Cupid's Invisible Hand

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Matching involves trade-offs

Marriage partners vary across several dimensions,
their preferences over partners also vary
and the analyst can only observe some of these dimensions.
What can we infer on “preferences over matches” from observed matching patterns?

Why do we care?

Impact of legalization on abortion on gains from marriage (**the original paper by Choo-Siow**)
Changes in returns to education (Chiappori-Salanié-Weiss)
Changes in group preferences (race, ethnic, castes.)
Policy questions: taxation, child care, welfare…

Framework

Static, frictionless matching market with perfectly transferable utility.

Each individual has a full type=an observable type + a type that is observed to all agents *but not to the econometrician*.

E.g. a man has full type $x = (I, \varepsilon)$.

We denote F the distribution over full types of men x, and $\hat{F}$ the induced distribution over observable types I.

For women: $y = (J, \eta)$, with distributions $G, \hat{G}$.

$\hat{F}(I)$ and $\hat{G}(J)$ have **discrete support**, (for a start) and there are large numbers of potential partners of each observable type.

Matching

A *matching* $\mu(x, y)$ is a set of *matches* and *singles*.
Feasibility constraints, say for y-women:

$$\sum_x \mu(x, y) + \mu(0, y) = F(y).$$

We denote $\mathcal{M}(F, G)$ the set of feasible matchings.
Similar notation for observable types: $\bar{\mu}(I, J) \in \mathcal{M}(\hat{F}, \hat{G})$.

Matching with Transferable Utilities

Matching a man with full type x and women with full type y produces a *joint surplus* $s(x, y)$, known to all participants.
Our goal is to estimate the function s (or bits of it) given that we have a theory (next slide) and that we observe:

- the distributions of observable types $\hat{F}$ and $\hat{G}$
- and the proportions $\bar{\mu}(I, J)$ of matches and singles on observable types
- but **not** $\mu(x, y)$, nor F or G.

Stable/optimal Matching

A *stable matching* allocates payoffs to men and women: $u(x)$ and $v(y)$
such that for all (x, y),

$$s(x,y) \leq u(x) + v(y),$$

with equality iff $\mu(x, y) > 0$.
An *optimal matching* maximizes social surplus

$$\mathcal{W} = \sup_{\mu \in \mathcal{M}(F,G)} E_\mu s(x,y).$$

Classical result: a stable matching is optimal.

The Separability Assumption

(Choo-Siow)
we restrict all complementarities in surplus to be between observable types:
If $I(x_1) = I(x_2)$ and $J(y_1) = J(y_2)$, then (" conditional non-modularity")

$$s(x_1, y_1) + s(x_2, y_2) = s(x_1, y_2) + s(x_2, y_1).$$

Then we can write

$$s(x, y) = \bar{s}(I, J) + \varepsilon_I(J) + \eta_J(I).$$

We normalize $\bar{s}(I, 0) \equiv 0$ and $\bar{s}(0, J) \equiv 0$.

Discrete choice of Partners

Separability turns the assignment problem into a discrete choice model from the analyst's viewpoint.
There exist functions $U(I,J) + V(I,J) = \bar{s}(I,J)$ such that $U(I,0) \equiv 0$ and $V(0,J) \equiv 0$ and
woman y chooses partner of **observable type** I (or 0)

$$\text{to maximize } V(I,J) + \eta_J(I),$$

and man x chooses partner of **observable type** J (or 0)

$$\text{to maximize } U(I,J) + \varepsilon_I(J).$$

Distributions of unobservables

Assume that $\varepsilon_I(.)$ is drawn from some $\mathcal{P}_I$
and $\eta_J(.)$ is drawn from some $\mathcal{Q}_J$
and they are independent across I, J
and take these distributions known for now.

Emax Utilities

The aggregate expected gain from marriage of J-women, $\bar{v}(J)$ is

$$H_J(\hat{G}(J), V(., J)) = \hat{G}(J) E_{Q_J} \max_{I,0} \left(V(I,J) + \eta_J(I) \right)$$

and

$$\frac{\partial H_J}{\partial V(I,J)}(V(., J)) = \bar{\mu}(I,J)$$

the proportion of J-women who end up with an I-man.

Legendre Transform: Intuition

H_J is convex (expectation of max of linear functions) in $V(.,J)$, so it has a convex Legendre-Fenchel transform: for any vector of probabilities $\bar{\mu}(.,J)$,

$$H_J^*(\hat{G}(J),\bar{\mu}(.,J)) = \sup_{V(.,J)} \left(\sum_{0,J} \bar{\mu}(I,J)V(I,J) - H_J(\hat{G}(J),V(.,J)) \right).$$

and

$$\frac{\partial H_J^*}{\partial \bar{\mu}(I,J)}(\bar{\mu}(.,J)) = V(I,J)$$

but $\bar{s}(I,J) = U(I,J) + V(I,J)$, so we identify $\bar{s}$ from $\bar{\mu}$ **given** $\mathcal{P}_I$ and $\mathcal{Q}_J$.

