## Matching, Sorting and Wages

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## Introduction

- Employment protection policies are pervasive and their effect is controversial.
- Labour market regulation has been blamed for Europe's problems (Bentollila and Bertola, Ljundqvist etc.)
- The debate is of course relevant for any country:
- Heckman and Pages present evidence that in Latin America labour market regulations can increase inequality as well as reduce efficiency.


## Introduction - Role for Policy

- To understand the potential costs and benefits of regulation we need a framework that allows for imperfections such as frictions and shocks
- These allow us to depart from the competitive paradigm and give scope for the data to determine whether regulation is beneficial or not.
- The key is to answer policy relevant questions based on evidence.


## Introduction - Search Frictions

- Among the many sorts of frictions, the labour macro literature has emphasized the role of search frictions, i.e. uncertainty on the availability-location of job offers.
- Policies like minimum wage or employment protection may improve efficiency
- Heterogeneity of workers and firms may affect the efficiency implications of policies.


## Earnings Dynamics - A structural approach

- The model has a further motivation:
- There has been a large literature on the nature of the earnings process.
- While consensus is not quite there yet, many believe that earnings have a unit route. as claimed by MaCurdy (1982), Abowd and Card (1989) and Meghir and Pistaferri (2005) and others.
- The model we present here is an attempt at providing a structural justification for for the kind of earnings processes we observe.
- The aim is to define the primitive sources of the shocks and demonstrate how these are then transmitted to wages.


## Earnings Dynamics - A structural approach

- We are aiming at an equilibrium model that is jointly consistent with data on

1. Employment dynamics
2. Wage dispersion
3. Wage dynamics

## Key Background Papers

- Key references:
- Mortensen \& Pissarides: Matching models and search
- Shimer \& Smith: Sorting and search
- Postel-Vinay \& Robin and Cahuc, Postel-Vinay \& Robin: Equilibrium wages with search and outside offers
- Postel-Vinay \& Turon: Search and earnings dynamics


## Features of the model

We build an equilibrium model that has the following features:

1. Search frictions: it takes time to locate jobs
2. Both workers and firm are heterogeneous.
3. There may be firm-worker complementarity - hence worker's pay can differ depending on their employer.
4. There are capacity constraints (one worker one job here) which means that firms may wish to wait for a better worker to arrive.
5. There are shocks to firms productivity, which means that the quality of matches change and firms shut down endogenously.
6. Workers can keep looking for work while on the job, which implies growth of wages over a career.
The frictionless limit of the model will involve perfect assortative matching (Becker)

## Sorting

- The presence of sorting has been controversial ever since matched employer employee data has allowed its direct evaluation.
- Understanding whether sorting leads to increases in output is important because it puts certain policy issues into perspective.
- In particular, in the presence of search frictions policies that encourage individuals to take any job can lead to large welfare loss through mismatch.


## Sorting

- So what is the evidence?
- Abowd, Kramarz and Margolis (2000) and Abowd, Kramarz, Lengermann and Roux (2003):
- They use French and U.S. matched employer-employee data to estimate

$$
\log w_{i t}=x_{i}^{\prime} \beta+\psi_{i}+\varphi_{J(i, t)}+e_{i t}
$$

- They find a small, and if any negative, cross-sectional correlation between firm ( $\varphi_{\jmath}$ ) and worker fixed effects ( $\psi_{i}$ )


## Sorting

- Does this rule out sorting?
- Eeckhout and Kircher (2008) show that this methodology will not (in general) identify sorting.
- They show that wages will not be monotonic in firm heterogeneity.
- Ignoring such non-monotonicity will "average out" the effects of sorting.
- Melo (2008), Bagger and Lentz (ongoing) and our own simulations confirm this result.


## Actual Versus Estimated Sorting

## An example of what can happen

- Such reduced form empirical results can be highly misleading.
- This is because low productivity firms may not be willing to pay as much for a very high productivity worker - she will leave soon and leave the job vacant.

Table: Actual and Estimated Sorting

| Production Function | $\operatorname{corr}(x, y)$ | $\operatorname{corr}\left(\hat{\phi}_{i}, \hat{\psi}_{j(i)}\right)$ |
| :--- | :---: | :---: |
| $f(x, y)=x y$ | 0.80 | 0.44 |
| $f(x, y)=x+y$ | 0.0 | -0.05 |
| $f(x, y)=1-(x-y)^{2}$ | 0.84 | -0.29 |
| $f(x, y)=1+\frac{1}{2} x-(x-y)^{2}$ | 0.82 | -0.06 |
| $f(x, y)=1+\frac{1}{2} y-(x-y)^{2}$ | 0.81 | -0.05 |

## Some evidence from matched data

Sorting in the data
$y$ earning conditional on job rank, education, age group and sex


## The Model

Market production, home production, and recruiting costs

- Workers of type (ability/human capital) $x \sim \mathbb{U}[0,1]$, with measure 1.
- Occupations/jobs/firms of type (productivity) $y \sim \mathbb{U}[0,1]$, with measure $N$.
- A match between a worker of type $x$ and a job of type $y$ produces

$$
f(x, y)
$$

with $f_{x}>0, f_{y}>0, f_{x y} \geq 0$.

