$ is the home production function for single women, which may depend on labour market productivity y. For simplicity, we only consider interior solutions, ruling out labor market non participation.

The problem is obviously recursive. Let $p_f^0(y)$ be the solution to the home production problem for single women:

$$C_f(y) = \max_p H_f(p,y) - yp. \tag{6}$$

Then, optimal leisure is $\ell_f^0(x) \equiv \ell_f\left(y, yT + C_f(y)\right)$ solving the problem

$$v_f(y, yT + C_f(y)) = \max_{\ell>0}\{V_f(c,\ell) \mid c = y(T-\ell) + C_f(y)\}. \tag{7}$$

Specifically, we assume the following functional form for the indirect utility function:

$$v_f(y,m) = \frac{m - A_f(y)}{B_f(y)}, \tag{8}$$

where $B_f(y)$ is an aggregate price index, with $B_f(0) = 1$, and $A_f(y)$ is a minimum expenditure level necessary to attain positive utility, with $A_f(0) = 0$. Linearity with respect to total expenditure m will generate a simple rent sharing mechanism. But other specifications are possible.

The leisure demand function is $\ell_f(y,m)$, for time price y and total expenditure m, follows from indirect utility by application of Roy's identity,

$$\begin{aligned}\ell_f(y,m) &= -\frac{\partial_1}{\partial_2} v_f(y,m) \\ &= A'_f(y) + b'_f(y)[m - A_f(y)],\end{aligned} \tag{9}$$

where ∂_1 and ∂_2 denote partial derivatives, a prime (such as in b'_f and A'_f) denotes a derivative, and $b_f(y) = \log B_f(y)$. A standard specification is $A_f(y) = ya_f$ and $B_f(y) = y^{b_f}$, yielding the linear expenditure system:

$$y\ell_f = ya_f + b_f(m - ya_f).$$

We use symmetric definitions for males.

2.4 Time Use for Married Individuals

Marriage allows individuals to benefit from economies of scale and task specialisation. At the household level, let home production be $H(p_m, p_f, x, y) + z$, a function of time spent in home production by both spouses, p_m, p_f, and relative productivities x, y. We also introduce a source of noise, z, which is a match-specific component that is drawn at the first meeting from a zero-mean distribution denoted G. It aims at capturing all other dimensions of mutual attractiveness but labor market productivity.[7] Designing empirically tractable multidimensional matching models with random search is definitely a promising area for further research.[8]

Let $p_m^1(x,y), p_f^1(x,y)$ deliver optimal home production, defined as the solution to the problem

$$C(x,y) = \max_{p_m, p_f} \{H(p_m, p_f, x, y) - xp_m - yp_f\}.$$

Determining leisure for each household member can be done if we know how home production, $C(x,y) + z$, is split between man and woman. Let t_m, t_f denote a particular allocation, with

$$t_m + t_f = C(x,y) + z.$$

Then, leisure follows as $\ell_m(x, xT + t_m)$ and $\ell_f(y, yT + t_f)$, as in equation (9), assuming that individual preferences for consumption and leisure are independent of the marriage status.

A model for optimal transfers is developed in the next subsection.

[7] From a technical point of view, it also allows to smooth out the discontinuity at the boundary of the matching set.

[8] Wong (2003) aggregates individual characteristics into one single index in a Shimer-Smith model with positive assortative mating. An obvious extension of our model would add to z a function of individual characteristics, using single indexes to model complementarity in a simple way.

2.5 Optimal Rent Sharing Between Spouses

Let $W_m(v,x)$ denote the present value of marriage for a married male of type x receiving a flow utility v, and let $W_m(x)$ denote the value of singlehood (derived in the next section). The flow value of a marriage contract delivering v utils is

$$rW_m(v,x) = v + \delta\left[W_m(x) - W_m(v,x)\right],$$

where r is the discount rate and the second term of the right-hand side is the option value of divorce. We define marriage surplus for males as

$$S_m(v,x) = W_m(v,x) - W_m(x) = \frac{v - rW_m(x)}{r+\delta}, \tag{10}$$

with a similar definition for females.

Spouses have to decide on a particular allocation, t_m, t_f, of the value of the public goods produced in the household. Although transfers can be positive or negative, both should be positive in equilibrium, otherwise one is better off remaining single. We assume that spouses share resources cooperatively using Generalized Nash Bargaining with bargaining coefficient β, whereby transfers t_m and t_f solve

$$\max_{t_m,,t_f} S_m\left(v_m(x, xT+t_m), x\right)^{\beta} S_f\left(v_f(y, yT+t_f), y\right)^{1-\beta}$$

subject to the condition

$$t_m + t_f \leq C(x,y) + z.$$

The solution is trivially found to be such that

$$t_m(x,y,z) = s_m(x) + \beta[C(x,y) + z - s_m(x) - s_f(y)], \tag{11}$$

$$t_f(x,y,z) = s_f(y) + (1-\beta)[C(x,y) + z - s_m(x) - s_f(y)], \tag{12}$$

where we denote

$$s_m(x) = B_m(x)rW_m(x) - xT + A_m(x), \tag{13}$$

$$s_f(y) = B_f(y)rW_f(y) - yT + A_f(y). \tag{14}$$

Functions s_m and s_f are the non labor incomes that exactly compensate singles for not being offered the marriage option.

Two dating bachelors decide to match if the total surplus is positive, *i.e.* $s(x,y) + z > 0$ with

$s(x,y) = C(x,y) - s_m(x) - s_f(y)$. The matching probability then follows as

$$\begin{aligned} \alpha(x,y) &= \Pr\{s(x,y) + z > 0 | x,y\} \\ &= 1 - G[-s(x,y)]. \end{aligned} \tag{15}$$

2.6 The Value of Singlehood

The (flow) value of being single for males is

$$\begin{aligned} rW_m(x) &= v_m\left[x, xT + C_m(x)\right] + \lambda_m \mathbb{E}_{(y,z)} \max\left\{S_m\left[v_m[x, xT + t_m(x,y,z)], x\right], 0\right\} \\ &= v_m\left[x, xT + C_m(x)\right] + \lambda_m \iint \max\left\{S_m\left[v_m[x, xT + t_m(x,y,z)], x\right], 0\right\} \mathrm{d}G(z) \frac{u_f(y)}{U_f} \mathrm{d}y. \end{aligned}$$

Using the expression for marriage surplus (10), and substituting $s_m(x)$ for $rW_m(x)$ using (13), we obtain:

$$s_m(x) = C_m(x) + \frac{\lambda\beta}{r+\delta} \iint \max\{C(x,y) + z - s_m(x) - s_f(y), 0\} \, \mathrm{d}G(z) \, u_f(y) \, \mathrm{d}y. \tag{16}$$

A similar expression can be derived for females:

$$s_f(y) = C_f(y) + \frac{\lambda(1-\beta)}{r+\delta} \iint \max\{C(x,y) + z - s_m(x) - s_f(y), 0\} \, \mathrm{d}G(z) \, u_m(x) \, \mathrm{d}x. \tag{17}$$

2.7 Equilibrium

An equilibrium is a fixed point (u_m, u_f, s_m, s_f) of the following system of equations, where the first two equations determine equilibrium wage distributions for singles (derived from (4) and (5)), and the last two equations determine equilibrium values of singlehood:

$$u_m(x) = \frac{v_m(x)}{1 + \frac{\lambda}{\delta} \int u_f(y) \alpha(x,y) \, \mathrm{d}y}, \tag{18}$$

$$u_f(y) = \frac{v_f(y)}{1 + \frac{\lambda}{\delta} \int u_m(x) \alpha(x,y) \, \mathrm{d}x}, \tag{19}$$

$$s_m(x) = \frac{C_m(x) + \frac{\lambda\beta}{r+\delta} \iint \max\{z + C(x,y) - s_f(y), s_m(x)\} \, \mathrm{d}G(z) \, u_f(y) \, \mathrm{d}y}{1 + \frac{\lambda\beta}{r+\delta} U_f}, \tag{20}$$

$$s_f(y) = \frac{C_f(y) + \frac{\lambda(1-\beta)}{r+\delta} \iint \max\{z + C(x,y) - s_m(x), s_f(y)\} \, \mathrm{d}G(z) \, u_m(x) \, \mathrm{d}x}{1 + \frac{\lambda(1-\beta)}{r+\delta} U_m}, \tag{21}$$

with $U_m = \int u_m(x) \, \mathrm{d}x$, $U_f = \int u_f(y) \, \mathrm{d}y$, $\lambda = \frac{M(U_m, U_f)}{U_m U_f}$, and

$$\alpha(x,y) = 1 - G[-C(x,y) + s_m(x) + s_f(y)].$$

We write equations (20), (21) in that form so that s_m and s_f become fixed points of contracting operators given u_m and u_f (see Shimer and Smith, 2000).

Shimer and Smith (2000) prove the existence of an equilibrium for a simpler version of the model. They consider a symmetric equilibrium with a quadratic matching function (*i.e.* λ constant). The common distribution of singles ($u = u_m = u_f$) is the solution to an equation similar to equations (4) or (5) :

$$u(x) = \frac{v(x)}{1 + \frac{\lambda}{\delta} \int u(y) \alpha(x,y) \, \mathrm{d}y}, \tag{22}$$

that can be shown to be contracting once u is reparameterized as $v = \log(u)$. However, the general equilibrium fixed-point operator that involves α as well as u is not globally contracting. Shimer and Smith show that an equilibrium exists but it is not necessarily unique. Tröger and Nöldeke (2009) prove the existence of an equilibrium in u for all α (the first step of Shimer and Smith's proof) for the linear matching case ($\lambda = 1/\sqrt{U}$).

3 Data

In this section we present the data used in estimation, and we emphasize a few salient facts on wage and hour distributions that the model is challenged to replicate.

3.1 Demography of Marriages and Divorces

In the US in 2001, 30.1% of men (24.6% of women), 15 years and plus, were not married and 21 % (23.1%) were divorcees (Kreider, 2005). The median age at first marriage was 24 for men and 21.8 for women. Table 1 displays the percents of men and women of various cohorts who had not married at different ages. People are generally getting married later, but women persistently earlier than men.

In 2001, the median duration of first marriages was 8.2 and 7.9 years, respectively, for men and women. The median duration between first divorce and remarriage, for those married two times, was 3.3 years and 3.5, and second marriages lasted 9.2 and 8.1 years on average. About 75-80% of first marriages, depending on cohorts, reached 10 years, 60-65% 20 years, 50-60% 30 years. This indicates a separation rate of around 2.5% per year. For second marriages, 70-80% reached 10 years, 55% 15 years, and 50% 20 years, consistently with a slightly higher separation rate, around 3% annual.