What does the optimal matching optimize?

$\sum_{x,y} \mu(x,y)s(x,y)$ over feasible matchings, but none of this is observable

not $\sum_{I,J} \bar{\mu}(I,J)\bar{s}(I,J)$, because partners match on unobservables as well

Convex duality: if $\bar{s} \longrightarrow U, V, \bar{\mu}$, then

$$H_J(\hat{G}(J), V(.,J)) + H_J^*(\hat{G}(J), \bar{\mu}(.,J)) = \sum_{I,0} \bar{\mu}(I,J)V(I,J)$$

so that expected utility of all women=$\sum_J H_J(\hat{G}(J), V(.,J)) = \sum_J \sum_{I,0} \bar{\mu}(I,J)V(I,J) - \sum_J H_J^*(\hat{G}(J), \bar{\mu}(.,J))$

Just-identification of the joint surplus

Under separability, the *observable* stable/optimal matching $\bar{\mu} \in \mathcal{M}\left(\hat{F}, \hat{G}\right)$ maximizes

$$\sum_{I,J} \bar{\mu}(I,J)\bar{s}(I,J) + \mathcal{E}(\bar{\mu}, \hat{F}, \hat{G}),$$

where $\mathcal{E}$ is the *generalized entropy*:

$$\mathcal{E}(\bar{\mu}, \hat{F}, \hat{G}) = -\sum_{I} G_I^*(\hat{F}(I), \bar{\mu}(I,.)) - \sum_{J} H_J^*(\hat{G}(J), \bar{\mu}(.,J)).$$

Intuition: without **unobserved** heterogeneity $\mathcal{E} \equiv 0$; with no **observed** heterogeneity $\bar{s} \equiv 0$.

Expected utilities in equilibrium

Intuition: each (average) woman of type J “contributes”

$$\frac{\partial \mathcal{E}}{\partial \hat{G}(J)}(\bar{\mu}, \hat{F}, \hat{G})$$

to the total surplus in equilibrium.
So she should get exactly that:

$$\bar{v}(J) = -\frac{\partial H_J^*}{\partial \hat{G}(J)}(\hat{G}(J), \bar{\mu}(., J)).$$

This is what she gets, and again it is just identified.

Example

Chiappori-Salanié-Weiss: $\varepsilon_I(J)$ and $\eta_J(I)$ are type-I EV, iid with scale parameters $\sigma(I)$ and $\tau(J)$;
then

$$\mathcal{E} \simeq -\sum_I \sigma(I) \left(\bar{\mu}(I,0) \log \bar{\mu}(I,0) + \sum_J \bar{\mu}(I,J) \log \bar{\mu}(I,J) \right) - \sum_J \tau(J) (\ldots)$$

and

$$\bar{s}(I,J) = \sigma(I) \log \frac{\bar{\mu}(I,J)}{\bar{\mu}(I,0)} + \tau(J) \log \frac{\bar{\mu}(I,J)}{\bar{\mu}(0,J)}.$$

CSW show how $\sigma(I)$ and $\tau(J)$ can be identified given restricted variation of surplus across cohorts.

Estimation

Choo-Siow-like: parameter-free distributions $\mathcal{P}_I$ and $\mathcal{Q}_J$, nonparametric joint surplus; use inversion formula

More generally: parameterize $\mathcal{P}_I$ and $\mathcal{Q}_J$ and $\bar{s}$ with parameter vector λ, use maximum likelihood

Requires a fast way of computing the optimal $\bar{\mu}$ for any λ.

Iterative Projection Fitting Procedure

The inversion formula gives us (at best) $\bar{\mu}(I, J)$ as a function of $\bar{\mu}(I, 0), \bar{\mu}(0, J), \hat{F}(I), \hat{G}(J)$
The difficulty: fitting the margins so that $\bar{\mu} \in \mathcal{M}(\hat{F}, \hat{G})$.
The solution: start from a well-chosen $\bar{\mu}^{(0)}$,
and project iteratively on $\mathcal{M}(\hat{F})$ and on $\mathcal{M}(\hat{G})$.

IPFP

Projecting requires a distance $\longrightarrow$ we get one from generalized entropy $\mathcal{E}$ (think of Küllback-Leibler)
$-\mathcal{E}$ is convex, so we construct a Bregman divergence from it:

$$D(\mu, \nu) = \mathcal{E}(\nu) - \mathcal{E}(\mu) + \langle \nabla \mathcal{E}(\nu), \mu - \nu \rangle.$$

Think of $y^2 - x^2 - 2x(y-x) = (y-x)^2$.
We find a $\bar{\mu}^{(0)}$ whose projection on $\mathcal{M}(\hat{F}, \hat{G})$ for D is $\bar{\mu}$, and we run the iterative algorithm.

Extensions

Chiappori-Galichon-Salanié: the roommate problem (no preexisting bipartition of population)
Galichon-Henry: hedonic models
continuous observed types.