Matching Markets with Endogenous Information

Tracy Lewis
Duke University

Li, Hao
University of British Columbia

Matching Problems in Chicago 2012

- Matching markets bring together agents from different sides to create a surplus maximizing match.
  - Matching marriage or dating partners, employees and employers, and research collaborators.
- Surplus is maximized by matching the "right" agents together.
- Often, however, an agent's "type" isn't known or publicly observed.
- How matching markets perform for those settings is not well understood.

- How do agents acquire and signal public information about themselves?
  - Privately informed agents take "tests" to reveal their abilities.
  - Tests may vary in accuracy, and accuracy is *unobserved*.
- How do intermediaries match agents of unknown abilities?
  - Intermediaries wish to maximize total surplus.
  - Observe only the agent's public information.
- What is the value of private information for matching efficiency?
  - Agents can acquire private information about their abilities.

- How do agents acquire and signal public information about themselves?
  - Depends on agent's perceived ability.
- How do intermediaries match agents of unknown abilities?
  - Use test scores to to match.
  - Test scores replace objective information in matching.
- What is the value of private information for matching efficiency?
  - Personal value is always non-negative, and often strictly positive.
  - Social value may be zero, and negative when average abilities are low.

- Two sided market: F (for female) and M (for male).
  - Both sides are identical with a mass of 1.
- Agents are of ability $T \in \{H = \textit{high ability}, L = \textit{low ability}\}$.
- Agent's T is unknown: $\pi \in [0, 1]$ of H ability known to exist on both sides.
- Agent's receive private signal $s \in \{g = \text{good}, b = \text{bad}\}$, with

$$\Pr(g \mid H) = \Pr(b \mid L) = p > \frac{1}{2}.$$

- Agents disclose public test cores of ability, $t \in \{h =$ high, $l =$ low$\}$.
  - Test $\tau \in \{a =$ accurate$, n =$ not accurate$\}$, with

$$\Pr(h \mid H) = \Pr(l \mid L) = \lambda^{\tau}.$$

  - $\lambda^{\tau} =$ probability test score is correct for test τ.
  - $\frac{1}{2} \leq \lambda^{n} < \lambda^{a} \leq 1$.

- The matching of a type T agent with a type T' agent produces a total surplus value of $v_{TT'}$.
  - Ability increases match surplus:

$$v_{HH} > v_{HL} > v_{LL} > 0.$$

  - Surplus is maximized by *positive assortative matching* if ability is observed:

$$\gamma = \frac{v_{HH} - v_{HL}}{v_{HL} - v_{LL}} > 1$$

- Intermediary matches agents to maximize expected match value.

- Agent learns his private signal, $s \in \{g, b\}$.
- Agent of each side $G = M, F$ selects disclosure strategy, $\sigma_s^\tau(G) =$ probability of disclosing with accuracy τ when private signal is s.
- Intermediary observes public signals, t, and matches agents to maximize expected surplus.

Model: Robustness

- Generalizable features
  - Multiple private signals.
  - Multiple public signals.
  - Correlation of private and public signals.
  - Biased private signals.
- Special feature
  - Unobserved disclosure.

- Lemma. *In any equilibrium of the matching game there is only positive assortative matching by public scores.*
  - Public test scores are informative. Likelihood that an agent is H given $t = h$ is higher than if $t = l$, regardless of agent's disclosure strategy.
- Generalizes assortative matching results.

- Lemma. *If an agent with a private signal s=b finds it weakly optimal to choose* $\tau = a$ *then it is strictly optimal for an agent on the same side with* $s = g$ *to choose* $\tau = a$.
  - By disclosing accurately, the true H ability agent maximizes his chance of getting public score h, whereas the true L ability agent minimizes his chance. Hence agents with g private signal prefer accurate disclosure as compared with agents with b.
- A version of single-crossing in signaling games.

- Implications of previous lemmas
  - When $\sigma_b^a(G) > 0$ then $\sigma_g^a(G) = 1$. Moreover this condition holds for $G = M$ if and only if it holds for $G = F$, so only symmetric equilibrium exists.
  - Only pooling equilibrium where both private types choose either to test accurately or inaccurately and separating equilibrium where g type agents signal accurately and b type agents test inaccurately are possible.

- Proposition. *Given a matching market* (p, γ), *there exist ability probabilities* $\pi^u = \left(\frac{p}{1-p}\gamma + 1\right)^{-1}$ and $\pi^s = \left(\frac{1-p}{p}\gamma + 1\right)^{-1}$ *such that*
  - (i) *For* $\pi < \pi^u$, *there exists a unique pooling equilibrium with* $\sigma_g^a = \sigma_b^a = 0$.
  - (ii) *For* $\pi > \pi^s$, *there exists a unique pooling equilibrium with* $\sigma_g^a = \sigma_b^a = 1$.
  - (iii) *For* $\pi^u < \pi < \pi^s$, *there exists a unique separating equilibrium with* $\sigma_g^a = 1, \sigma_b^a = 0$.

- Intuition behind characterization
  - (i) In a *very high ability* market, all agents believe they are most likely H, so accurate disclosure is best.
  - (ii) In a *very low ability market*, all agents believe they are most likely L, so inaccurate disclosure is best.
  - (iii) In a *moderate ability market*, private signal informs agents of their probable ability, so b types choose n test and g types choose a test.

- Implications of equilibrium signaling
  - Matching is imperfectly positive assortative, because privately informed agents may choose inaccurate testing.
  - Public information is endogenous, and is more accurate in a higher ability market.
  - More accurate disclosure in markets with greater complementarity in match surplus.