According to survival data the median marriage duration should therefore be of 23-28 years instead of 8-9 years. The Poisson assumption is at odds with the data because a large proportion of marriages never end, and those who end in divorce do it relatively fast, in the first two years. One way of making divorce rates non stationary in the model is to permit z to change rapidly.

Table 1: Percent never married by age

	Men				Women			
Age	1945 to 1949	1950 to 1954	1955 to 1959	1960 to 1964	1945 to 1949	1950 to 1954	1955 to 1959	1960 to 1964
20	20.4	23.0	17.6	15.8	44.8	40.5	36.6	30.2
25	66.6	59.2	49.9	45.0	78.7	70.1	66.0	59.5
30	79.7	74.0	68.8	65.6	85.4	80.7	78.1	74.4
35	86.2	81.7	78.5	76.6	88.3	86.2	84.5	83.0
40	89.6	85.9	83.6		90.9	89.1	87.7	
45	91.5	88.2			92.1	90.6		
50	93.1				93.0			

Note: The Table reports the percentage of men and women who had never married, by age (in row) and cohort (in column).
Source; Kreider (2005), US Census Bureau, SIPP, 2001 Panel, Wave 2 Topical Module.

Thus, marriages resulting from a very large z would end fast if new, likely lower values are drawn soon.

3.2 Wage and Labor Supply Data

We use the US Survey of Income and Program Participation (SIPP) from 1996-1999. For every quarter that an individual is in the panel we collect information on the labor market state at the time of the survey, wages if employed, the number of hours worked, gender, and the corresponding information for the respondent's spouse if married. Our sample is restricted to individuals who are not self-employed or in the military, between the ages of 21 and 65.

We assume the environment stationary and calculate individual mean wages over employment spells, and mean hours worked over all quarters including non-employment spells. By this way, we somewhat reduce the transitory noise in wages and hours, and we reduce the number of labor-market non-participation spells (with declared hours equal to zero). Then we drop all observations with zero hours worked (individuals and individuals' spouses never employed in the 4-year period). This is definitely not a satisfactory procedure but the model cannot deal (for the moment) with both the extensive and the intensive margins of labor market participation. We also trim the 1% top and bottom wages to clean the data. In our sample, $2N/(2N+U_m+U_f) = 50.3\%$ of the population is married, out of $L_m = 13,223$ males and $L_f = 13,925$ females. There is a slight deficit of single males vis-a-vis single females: $U_m/U_f = 0.9$ ($N = 6,827, U_m = 6,386, U_f = 7,098$).

3.3 Wage Distributions

Figure 1a shows the Gaussian kernel density estimates of wage distributions by gender and marriage status. Married males have higher and more dispersed wages than single males. Single males, and single and married females exhibit strikingly similar wage distributions. Panel (b)

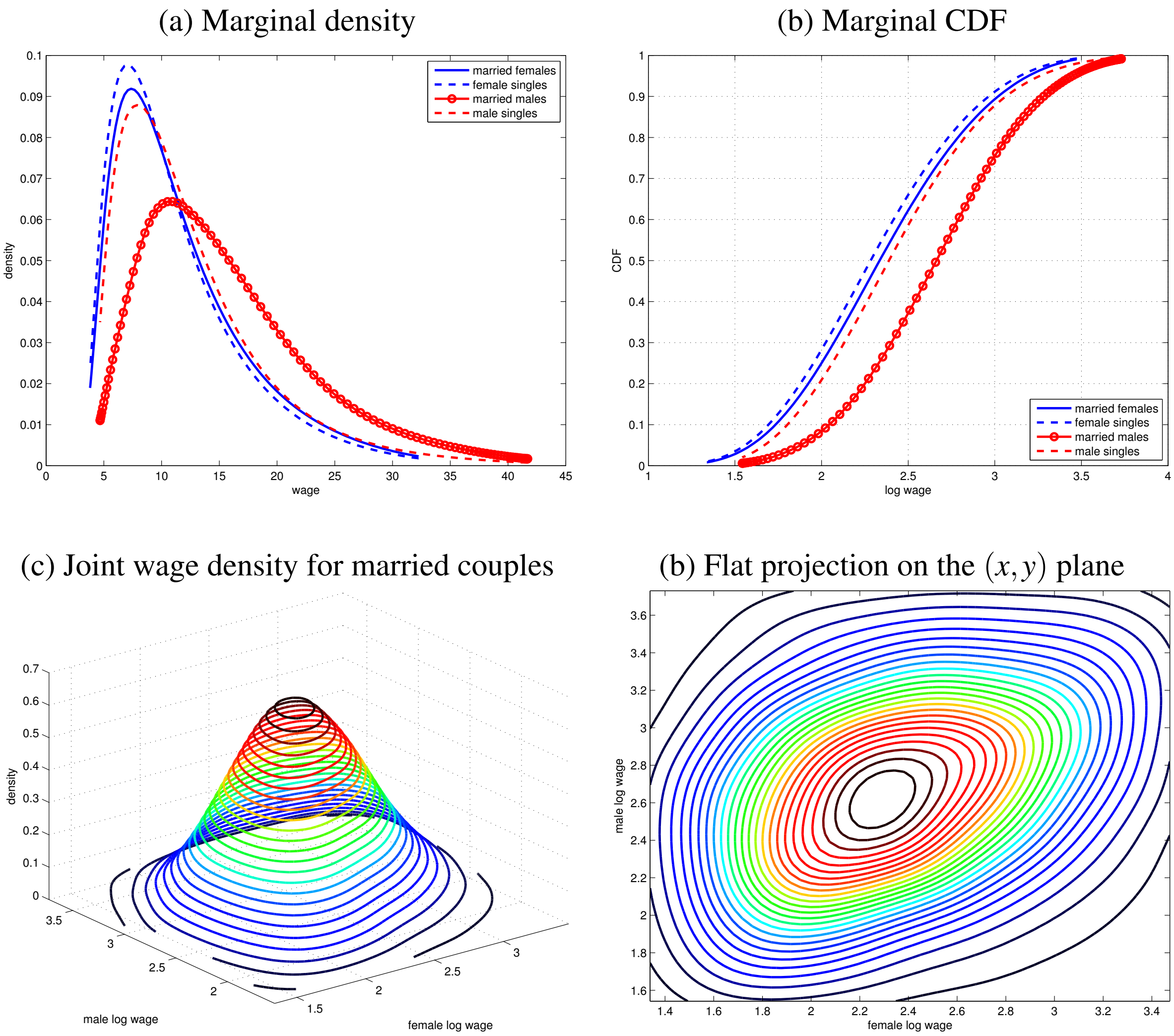


Figure 1: Wage distributions

displays the corresponding CDFs. The wage scale is in logs so as to emphasize the non normality of the distributions: both tails are fatter than for a normal distribution.

The figure in Panel (c) plots the joint distribution of wages among married couples, also estimated using a Gaussian kernel density. The most salient feature of this distribution is its very large support. Virtually all wage configurations, like a low male wage and a high female wage or vice versa, exist in the sample. Spouses' wages are only weakly correlated (25%), but the wage density is clearly oriented along the dominant diagonal (see the flat projection in Panel (d)). These patterns (wide support, low correlation) justify the introduction of the match-specific externality component (z): it allows for imperfect sorting due to unobserved matching characteristics and explains both the low correlation and the large support.

At this stage, the data seem like an impossible challenge for the theory. Such a low correlation between x and y tends to indicate a very little amount of sorting based on wages. However, the estimation of the model has some interesting surprises in store.

3.4 Hours

Figure 2a displays nonparametric kernel estimates of mean hours given own wage for single and married individuals. A clear ordering appears: married males work more than single males, who work more than single females, who work much more than married females. Marriage allows men to specialize in wage-work and women in household production. Note that the labor supply profiles of singles tend to tilt upwards, being more like married females' at low wages and more like married males' at high wages.

Panel (b) plots conditional mean hour estimates given both wages for married males and married females. There is some evidence of complementarity: male hours are higher and female hours lower for high wage men married to low wage women, and male hours are lower and female hours higher for high wage women married to low wage men.

4 Steady State and Matching Probabilities

In this section, we use the steady-state restriction (3) to estimate the shape of the matching probability $\alpha(x,y)$ as a function of wages. Before showing what it is, we first argue that the steady-state assumption is an acceptable first-order approximation, even in a changing macro-environment.

4.1 Divorce Rates

The steady-state equation (3), by relating marriages to divorces, makes it possible to use data on first marriage ages to learn about marriage duration and divorce frequency. In addition it