- When unmatched, workers produce $b(x)$
- Unmatched firms may pay $c$ to post a vacancy and fill the job.


## The Model

Shock to productivity

- Firm productivity is subject to shocks.
- They arrive with probability $\delta$ and are drawn from $y^{\prime} \sim \mathbb{U}[0,1]$.
- Persistence comes through $\delta$.
- At present we have not allowed individual worker productivity $x$ to be subject to shocks.


## The Model

Meeting technology

- Throughout we will assume that firms have one job.
- Workers meet jobs both while unemployed and employed (on-the-job search)
- Number of meetings per period (meeting function) $=$ $a\left(s_{0} U+s_{1}(1-U)\right)^{0.5} V^{0.5}$
- U number of unemployed; $s_{0}$ search effort of unemployed
- 1 - U number of employed workers; $s_{1}$ search effort of employed
- $V$ number of (open) vacancies
- When $x$ and $y$ meet they must decide whether to produce or continue searching.
- This decision generates an endogenous distribution of matches $h(x, y)$, unemployed workers $u(x)$ and job vacancies $v(y)$.


## Values and Match Surplus

- $W_{0}(x)$ : Value to worker of unemployment
- $W_{1}(w, x, y)$ : Value to worker of wage $w$ in an $(x, y)$ match
- $\Pi_{0}(y)$ : Value to firm of an unfilled post
- $\Pi_{1}(w, x, y)$ : Value to firm of paying wage $w$ in an $(x, y)$ match
- The match surplus is

$$
S(x, y)=W_{1}(w, x, y)-W_{0}(x)+\Pi_{1}(w, x, y)-\Pi_{0}(y)
$$

- Match is formed iff $S(x, y)>0$ (individual rationality)


## Wage Contracts

Dey \& Flinn (2005), Cahuc, Postel-Vinay \& Robin (2006)

- In general there is a set of admissible wages:

$$
\left\{w \mid 0 \leq W_{1}(w, x, y)-W_{0}(x) \leq S(x, y)\right\}
$$

- We adopt a mix of Bertrand competition and rent sharing to pin down the wage at any point in time


## Wage Contracts

For workers transiting from unemployment

Initial wage contract out of unemployment is set by Nash Bargaining, with unemployment as the outside option:

$$
w=\phi_{0}(x, y)
$$

such that

$$
W_{1}\left(\phi_{0}(x, y), x, y\right)-W_{0}(x)=\beta S(x, y)
$$

- The firm is competing with the worker's home production technology, and the wage depends only on $x$ and $y$.


## Wage Contracts

Currently employed workers who are contacted by another firm
Firms counter outside offers, leading to bargaining with full surplus extraction from the lower surplus match as the outside option:

$$
w=\phi_{1}\left(x, y, y^{\prime}\right)
$$

such that

$$
W_{1}\left(\phi_{1}\left(x, y, y^{\prime}\right), x, y\right)-W_{0}(x)=S\left(x, y^{\prime}\right)+\beta\left[S(x, y)-S\left(x, y^{\prime}\right)\right]
$$

where the firm $y$ gets the worker and $S(x, y)>S\left(x, y^{\prime}\right)$.

- If $\beta=0$ the worker always receives her reservation wage.
- If $\beta=1$ it is as if the worker always has access to a close competitor.
- The winning firm $y$ is competing with $y^{\prime}$, which influences the wage contract.


## Productivity Shocks and Wage Renegotiation

## Postel-Vinay \& Turon (2010)

If the firm receives a shock from $y$ to $y^{\prime}$, the current wage may move outside the bargaining set.
Upon realization of a new $y^{\prime}$ :

1. If $S\left(x, y^{\prime}\right)<0$ the match separates
2. If $S\left(x, y^{\prime}\right) \geq 0$ but $W_{1}\left(w, x, y^{\prime}\right)-W_{0}(x)<0$ the wage is renegotiated to $w^{\prime}=\psi_{0}\left(x, y^{\prime}\right)$ such that

$$
W_{1}\left(\psi_{0}\left(x, y^{\prime}\right), x, y^{\prime}\right)-W_{0}(x)=0
$$

3. If $S\left(x, y^{\prime}\right) \geq 0$ but $S\left(x, y^{\prime}\right)<W_{1}\left(w, x, y^{\prime}\right)-W_{0}(x)$ the wage is renegotiated to $w^{\prime}=\psi_{1}\left(x, y^{\prime}\right)$ such that

$$
W_{1}\left(\psi_{1}\left(x, y^{\prime}\right), x, y^{\prime}\right)-W_{0}(x)=S\left(x, y^{\prime}\right)
$$

4. If $0 \leq W_{1}\left(w, x, y^{\prime}\right)-W_{0}(x)<S\left(x, y^{\prime}\right)$ neither the worker nor the firm has a credible threat to force renegotiation.