- Comparative statics with respect to p, quality of private information.
  - Social value: effects on efficiency of matching.
  - Personal value: effects on private incentives for agents to acquire private signals to make better testing decisions.

- Let W be matching surplus for one side of the market from matching:

$$W = N_{HH} v_{HH} + N_{HL} v_{HL} + N_{LL} v_{LL}$$

where $N_{TT'}$ is mass of TT' matches in equilibrium.

  – $N_{HH} + N_{HL} + N_{LL} = 1$.
  – $N_{HH} - N_{LL} = 2\pi - 1$.

- Complementarity implies N_{HH} measures total matching surplus.

  – $dW = (v_{HH} + v_{LL} - 2v_{HL})\, dN_{HH}$.

- Bounds on matching surplus.

  – Maximum W^a in a-pooling, and minimum W^n in n-pooling.

□ Proposition. *In a low ability market with* $\pi < \frac{1}{1+\gamma}$,

- (i) *For* $p < p^u = \left(\frac{\pi}{1-\pi}\gamma + 1\right)^{-1}$ *private information has no social value and the surplus is minimized with* $W(p) = W^n$.
- (ii) *For* $p > p^u$, *the surplus is greater with* $W(p) > W^n$ *and,*

$$\frac{dW}{dp} = \begin{cases} < 0 \text{ for } p^u < p < \hat{p} \\ > 0 \text{ for } \hat{p} < p < 1 \end{cases}$$

where $\hat{p} = \min\left\{\frac{\xi - \pi(1+\xi)}{\xi - 1}, 1\right\}$ *and* $\xi = \frac{2\lambda^a - 1}{2\lambda^n - 1}$.

- Results
  - Social value of private information is zero in a pooling equilibrium: discrete change in quality of information needed to impact matching efficiency.
  - Social value of private information can be non monotonic in a separating equilibrium.
- Intuition
  - As quality of private information improves, both more true H and more true L agents will get an h score. Size of h pool increases which increases surplus, but fraction of true H agents in h pool decreases which decreases surplus.

□ Proposition. *In a high ability market with* $\pi > \frac{1}{1+\gamma}$,

- (i) *For* $p < p^s = \left(\frac{1-\pi}{\pi}\frac{1}{\gamma} + 1\right)^{-1}$ *private information has no social value and the surplus is maximized with* $W(p) = W^a$.
- (ii) *For* $p > p^s$, *the surplus is smaller with* $W(p) < W^a$ *and,*

$$\frac{dW}{dp} = \begin{cases} < 0 \text{ for } p^s < p < \hat{p}, \text{ if } p^s < \hat{p} \\ > 0 \text{ for } \max\{p^s, \hat{p}\} < p < 1. \end{cases}$$

- Proposition. *The personal value of private information is always non negative and it is strictly positive when* $p > p^u$ *in low ability markets and when* $p > p^s$ *in high ability markets.*
  - An increase in the quality of a single agent's private signal has no affect on equilibrium and therefore can not harm the agent. Moreover by revealed preference an agent who uses private information to make a decision must strictly benefit from better information in a separating equilibrium.

- Implications for acquiring private information
  - No incentives in very high or low ability markets: public information about ability substitutes for private information.
  - In high ability markets information acquisition by different agents are strategic substitutes.
  - In low ability markets, information acquisition by different agents are strategic complements.

- Agents are observed to have two characteristics $(\tau = a, n; t = h, l)$.
- For symmetric test taking strategies $\sigma_g^a, \sigma_b^a \in [0,1]$ there are maximum four pools of agents to draw from: $\{(a; h), (a; l), (n; h), (n; l)\}$.
  - Positive assortative matching by test and score is optimal for intermediary.
- Equilibrium refinement is needed if all agents take same test.

- Same trade-off in choosing test accuracy
  - Benefit of testing accurately is greater chance of getting h and matching with H agents, if agent himself is truly H.
  - Cost is smaller chance, if agent himself is truly L.
- We make assumptions on p, λ^a and λ^n such that both benefit and cost are positive regardless of testing strategies σ_g^a, σ_b^a.
  - Suffices if p is low, so that private signal is relatively uninformative, and $\lambda^a - \lambda^n$ is large, so that test accuracies are sufficiently different.
- Ordering property holds again: $\sigma_b^a > 0$ implies $\sigma_g^a = 1$.
  - Ordering property allows us to apply standard belief refinement when all choose same test.

- Proposition. Suppose that the ordering property holds. Then in the unique equilibrium that satisfies belief refinement, both private types test accurately.
  - Can also have a semi-pooling with $\sigma_g^a = 1$ and $\sigma_b^a \in (0, 1]$, and n semi-pooling with $\sigma_g^a \in [0, 1)$ and $\sigma_b^a = 0$. But we need degree of complementarity γ to be low enough to support a pooling with private type b indifferent between two tests, and n pooling with private type g indifferent respectively. Neither is possible: in the first case type b reveals himself by testing inaccurately, and in the second case type g reveals himself by testing accurately.

- Implications: Both test and grade are public signals of agents' abilities.
  - An unraveling argument means test accuracy can't separate private types.
  - Maximum surplus W^a is attained.

- Surplus devision in matches
  - Agents may signal with the match they pursue.
- Acquisition of abilities in matching markets
  - How do information systems impact on investment in abilities.
- Using matching procedures to incentivize accurate disclosure
  - Can commit to coarse matching and other deviations from ex post optimal matching.