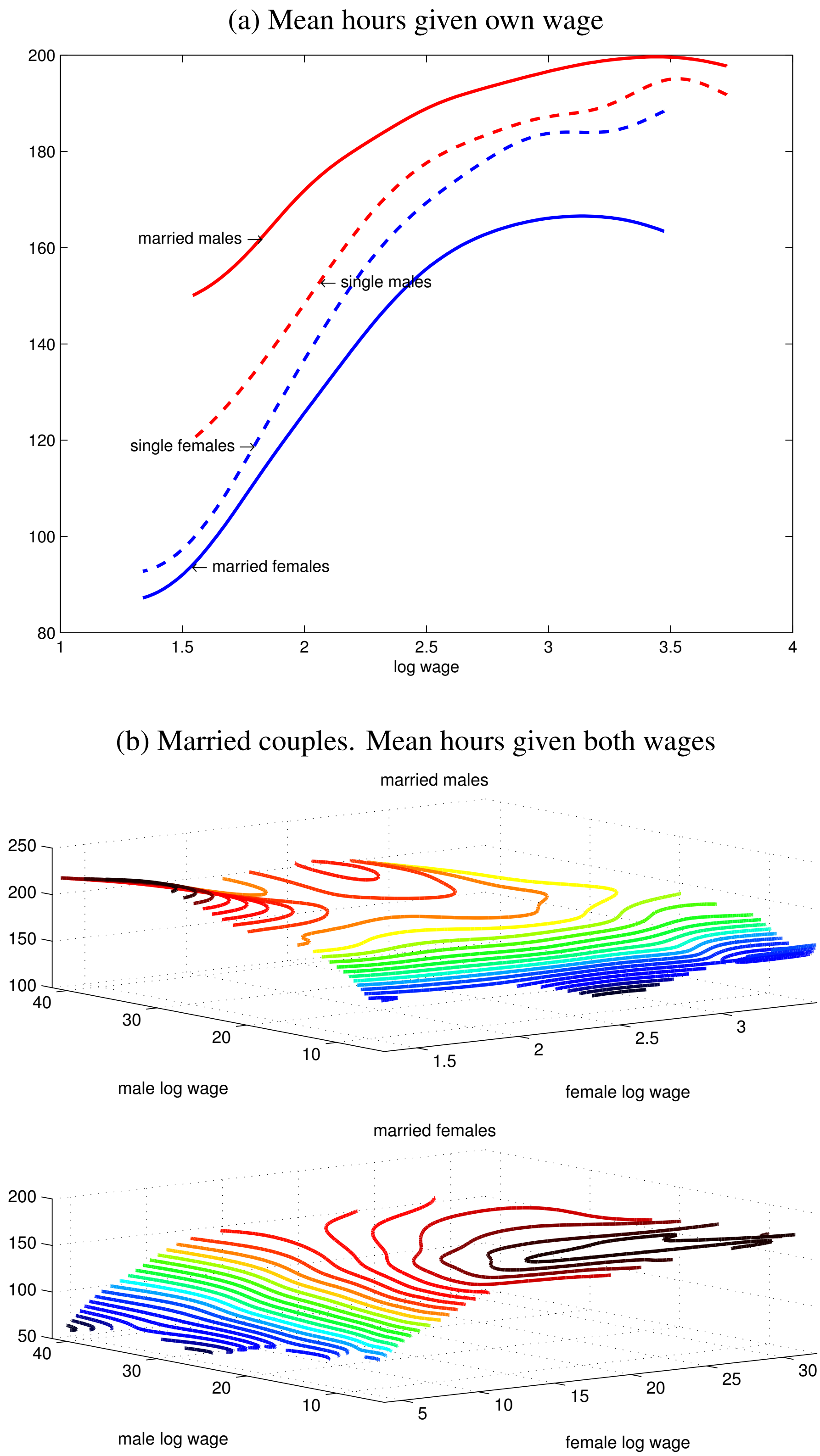
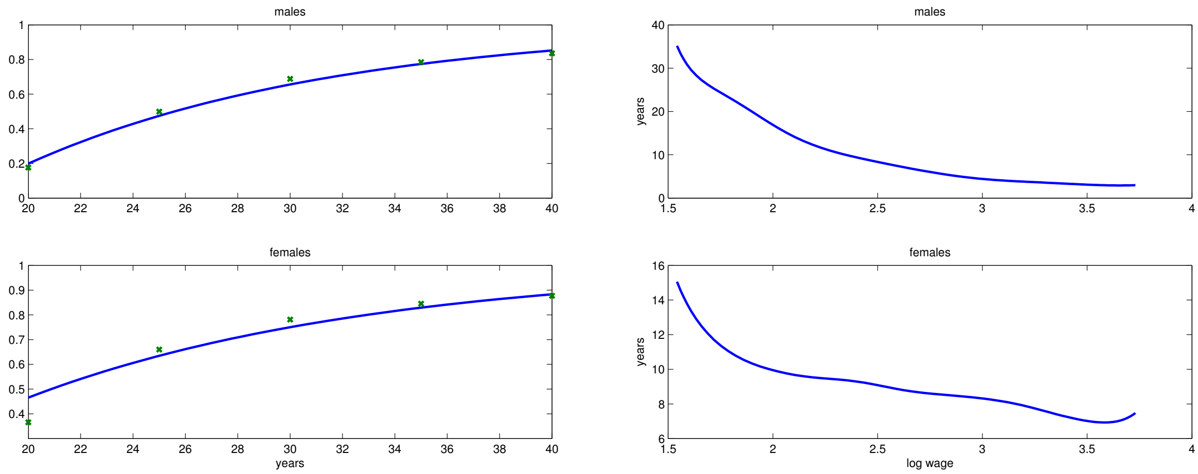


Figure 2: Mean hours

(a) Percent never married by age

(b) Median search duration before marriage



Note: x-marks show 1955-59 cohort data.

Figure 3: Duration of singlehood

can help to tell us which estimate in the bracket $[2.5, 8]$ percent per year to choose for δ in the estimation of other structural parameters.

The average probability for a single man of type x to randomly meeting a single woman and marry her is equal to

$$\mu_m(x) \equiv \lambda_m \int \frac{u_f(y)}{U_f} \alpha(x,y)\, \mathrm{d}y.$$

At the steady-state equilibrium described by equation (3), we have

$$\mu_m(x) = \delta \int \frac{n(x,y)}{u_m(x)}\, \mathrm{d}y = \delta \frac{\ell_m(x) - u_m(x)}{u_m(x)},$$

with a similar formula for single women. The average marriage rate among single men is the expectation of this quantity:

$$\mu_m \equiv \int \mu_m(x) \frac{u_m(x)}{U_m}\, \mathrm{d}x = \delta \frac{L_m - U_m}{U_m}.$$

Using the data displayed in Table 1 on the distribution of first marriage age of the 1955-1959 cohort we estimate both an age at which individuals start searching for a partner (age_{0m} and age_{0f}) and δ by running jointly the regressions of log survival probabilities on search durations:

$$\log S_m(t) = -\delta(t - age_{0m}),$$
$$\log S_f(t) = -\delta(t - age_{0f}),$$

where $S_m(t), S_f(t)$ denotes the probability of not being married by age t respectively for men and women. We estimate starting ages $age_{0m} = 17.3$ years and $age_{0f} = 11.8$ years. The estimated divorce rate is $\delta = 8.0\%$ annual. Figure 3a shows the fit of this simple model, which is good. The implied median first marriage duration is 8.7 years (mean of 12.6 years), which is remarkably similar to the value that can be directly estimated from divorce data.

The median waiting time before marriage is estimated 8.1 years for men (mean of 11.7) and 9.0 years for women (mean of 13.0). Figure 3b plots the implied average search durations by gender and wage (*i.e.* $\log(2)/\mu_i(x), i = m, f$). Low wage individuals have to wait for a very long time, and women more than men. The waiting time decreases with the wage. So, women get married before men but start searching earlier and take longer.

We obtain this result because there are more female singles (7,098) than male (6,386) in the sample. Given its simplicity, the model can only explain this difference from different wage distributions and different effects of wages on preferences and marriage externality. Of course, many other factors can explain the relative male scarcity in the marriage market, such as a higher mortality rate.

In the end, we conclude that the steady-state assumption is a reasonable approximation because estimates of marriage duration from data on wedlock age are consistent with direct observation. However, Table 1 seems to indicate that marriage and divorce habits do change over time, younger cohorts marrying both later and divorcing more often. This also happens with unemployment rates, for example, which do fluctuate over time (between 4% and 10% for the US). Still the steady-state approximation—job destruction rate divided by job destruction rate plus job finding rate—is a very good approximation. This is because inflows and outflows may fluctuate over time, yet they do so in tandem, so that net flows remain small compared to gross flows in all circumstances, which is what the steady-state restriction effectively means.

4.2 The Matching Probability

The equilibrium flow condition (3) implies that

$$\lambda\alpha(x,y) = \delta\frac{n(x,y)}{u_m(x)u_f(y)}. \tag{23}$$

So the matching probability is identified up to the multiplicative factor λ (or λ/δ) from the observed distributions of wages among singles and couples.

In absence of (good) data on datings, it is extremely difficult to separate the meeting probability from the probability of matching given meeting. In order to show the implied shape of $\alpha(x,y)$, we thus arbitrarily chose λ so that the meeting rate would be twice a year for men ($\lambda_m = 1/6$). The shape of the implied matching probability, as a function of wages, is unaffected by this choice. Note that it is likely that no wage combination (x,y) can induce marriage for sure: $\alpha(x,y) \leq 1$ for all (x,y). This condition imposes the lower bound $\max_{x,y}\frac{n(x,y)}{u_m(x)u_f(y)} = 1.37e-03$ on λ/δ, or a minimum meeting rate for men of $\lambda_m = .065$, or again a maximum of 15 datings per year.

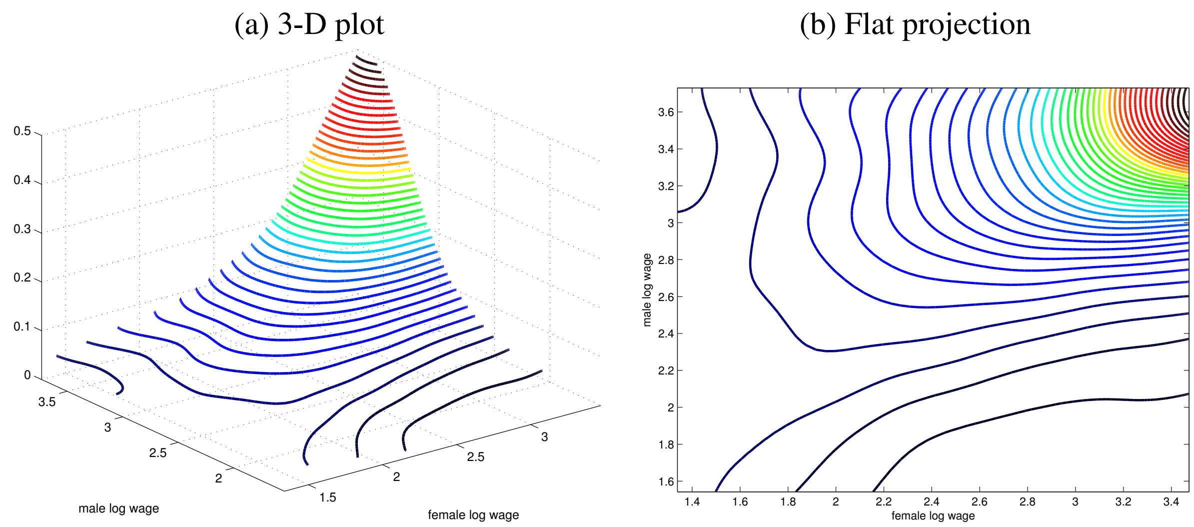


Figure 4: Matching probability, $\alpha(x,y)$ (with dating frequency twice a year for men)

Figure 4 displays the estimated matching probability function obtained by replacing wage densities in equation (23) by the estimates plotted in Figure 1. It is unambiguously increasing in both wages. More precisely, the matching probability increases exponentially with both wages and is rather flat for wages below the median with high-wage women having a very low probability of matching with anybody else but a high-wage man. This pattern indicates that positive assortative mating is definitely at work in the marriage market albeit mostly in the upper tail of the wage distribution. This explains why we find such a low overall correlation between spouses' wages.

5 Transfers and Sharing Rule

We now turn to the identification of transfers t_m, t_f, and the sharing rule in particular, defined by the proportion $t_m/(t_m+t_f)$. We first consider this problem using data on wage distributions only. We shall address the issue of the information that is provided by labor hours in the next section.

Let σ be the standard deviation of the distribution of the match-specific component z, and define G_0 as the distribution of z/σ, that is $G(z) = G_0(z/\sigma)$.

5.1 Average Transfers

By inverting the equilibrium relationship between $\alpha(x,y)$ and $s(x,y)$ in equation (15), we obtain

$$s(x,y) = -\sigma G_0^{-1}(1-\alpha(x,y)). \tag{24}$$

Then, noting that

$$\int \max\{z+s,0\}\,\mathrm{d}G(z) = s[1-G(-s)] + \int_{-s}^{+\infty} z\,\mathrm{d}G(z),$$

for all s, equation (16) relates $s_m(x)$ to total surplus $s(x,y)$ as

$$s_m(x) = C_m(x) + \beta\sigma\frac{\lambda}{r+\delta}\int \mu_{G_0}(\alpha(x,y'))u_f(y')\,\mathrm{d}y', \tag{25}$$

with

$$\mu_{G_0}(\alpha) = -\alpha G_0^{-1}(1-\alpha) + \int_{G_0^{-1}(1-\alpha)}^{+\infty} v\,\mathrm{d}G_0(v).$$

And by symmetry,

$$s_f(y) = C_f(y) + (1-\beta)\sigma\frac{\lambda}{r+\delta}\int \mu_{G_0}(\alpha(x',y))u_m(x')\,\mathrm{d}x'. \tag{26}$$

It follows that $\frac{s(x,y)}{\sigma}$, $\frac{s_m(x)-C_m(x)}{\beta\sigma}$ and $\frac{s_f(y)-C_f(y)}{(1-\beta)\sigma}$ are identified given G_0 and λ.

Actual transfers depend on the realized value of the match specific component of the public good, z, which we never observe. Define instead expected transfers as

$$\begin{aligned}\bar{t}_m(x,y) &\equiv \mathbb{E}[t_m(x,y,z)|x,y,s(x,y)+z>0],\\ \bar{t}_f(x,y) &\equiv \mathbb{E}[t_f(x,y,z)|x,y,s(x,y)+z>0],\end{aligned}$$

where the expectation is of course conditional on matching, $s(x,y)+z>0$. Equations (11) and (12) imply that

$$\begin{aligned}\bar{t}_m(x,y) &= s_m(x) + \beta\mathbb{E}[s(x,y)+z|x,y,s(x,y)+z>0]\\ &= C_m(x) + \beta\sigma\left[\frac{\lambda}{r+\delta}\int \mu_{G_0}(\alpha(x,y'))u_f(y')\,\mathrm{d}y' + \frac{\mu_{G_0}(\alpha(x,y))}{\alpha(x,y)}\right]\end{aligned} \tag{27}$$

and

$$\begin{aligned}\bar{t}_f(x,y) &= s_f(y) + (1-\beta)\mathbb{E}[s(x,y)+z|x,y,s(x,y)+z>0]\\ &= C_f(y) + (1-\beta)\sigma\left[\frac{\lambda}{r+\delta}\int \mu_{G_0}(\alpha(x',y))u_m(x')\,\mathrm{d}x' + \frac{\mu_{G_0}(\alpha(x,y))}{\alpha(x,y)}\right].\end{aligned} \tag{28}$$

Hence, $\frac{\bar{t}_m(x,y)-C_m(x)}{\beta\sigma}$ and $\frac{\bar{t}_f(x,y)-C_f(y)}{(1-\beta)\sigma}$ are in turn also identified given G_0 and λ on the support of $\alpha(x,y)$.

Mean transfers are proportional to the bargaining power coefficient (β for men, $1-\beta$ for women). Clearly enough, the same transfer can be obtained with a better outside option and a lower β. Collective models do not separate these two sources of bargaining power within the family. Indeed, there is a one-to-one relationship between the minimal utility that the Pareto program assigns to household members and the equivalent utility weight (or the Kuhn-Tucker multiplier). In a bargaining model, however, the weight of each individual (log) surplus in the Nash program (β and $1-\beta$) is structurally independent of the minimal utility levels (or outside options). This superior flexibility calls for more data, as identifying β effectively requires separate data on the size of the cake to be shared between parties and the shares themselves.[9]

With data only on wages and matching, it is not possible to identify σ either. This is because the only information that is used to identify transfers is the frequency of marriage for any particular wage configuration. Marriage occurs in the model when

$$t_m - C_m + t_f - C_f = C - C_m - C_f + z > 0.$$

In absence of any additional information on the component $C(x,y)$ of household production, allowing to anchor it at some known level, we can divide all terms of this equation by any positive number and the inequality remains true for all x,y. In the next section, we shall ask whether labor supply data can help to improve the inference on β and σ given our model. The answer is no.

5.2 Average Total Net Transfers

Average total net transfers

$$\bar{t}_m - C_m + \bar{t}_f - C_f = C - C_m - C_f + \mathbb{E}[z|x,y,s(x,y)+z>0]$$

are identified up to the scale factor σ given β,λ and G_0. To use equations (27) and (28) for estimation, we set $\beta = 0.5$ and $\sigma = 1000$, the order of magnitude of monthly earnings. Moreover, we set G_0 equal to the CDF of a standard normal distribution. We shall later address the issue of the separate identification of G_0. For the time being, let us assume that it is not too far from normal. Details on empirical implementation can be found in the appendix.

Figure 5a shows an estimate of the average total transfer function, $\bar{t}_m - C_m + \bar{t}_f - C_f$, and Panel (b) shows total household earnings (wage times hours worked) for comparison. The two functions have very different shapes. Household earnings are more or less linearly increasing in both wages (the mapping is approximately a linear plane). Total transfers, however, seem largely independent of female wages for all but the highest male wages. Thus, in the marriage market, a higher wage makes a woman more attractive only to high-wage men (say above the median wage). By contrast, a higher wage always makes a man more attractive.

The informational source of this inference is two-fold. First, single men do not have very different wages as single women, whereas married men earn more. This can only reflect a greater

[9] For example, Cahuc, Postel-Vinay, and Robin (2006) use data on firm value-added and wages to identify the bargaining power of workers in an equilibrium search-matching model.

(a) Mean total net transfers, $\bar{t}_m - C_m + \bar{t}_f - C_f$ (b) Mean total earnings

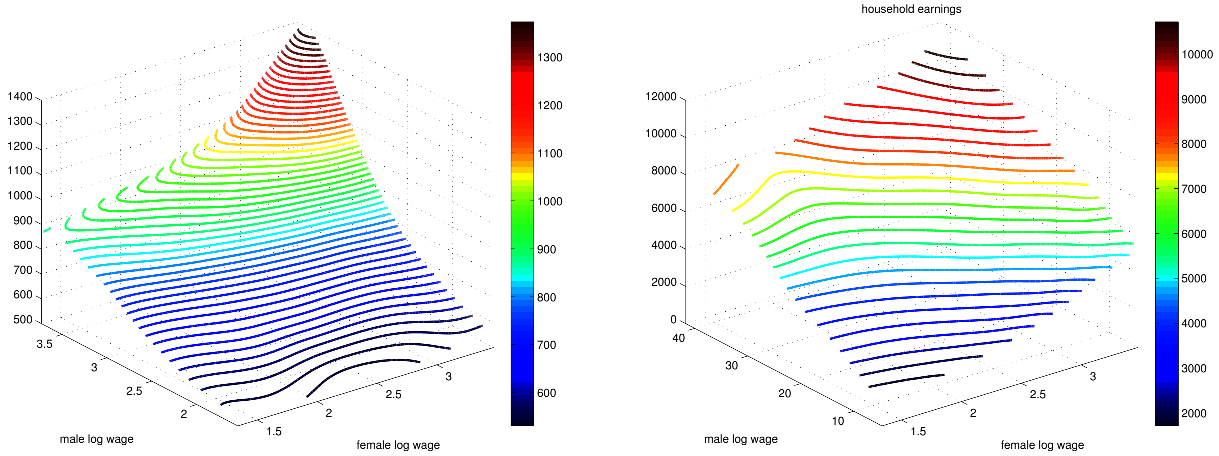


Figure 5: Total transfers and earnings

demand for high-wage men in the labor market. Second, this general effect being factored out, the matching probability strongly increases in both wages above the median. The joint density of married couples' wages is thus much fatter in that region than the product of wage densities for singles, signaling another selection effect: the configuration of two high wages produces a lot of public goods and is therefore highly demanded.

Of course there may exist alternative explanations, possibly along the line: married men are older, have accumulated more human capital and are thus better paid. A better model would indeed be non stationarity, but this is for the moment, analytically and empirically, out of reach. In addition, it is striking that the marriage gap in wages applies to men and not to women. Now, wage differentials may reflect other differences, like differences in education, and it may be that women value men's education more than most men value women's education, with the exception of highly educated men. This explanation calls for another type of extension, with multidimensional heterogeneity. This one seems easier to grasp.

5.3 The Sharing Rule

The sharing rule $\frac{\bar{t}_m - C_m}{\bar{t}_m - C_m + \bar{t}_f - C_f}$ is identified given β, λ and G_0, irrespective of σ. Figure 6a plots its estimate using the same calibrations of β, λ and G_0 as in the preceding sections. The median share of total transfers that goes to a married man with a median wage (2.6 in logs) is about one half, with some variance depending on his wife's wage. The same is true for a married woman with a median wage (around 2.4 in logs). This is expected given the arbitrary choice of $1/2$ for β. However, it is most remarkable that the variance of transfer shares given one's own wage is much larger for females than for males. This can be seen most clearly in Panels (b) and (c) which plot flat projections in (x,z) and (y,z) planes. Looking at the picture more closely, we see that a linear function of x and y would not be a bad approximation of the sharing rule, with a positive and steep slope in the x-direction (male wages), and a negative and relatively flatter

(a) 3-D plot (b) Flat x,z-projection (c) Flat y,z-projection

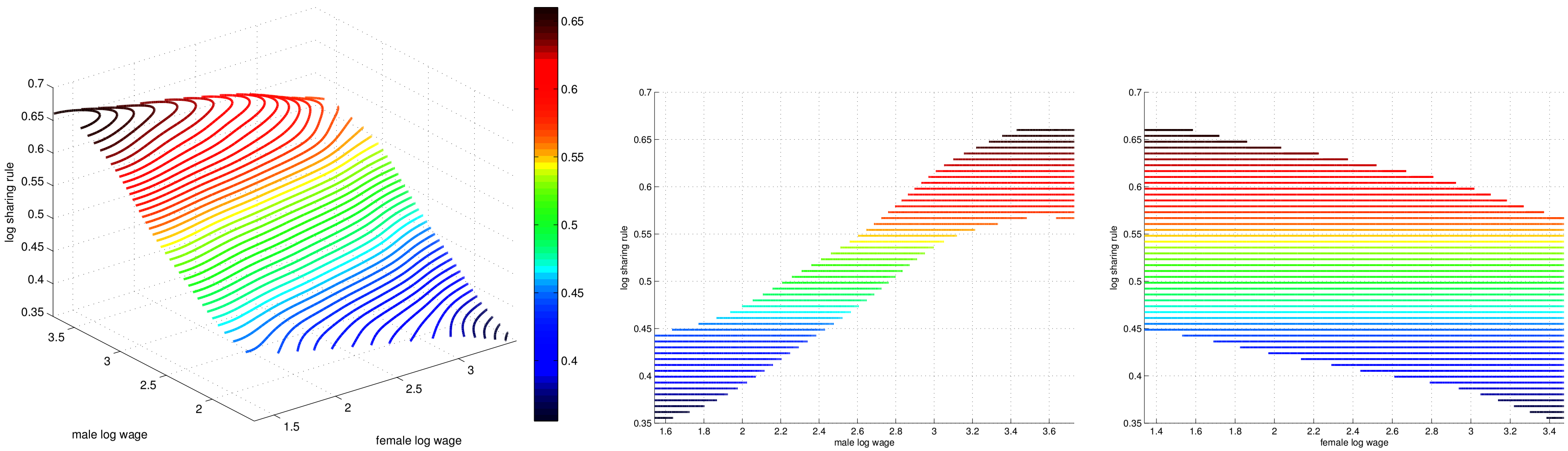


Figure 6: Sharing rule. Share of total net transfers that goes to man, $\frac{\bar{t}_m - C_m}{\bar{t}_m - C_m + \bar{t}_f - C_f}$

slope in the y-direction (female wages).

Summing up, a man who is more productive in the labor market is also more attractive in the marriage market both by the higher income that he brings to the family, and also by the larger public good value that he generates. Women contribute to family income and marriage surplus in the same way, although female wage is a less efficient input of household production than male wage. This makes a high wage a less attractive trait in a woman than in a man. Consequently, high-wage men are over-represented among married couples and they are in a better position to bargain a higher share of the rent.

6 Inference from Hours

Let $\ell_m^0(x), h_m^0(x), p_m^0(x)$, with $\ell_m^0 + h_m^0 + p_m^0 = T$, denote leisure hours, hours worked and time spent in home production by sing men with productivity x. Let $\ell_f^0(y), h_f^0(y), p_f^0(y)$ be the notation for females, and let $\ell_m^1(x,y,z), h_m^1(x,y,z), p_m^1(x,y)$ by the notation for married males (use a subscript f instead of m for married females).

First, from equation (9), we deduce that

$$\ell_m^1(x,y,z) - \ell_m^0(x) = b_m'(x)\left[t_m(x,y,z) - C_m(x)\right]. \tag{29}$$

The match component z being not observed, let us average it out and define

$$\Delta\ell_m(x,y) \equiv \mathbb{E}(\ell_m^1|x,y) - \ell_m^0(x) = b_m'(x)\beta\sigma\frac{\bar{t}_m(x,y) - C_m(x)}{\beta\sigma}. \tag{30}$$

The normalized net male transfer $\frac{\bar{t}_m(x,y)-C_m(x)}{\beta\sigma}$ being already identified, this equation identifies $\beta\sigma$ and $b_m'(x)\beta\sigma$. As there is no obvious normalization that can be applied to the income effect $b_m'(x)$, hours do not help to identify β and σ.

We also have,

$$\begin{aligned} H_m(p_m^0, x) &= C_m(x) + xp_m^0 \\ H_f(p_f^0, y) &= C_f(y) + yp_f^0 \\ H(p_m^1, p_f^1, x, y) &= C(x,y) + xp_m^1 + yp_f^1 \end{aligned}$$

6.1 Partial Identification of Individual Preferences

For a given calibration of β and σ, Chiappori's fundamental result holds: one private good (labor) is enough to identify individual preferences. Indeed, $B_m(x), B_f(y)$ follow as

$$B_m(x) = \exp \int_0^x b'_m(x')\,\mathrm{d}x', \tag{31}$$

$$B_f(y) = \exp \int_0^y b'_f(y')\,\mathrm{d}y'. \tag{32}$$

Moreover, the two remaining unknowns $A_m(x), A_f(y)$, can easily be recovered from hours worked by single individuals. Setting the transfer equal to 0 in equations (9) and (**??**), these equations can be rewritten as linear ordinary differential equations:

$$\frac{\mathrm{d}[xT - A_m(x)]}{\mathrm{d}x} - b'_m(x)[xT - A_m(x)] = h^0_m(x) + p^0_m(x),$$

$$\frac{\mathrm{d}[yT - A_f(y)]}{\mathrm{d}y} - b'_f(y)[yT - A_f(y)] = h^0_f(y) + p^0_f(y).$$

The solution is

$$xT - A_m(x) = B_m(x) \int_0^x \frac{h^0_m(x') + p^0_m(x')}{B_m(x')}\,\mathrm{d}x', \tag{33}$$

$$yT - A_f(y) = B_f(y) \int_0^y \frac{h^0_f(y') + p^0_f(y')}{B_f(y')}\,\mathrm{d}y', \tag{34}$$

using initial conditions $A_m(0) = A_f(0) = 0$ and $B_m(0) = B_f(0) = 1$.

6.2 Estimation of Preference Parameters

We estimate income effects, $b'_m(x)$, by regressing $\Delta_m(x,y)$ on $\bar{t}_m(x,y)$. The corresponding population parameter is

$$b'_m(x) = \frac{\int \Delta_m(x,y)\bar{t}_m(x,y)n(x,y)\,\mathrm{d}y}{\int \bar{t}_m(x,y)^2 n(x,y)\,\mathrm{d}y},$$

and similarly for women:

$$b'_f(x) = \frac{\int \Delta_f(x,y)\bar{t}_f(x,y)n(x,y)\,\mathrm{d}x}{\int \bar{t}_f(x,y)^2 n(x,y)\,\mathrm{d}x}.$$

Then preference parameters A and B can be calculated using equations (33), (34), (31) and (32).[10]

Figure 7a shows the estimated income effects, $-b'_m(x)$ and $-b'_f(y)$, obtained with $\beta = 0.5$ and $\sigma = 1000$, together with $B_m(x)$ and $B_f(y)$ (see appendix for estimation details). Panel (b) shows estimates of price effects $T - A'_m(x)$ and $T - A'_f(y)$, as well as $A_m(x)$ and $A_f(y)$. A

[10] We set the maximal number of hours T equal to the upper bound of hours in the sample, *i.e.* $T = 667$ hours per month (28 full 24-hour days!).

low-order polynomial approximation is shown (dashed curves) for comparison.

For men, leisure (household production) is an inferior good—higher transfers increase hours worked—whereas for women, it is a normal good. This seems necessary to explain why married men work more than singles and married women less. Note that this is not an uninteresting economic result (as it looks like an explanation by exogenous differences in preferences). We could have found that household transfers are unfavorable to married men, forcing them to work more to compensate. This is not the way it works. Married men have higher wages than other categories of individuals because they are more attractive (selection effect). Better outside options let them draw a bigger share of the surplus. Receiving more transfers, if they also work more, it must be because they like it. The specialization story where men work more because they have a comparative advantage in wage-work—indeed they are better paid—leaving women to perform more household tasks, does not seem to provide a satisfactory description of both wage and hour distributions when all equilibrium restrictions are respected.

6.3 Identification of G_0

The identification of the standardized distribution of the match specific component of the public good, G_0, comes from the residuals of the labor supply equations for couples. Thus, equation (29) and (30) imply that

$$\begin{aligned}\frac{h_m^1(x,y,z) - \mathbb{E}(h_m^1|x,y)}{-b_m'(x)\beta\sigma} &= \frac{t_m(x,y,z) - \bar{t}_m(x,y)}{\beta\sigma} \\ &= \frac{z}{\sigma} - \mathbb{E}\left(\frac{z}{\sigma}\middle| x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right), \end{aligned} \tag{35}$$

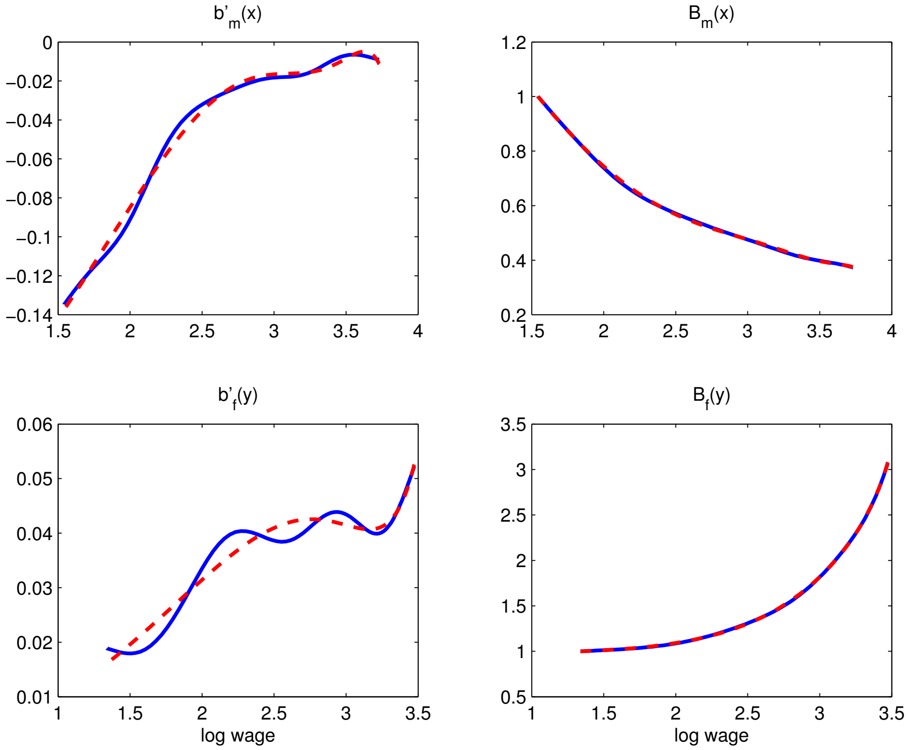
and similarly for females:

$$\frac{h_f^1(x,y,z) - \mathbb{E}(h_f^1|x,y)}{-b_f'(y)(1-\beta)\sigma} = \frac{z}{\sigma} - \mathbb{E}\left(\frac{z}{\sigma}\middle| x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right). \tag{36}$$

Suppose for a moment that $b_m'(x)\beta\sigma$ and $s(x,y)/\sigma$ can be identified without knowledge of G_0, then the first equation identifies the distribution of z/σ given x,y and conditional on matching: $\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}$. It follows that the unconditional distribution of z/σ, alias G_0, is identified if $\alpha(x,y)$ tends to one for some limiting value of (x,y). However, it is unlikely that the decision to marry could be entirely determined by wages, so $\alpha(x,y)$ should be less than one for all x,y. Moreover, the estimators of $b_m'(x)\beta\sigma$ and $s(x,y)/\sigma$ that we have previously designed depend on G_0. So, equation (35) identifies G_0 as the solution to a complicated inverse problem. It is thus difficult, if not impossible, to construct a nonparametric estimator for the distribution of z/σ.

For this reason, we did not attempt to estimate G_0 and postulated a standard normal distri-

(a) Income effects



(b) Price effects

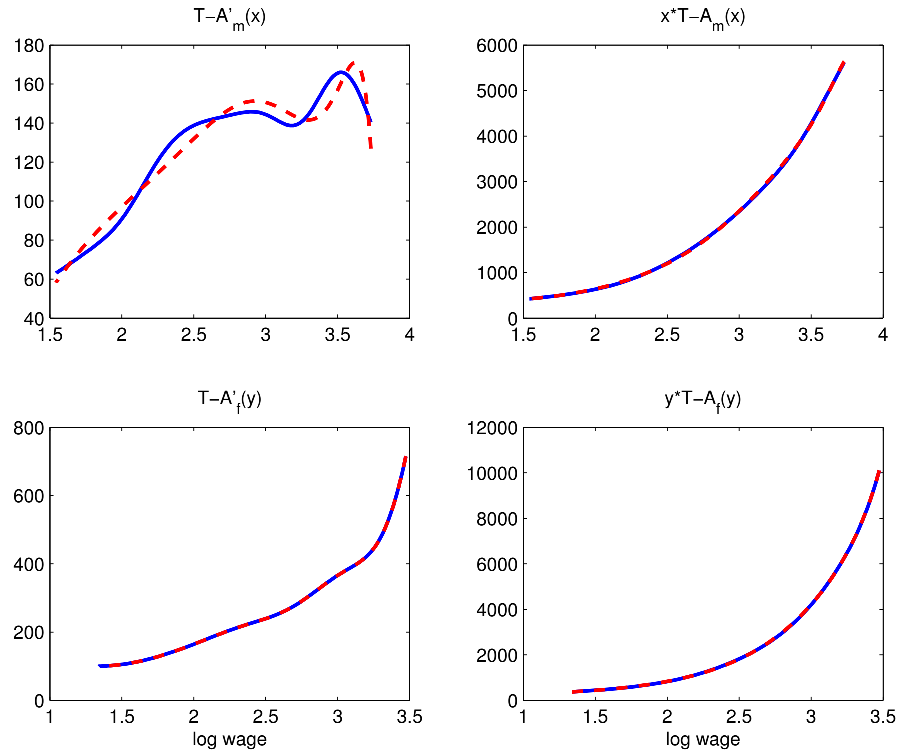


Figure 7: Preference parameters (the dotted line correspond to 4th order approximations)

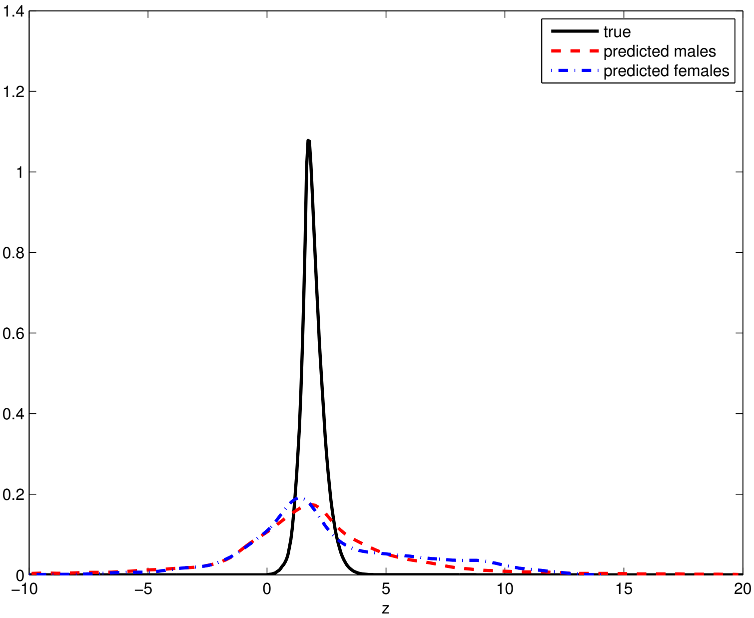


Figure 8: Theoretical and predicted truncated distributions of z/σ for G_0 normal

bution instead; in which case,

$$\mu_{G_0}(\alpha) = -\alpha\Phi^{-1}(1-\alpha) + \phi \circ \Phi^{-1}(1-\alpha),$$

where ϕ is the PDF of the standard normal distribution.[11] Figure 8 shows the estimated distribution density of

$$\frac{h_m^1(x,y,z) - \mathbb{E}(h_m^1|x,y)}{-b_m'(x)\beta\sigma} + \mathbb{E}\left(\frac{z}{\sigma}\middle| x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right) = \frac{h_m^1(x,y,z) - h_m^0(x)}{-b_m'(x)\beta\sigma} - \frac{s_m(x)}{\beta\sigma} - \frac{s(x,y)}{\sigma}$$

for males (labeled "predicted males" in the plot) and the symmetric PDF for females (labeled "predicted females"):

$$\frac{h_f^1(x,y,z) - \mathbb{E}(h_f^1|x,y)}{-b_f'(y)(1-\beta)\sigma} + \mathbb{E}\left(\frac{z}{\sigma}\middle| x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right) = \frac{h_f^1(x,y,z) - h_f^0(y)}{-b_f'(y)(1-\beta)\sigma} - \frac{s_f(y)}{(1-\beta)\sigma} - \frac{s(x,y)}{\sigma}$$

The solid line plots the truncated PDF of z/σ for married couples (*i.e.* given $\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}$), assuming z/σ drawn from a normal distribution at meeting time.

Two remarks are in order. First the distributions of the thus standardized hours of married

[11] We also experimented with mixtures of two normals with varying kurtosis, and obtained similar results.

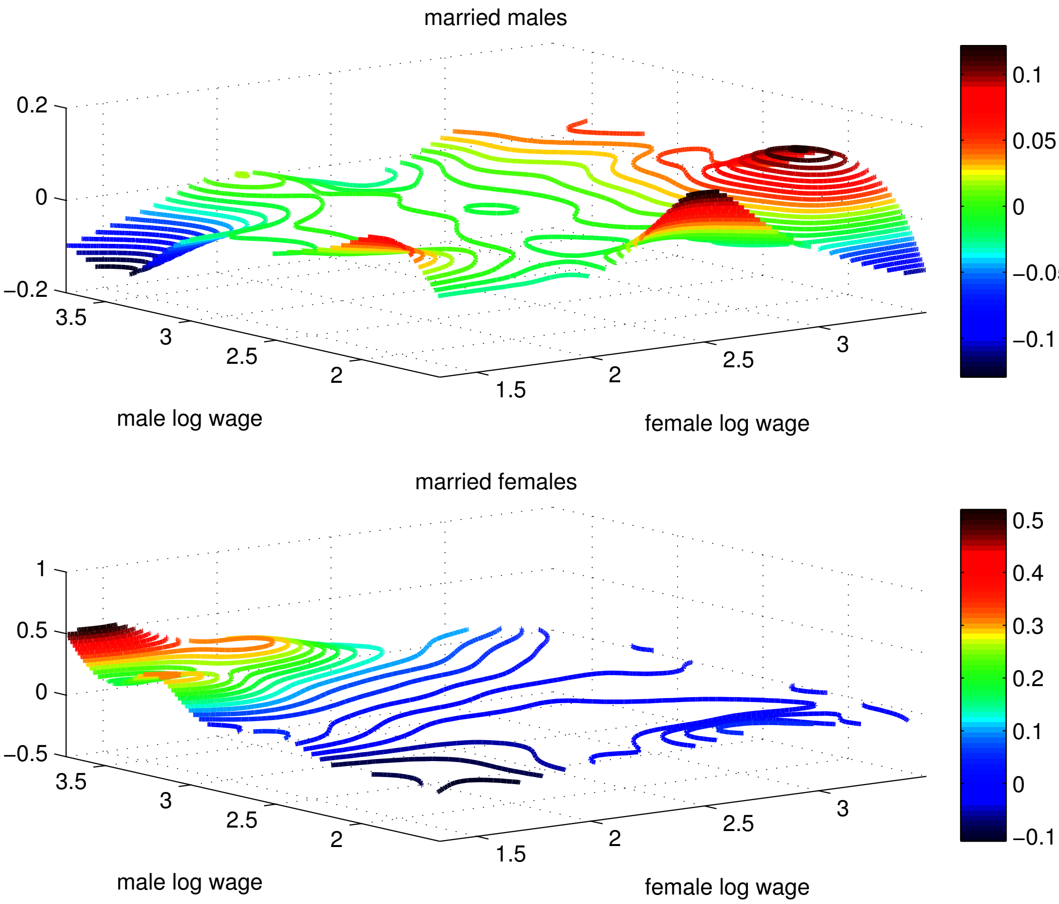


Figure 9: Fit of conditional mean hours, $\widehat{h}_i^1(x,y)/\mathbb{E}(h_i^1|x,y)-1, i=m,f$

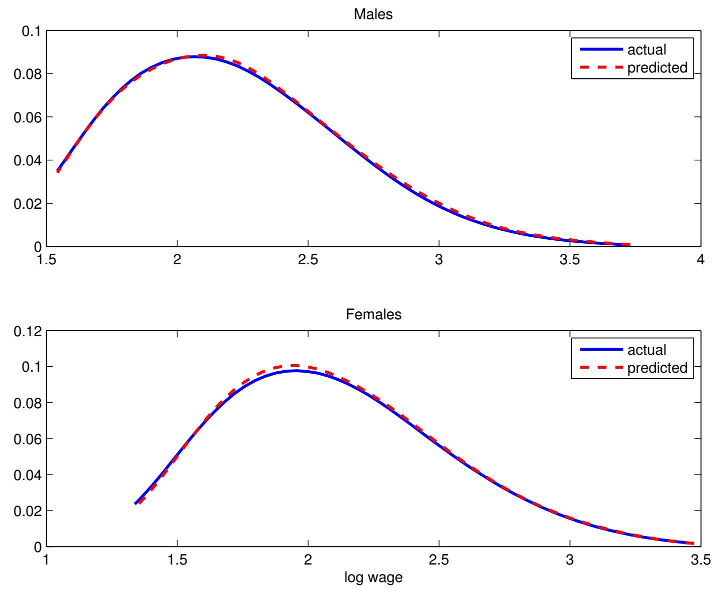
men and women, h_m^1 and h_f^1, are remarkably aligned. Second, the truncated distribution of z/σ has a much smaller variance (two orders of magnitude smaller!). This discrepancy reveals the presence of two classical measurement errors with independent and identical distribution.

7 Equilibrium Computation and Low Order Approximation

In this section we want to verify that we can go backwards, that is, calculate the equilibrium wage distributions and labor supply functions from the previous nonparametric estimates of the structural parameters, namely, the marriage externality function $C(x,y)$ and the preference parameters. To calculate the equilibrium (equations (18), (19), (20), (21)) we postulated a Cobb-Douglas meeting function $M(U_m,U_f)=M_0U_m^{1/2}U_f^{1/2}$, and estimated M_0 as $M_0=\lambda U_m^{1/2}U_f^{1/2}$ for the calibrated value of λ. Despite the lack of a global contraction mapping property, we found that the standard fixed-point iteration algorithm, $x_{n+1}=Tx_n$, worked well in practice, even starting far from the equilibrium (like with $s_m(x)=0$ and $u_m(x)=\ell_m(x)$). Details about the numerical computation of the equilibrium (discretization plus standard fixed point algorithm) can be found in the appendix.

The just-identified nonparametric estimates unsurprisingly deliver a perfect fit of wage dis-

(a) Wage densities for singles



(b) Mean hours given own wage

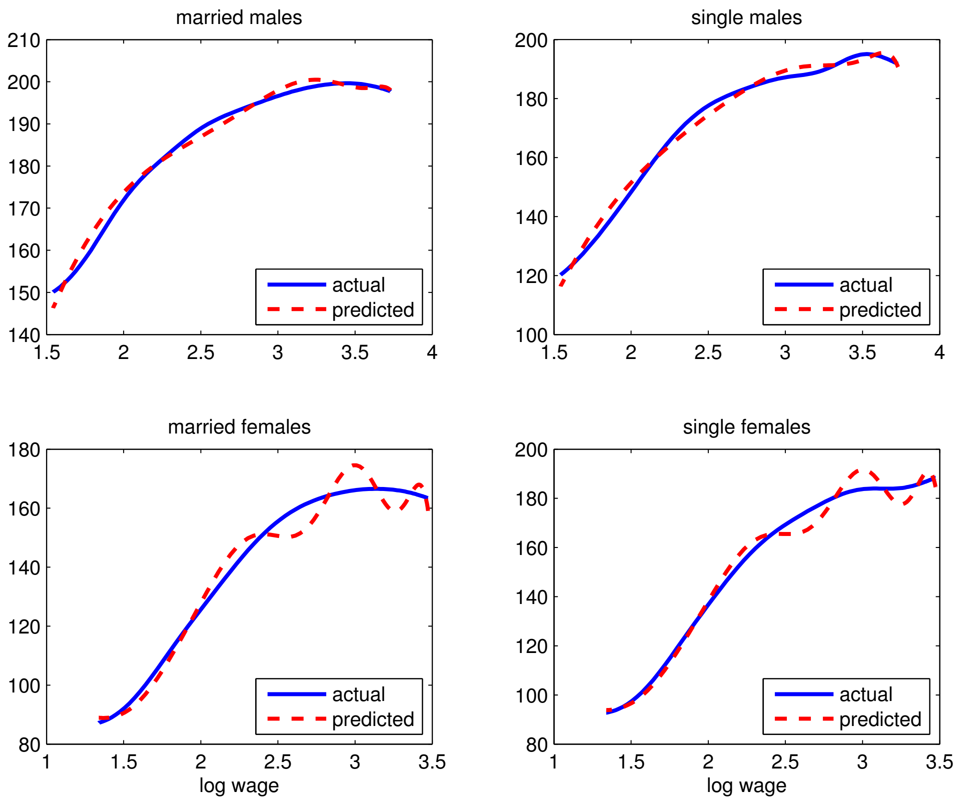


Figure 10: Fit of the model. 4th order polynomial approximation

tributions and own wage-hour supply functions. However the ability of the model to fit the conditional mean hours given both spouses' wages is limited by the form of the transfer functions, and the model fails to some extent to fit hours for high wage men and low wage women (see Figure 9).

Simulating the model using the full nonparametric estimates of the structural parameters yields, as expected, a perfect fit. What is more challenging is to use a small-order polynomial approximation of the functional parameters (technical details in appendix). Figure 10 shows the fit of a 4th order polynomial approximation. It is quite good, despite unwanted undulations of hours at higher wages.

8 Conclusion

In this paper, we have developed a prototypical version of a search-matching model of the marriage market with labor supply, extending the model in Shimer and Smith (2000) to allow for labor supply. We study its identification and estimation from cross-section data.

The model is rich of interesting lessons. We first show that wage distributions provide useful information on matching patterns. Despite a low correlation between spouses' wages, we estimate a matching probability function that is strongly increasing in both wages. However, if a high wage is always increasing male attractiveness, a high wage makes a female attractive only to high wage males. In consequence, a higher wage provides men with a greater share of the marriage surplus but not women. Finally, with only one private good (labour supply), hours give no additional information on matching and rent sharing. Differences in hours between single and married individuals suggest that labor is a normal good for men, but an inferior good for women.

Many possible extensions of the model easily come to mind, like endogenizing divorce, either through shocks to z or via on-the-marriage search, or like allowing for other dimensions of heterogeneity but wages, or introducing children.

Appendix

Computational Details

This appendix shortly describes the numerical tools used for estimation.

First, we discretize continuous functions on a compact domain using Chebyshev grids.[12] For example, let $[\underline{x},\overline{x}]$ denote the support of male wages, we construct a grid of $n+1$ points as

$$x_j = \frac{\overline{x}+\underline{x}}{2} + \frac{\overline{x}-\underline{x}}{2}\cos\frac{j\pi}{n},\ j=0,...,n.$$

[12] It can be shown that the error associated to a polynomial approximation (of any order) of an unknown function at any point x is proportional to $\prod_{j=0}^{n}(x-x_j)$. The Chebyshev points are the $\{x_j\}_{j=0,...n}$ minimizing this quantity.

Second, to estimate wage densities $n(x,y)/N$, $u_m(x)/U_m$ and $u_f(y)/U_f$ on those grids we use kernel density estimators with twice the usual bandwidth to smooth the density functions in the tails. This is important as, for instance, we divide n by $u_m u_f$ to calculate α according to (23). Additional smoothing is thus required.

Third, many equations involve integrals. Given Chebyshev grids, it is natural to use Clenshaw-Curtis quadrature to approximate these integrals:

$$\int_{\underline{x}}^{\overline{x}} f(x)\,\mathrm{d}x \simeq \frac{\overline{x}-\underline{x}}{2}\sum_{j=0}^{n} w_j f(x_j),$$

where the weights w_j can be easily computed using Fast Fourier Transform (FFT). The following MATLAB code can be used to implement CC quadrature (Waldvogel, 2006):

```
function [nodes,wcc] = cc(n)
nodes = cos(pi*(0:n)/n);
N=[1:2:n-1]'; l=length(N); m=n-l;
v0=[2./N./(N-2); 1/N(end); zeros(m,1)];
v2=-v0(1:end-1)-v0(end:-1:2);
g0=-ones(n,1); g0(1+l)=g0(1+l)+n; g0(1+m)=g0(1+m)+n;
g=g0/(n^2-1+mod(n,2)); wcc=real(ifft(v2+g));
wcc=[wcc;wcc(1)];
```

Note that, although Gaussian quadrature provides exact evaluations of integrals for higher order polynomials than CC, in practice CC works as well as Gaussian. On the other hand, quadrature weights are much more difficult to calculate for Gaussian quadrature. See Trefethen (2008).

Fourth, we need to solve functional fixed point equations. The standard algorithm to calculate the fixed point $u(x) = T[u](x)$ is to iterate $u_{p+1}(x) = Tu_p(x)$ on a grid. If the fixed point operator T involves integrals, we simply iterate the finite dimensional operator $\widehat{T}$ obtained by replacing the integrals by their approximations at grid points. For example, an equation like

$$u(x) = T[u](x) = \frac{\ell(x)}{1+\rho\int_{\underline{x}}^{\overline{x}} u(y)\alpha(x,y)\,\mathrm{d}y}$$

becomes

$$u = [u(x_j)]_{j=0,\dots,n} = \widehat{T}(\mathbf{u}) = \left[\frac{\ell(x_j)}{1+\rho\sum_{k=0}^{n} w_k u(x_k)\alpha(x_j,x_k)}\right]_{j=0,\dots,n}.$$

It was sometimes necessary to "shrink" steps by using iterations of the form $u_{p+1} = u_p + \theta(Tu_p - u_p)$ with $\theta \in (0,1]$. A stepsize $\theta < 1$ may help if T is not everywhere strictly contracting.

Fifth, the fact that CC quadrature relies on Chebyshev polynomials of the first kind also allows to interpolate functions very easily between points $y_0 = f(x_0),\dots,y_n = f(x_n)$ using Discrete Cosine Transform (DCT):

$$f(x) = \sum_{k=0}^{n} Y_k \cdot T_k(x), \tag{A.1}$$

where Y_k are the OLS estimates of the regression of $y = (y_0,\dots,y_n)$ on Chebishev polynomials

$$T_k(x) = \cos\left(k\arccos\left(\frac{x-\frac{\overline{x}+\underline{x}}{2}}{\frac{\overline{x}-\underline{x}}{2}}\right)\right),$$

but are more effectively calculated using FFT. A MATLAB code for DCT is, with $y = (y_0, ..., y_n)$:

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
f = @(x) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))
*(0:n))*Y(1:n+1);
```

A bidimensional version is

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
Y = Y(:,[1:n+1 n:-1:2]);
X = real(fft(X'/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)]';
f=@(x,y) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))*(0:n))...
*Y(1:n+1,1:n+1)...
*cos((0:n)'*acos((2*y'-(ymin+ymax))/(ymax-ymin)));
```

The fact that the grid $(x_0, ..., x_n)$ is not uniform and is denser towards the edges of the support interval allows to minimize the interpolation error and thus avoids the standard problem of strong oscillations at the edges of the interpolation interval (Runge's phenomenon).

Another advantage of DCT is that, having calculated $Y_0, ..., Y_n$, then polynomial projections of $y = (y_0, ..., y_n)$ of any order $p \leq n$ are obtained by stopping the summation in (A.1) at $k = p$. Finally, it is easy to approximate the derivative f' or the primitive $\int f$ simply by differentiating or integrating Chebyshev polynomials using

$$\cos(k \arccos x)' = \frac{k \sin(k \arccos x)}{\sin(\arccos x)},$$

and

$$\int \cos(k \arccos x)\, \mathrm{d}x = \begin{cases} x & \text{if } k = 0, \\ \frac{x^2}{2} & \text{if } k = 1, \\ \frac{\cos(k+1)x}{2(k+1)} - \frac{\cos(k-1)x}{2(k-1)} & \text{if } k \geq 2. \end{cases}$$

In calculating an approximation of the derivative, it is useful to smoothen the function by summing over only a few polynomials. Derivatives are otherwise badly calculated near the boundary. Moreover, our experience is that the approximation:

$$\int_{\underline{x}}^{x} \mathbf{1}\{t \leq x\} f(x)\, \mathrm{d}x \simeq \sum_{k=0}^{n} w_k \mathbf{1}\{t \leq x_k\} f(x_k)$$

gave similar results as integrating the interpolated function.

We implemented these procedures with numbers of grid points such as $n = 50, 100, 500$ on a laptop without running into any memory or computing time difficulty.

References

Aiyagari, S., J. Greenwood, and N. Guner (2000): "On the State of the Union," *Journal of Political Economy*, 108(2), 213–244.

Amuedo-Dorantes, C., and S. Grossbard (2007): "Cohort-level sex ratio effects on women's labor force participation," *Review of Economics of the Household*, 5(3), 249–278.

Becker, G. S. (1981): "Altruism in the Family and Selfishness in the Market Place," *Economica*, 48(189), 1–15.

Blundell, R., P.-A. Chiappori, T. Magnac, and C. Meghir (2007): "Collective Labour Supply: Heterogeneity and Non-Participation," *Review of Economic Studies*, 74(2), 417–445.

Brien, M. J. (1997): "Racial Differences in Marriage and the Role of Marriage Markets," *Journal of Human Resources*, 32, 741–778.

Browning, M., F. Bourguignon, P.-A. Chiappori, and V. Lechene (1994): "Income and Outcomes: A Structural Model of Intrahousehold Allocation," *Journal of Political Economy*, 102(6), 1067–96.

Browning, M., and P. A. Chiappori (1998): "Efficient Intra-Household Allocations: A General Characterization and Empirical Tests," *Econometrica*, 66(6), 1241–1278.

Burdett, K., and M. G. Coles (1997): "Marriage and Class," *Quarterly Journal of Economics*, 112(1), 141–168.

Cahuc, P., F. Postel-Vinay, and J.-M. Robin (2006): "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, 74(2), 323–364.

Caucutt, E. M., N. Guner, and J. Knowles (2002): "Why Do Women Wait? Matching, Wage Inequality, and the Incentives for Fertility Delay," *Review of Economic Dynamics*, 5(4), 815–855.

Chiappori, P.-A. (1988): "Rational Household Labor Supply," *Econometrica*, 56(1), 63–90.

——— (1992): "Collective Labor Supply and Welfare," *Journal of Political Economy*, 100(3), 437–467.

Chiappori, P.-A., and O. Donni (2009): "Non-unitary Models of Household Behavior: A Survey of the Literature," *IZA DP*, (4603).

Chiappori, P.-A., B. Fortin, and G. Lacroix (2002): "Marriage market, divorce legislation, and household labor supply," *Journal of Political Economy*, 110(1), 37–72.

Chiappori, P.-A., M. Iyigun, and Y. Weiss (2008): "An Assignment Model with Divorce and Remarriage," *IZA*, (3892).

Chiappori, P.-A., M. Iyigun, and Y. Weiss (2009): "Investment in Schooling and the Marriage Market," *American Economic Review*, 99(5), 1689–1713.

Chiappori, P.-A., and S. Oreffice (2008): "Birth Control and Female Empowerment: An Equilibrium Analysis," *Journal of Political Economy*, 116(1), 113–140.

Chiappori, P.-A., B. Salanie, and Y. Weiss (2010): "Partner Choice and the Marital College Premium," *Mimeo*.

Chiappori, P.-A., and Y. Weiss (2006): "Divorce, Remarriage, and Welfare: A General Equilibrium Approach," *Journal of the European Economic Association*, 4(2-3), 415–426.

Chiappori, P.-A., and Y. Weiss (2007): "Divorce, Remarriage, and Child Support," *Journal of Labor Economics*, 25, 37–74.

Choo, E., S. Seitz, and A. Siow (2008a): "The Collective Marriage Matching Model: Identification, Estimation and Testing," *University of Toronto WP*, (340).

——— (2008b): "Marriage matching, risk sharing and spousal labor supplies," *University of Toronto WP*, (332).

Choo, E., and A. Siow (2006): "Who Marries Whom and Why," *Journal of Political Economy*, 114(1), 175–2001.

DelBoca, D., and C. Flinn (2009): "Endogeneous Household Interaction," *IZA DP*, (4377).

DelBoca, D., and C. J. Flinn (2005): "Household Time Allocation and Modes of Behavior: A Theory of Sorts," (1821).

——— (2006): "Modes of Spousal Interaction and the Labor Market Environment," *IZ*, (2005).

Eeckhout, J. (1999): "Bilateral Search and Vertical Heterogeneity," *International Economic Review*, 40(4), 869–887.

Eeckhout, J., and P. Kircher (2010a): "Sorting and Decentralized Price Competition," *Econometrica*, 78(2), 539–574.

——— (2010b): "Sorting versus screening: Search frictions and competing mechanisms," *Journal of Economic Theory*, 145(4), 1354–1385.

Eissa, N., and H. W. Hoynes (2004): "Taxes and the labor market participation of married couples: the earned income tax credit," *Journal of Public Economics*, 88(9-10), 1931–1958.

Fernandez, R., N. Guner, and J. Knowles (2005): "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality," *Quarterly Journal of Economics*, 120(1), 273–344.

Fortin, B., and G. Lacroix (1997): "A Test of the Unitary and Collective Models of Household Labour Supply," *Economic Journal*, 107(443), 933–955.

Gould, E. D., and M. D. Paserman (2003): "Waiting for Mr. Right: rising inequality and declining marriage rates," *Journal of Urban Economics*, 53(2), 257–281.

Greenwood, J., N. Guner, J. A. Knowles, J. Greenwood, N. Guner, and J. A. Knowles (2000): "Women on Welfare: A Macroeconomic Analysis," *American Economic Review*, 90(2), 383–388.

Grossbard-Shechtman, S. A. (1984): "A Theory of Allocation of Time in Markets for Labour and Marriage," *Economic Journal*, 94(376), 863–82.

Kreider, R. M. (2005): "Number, Timing, and Duration of Marriage and Divorces: 2001," *US Census Bureau, Current Population Reports*, (P70-97).

Lu, M., and R. P. McAfee (1996): "Matching and Expectations in a Market with Heterogeneous Agents," in *Advances in Applied Microeconomics*, ed. by M. Baye, vol. 6, pp. 121–156. JAI Press, Greenwich (CT).

Lundberg, S., and R. A. Pollak (1993): "Separate Spheres Bargaining and the Marriage Market," *Journal of Political Economy*, 101(6), 988–1010.

Lundberg, S., and R. A. Pollak (1996): "Bargaining and Distribution in Marriage," *Journal of Economic Perspectives*, 10(4), 139–58.

Lundberg, S. J., R. A. Pollak, and T. J. Wales (1997): "Do Husbands and Wives Pool Their Resources? Evidence from the United Kingdom Child Benefit," *The Journal of Human Resources*, 32(3), 463–480.

Manser, M., and M. Brown (1980): "Marriage and Household Decision-Making: A Bargaining Analysis," *International Economic Review*, 21(1), 31–44.

Mazzocco, M. (2004): "Saving, Risk Sharing, and Preferences for Risk," *American Economic Review*, 94(4), 1169–1182.

——— (2007): "Household Intertemporal Behaviour: A Collective Characterization and a Test of Commitment," *Review of Economic Studies*, 74(3), 857–895.

McElroy, M. B., and M. J. Horney (1981): "Nash-Bargained Household Decisions: Toward a Generalization of the Theory of Demand," *International Economic Review*, 22(2), 333–349.

Sattinger, M. (1995): "Search and the Efficient Assignment of Workers to Jobs," *International Economic Review*, 36(2), 283–302.

——— (2003): "A Search Version of the Roy Model," (03-08).

Seitz, S. (2009): "Accounting for Racial Differences in Marriage and Employment," *Journal of Labor Economics*, 27(3), 385–437.

Shapley, L. S., and M. Shubik (1971): "The assignment game I: The core," *International Journal of Game Theory*, 1, 111–130, 10.1007/BF01753437.

Shimer, R., and L. Smith (2000): "Assortative Matching and Search," *Econometrica*, 68(2), 343–369.

Siow, A. (2009): "Testing Becker's Theory of Positive Assortative Matching," *University of Toronto WP*, (356).

Trefethen, L. N. (2008): "Is Gauss Quadrature Better than Clenshaw–Curtis?," *SIAM Review*, 50(1), 67–87.

Tröger, T., and G. Nöldeke (2009): "Matching Heterogeneous Agents with a Linear Search Technology," *Bonn Econ DP*, (01/2009).

Waldvogel, J. (2006): "Fast Construction of the Fejér and Clenshaw-Curtis Quadrature Rules," *BIT Numerical Mathematic*, 46, 195–202.

Wong, L. Y. (2003): "Structural Estimation of Marriage Models," *Journal of Labor Economics*, 21(3), 699–727.