## The value of being out of work

An unemployed worker of type $x$ has flow value comprising home production and the expected gain due to employment:

$$
r W_{0}(x)=b(x)+s_{0} \kappa \beta \int S(x, y)^{+} v(y) d y
$$

- where $\kappa$ incorporates the equilibrium meeting rate as a function of total search on both sides of the market

$$
\kappa \equiv \frac{M\left(s_{0} U+s_{1}[1-U], V\right)}{\left(s_{0} U+s_{1}[1-U]\right) V}
$$

- We use the notation $S(x, y)^{+} \equiv \max \{0, S(x, y)\}$


## The Value of a Vacancy and the Marginal Firm

$$
\begin{aligned}
& r \Pi_{0}(y)=\left[-c+(1-\beta) s_{0} \kappa \int S(x, y)^{+} u(x) d x\right. \\
& \left.\quad+(1-\beta) s_{1} \kappa \int\left[S(x, y)-S\left(x, y^{\prime}\right)\right]^{+} h\left(x, y^{\prime}\right) d x d y^{\prime}\right]^{+} \\
& \quad+\delta \int\left[\Pi_{0}\left(y^{\prime}\right)-\Pi_{0}(y)\right] d y^{\prime}
\end{aligned}
$$

where $c$ is the flow cost of posting a vacancy.

- The marginal firm makes zero expected profit from the posting decision. Firms with productivity below this level remain idle.
- An unmatched firm is also subject to $y$ shocks and some may find it profitable to post a vacancy.
- Note that firms who are matched with a worker may have productivity lower than the level that justifies posting a vacancy if the worker departs.


## Match Surplus: $S(x, y)$

$$
\begin{aligned}
\rho S(x, y)= & f(x, y)-r W_{0}(x)-r \Pi_{0}(y) \\
& +s_{1} \kappa \int \beta\left[S\left(x, y^{\prime}\right)-S(x, y)\right]^{+} v\left(y^{\prime}\right) d y^{\prime} \\
& +\delta \int S\left(x, y^{\prime}\right)^{+} d y^{\prime}
\end{aligned}
$$

- Discounting due to time preference, shocks to $y$, and exogenous separation: $\rho=(r+\delta+\xi)$
- Part of the surplus of an $(x, y)$ match comes from the ability of the worker to extract surplus from future employers.
- The surplus does not depend on the wage (or on renegotiations of the wage).


## Steady State Flow Equations

- To solve for equilibrium we need to define the steady-state flows.
- The flow into and out of matches of type $(x, y)$ are equal :

$$
\begin{aligned}
&\left(\xi+\delta+s_{1}\right.V(\bar{B}(x, y))) h(x, y) \\
&=\delta \int h\left(x, y^{\prime}\right) d y^{\prime} \\
&+\left(s_{0} u(x)+s_{1} \int_{B(x, y)} h\left(x, y^{\prime}\right) d x d y^{\prime}\right) \kappa v(y)
\end{aligned}
$$

where $B(x, y)=\left\{y^{\prime} \mid S(x, y)>S\left(x, y^{\prime}\right)\right\}$, is the set of jobs that would lead to an increased surplus. Its complement is $\bar{B}(x, y)$

## Equilibrium Definition

Given values for the primitives: $N, \delta, \xi, s_{0}, s_{1}, M(\cdot, \cdot), r, b, c$, $f(x, y)$, and $\beta$ the stationary equilibrium is fully characterized by knowledge of

- $S(x, y)$ : Fixed point in the surplus function
- $h(x, y)$ : Implied stationary distribution of matches
- V: Number of posted vacancies

Wage paths are history dependent and can be simulated after solving for the equilibrium allocations.

Surplus, Value and Wage dynamics




## A Framework for Policy Analysis

- We consider three labour market policies:
- Experience rating
- Minimum wage
- Severance pay
- They require some modification to the model.


## Experience rating

- This policy, prevalent in the US imposes an increased cost to firms with high turnover
- We model this as a tax on endogenous separations.
- Define $\delta Q\left(\overline{\mathcal{M}}_{0}(x) \mid y\right)$ to be the probability that a productivity shock arrives and leads to an infeasible match.
- We thus subtract from the surplus of the match the term $\tau^{E X P} \delta Q\left(\overline{\mathcal{M}}_{0}(x) \mid y\right)$
- The GE effects of funding unemployment benefits through experience rating are ignored.


## Minimum wages

1. Minimum wage: Feasible matches are those such that

$$
S(x, y)>0
$$

and

$$
\Pi_{1}\left(w^{\min }, x, y\right)>\Pi_{0}(y)
$$

2. This changes both the jobs that are posted and the matches that are feasible. It changes the matching set for the unemployed
3. The practical issue is that we now need to simultaneously solve for the surplus and the value of the wage contract

## Severance Pay

- Severance pay is a transfer of payments from the firm to the worker on separation
- On their own severance payments have no effect on the equilibrium, they simply change the timing of wage payment to the worker (in expectation) (see Lazear)
- But, in combination with minimum wages they do by preventing up-front payments from the worker
- We model this by adding $\tau^{S E V} \delta Q\left(\overline{\mathcal{M}}_{0}(x) \mid y\right)$ to the workers' value and subtracting it from the firm value
- The effect comes indirectly through profits $\Pi_{1}\left(w^{\text {min }}, x, y\right)$ which in turn affects the vacancies that are posted


## Data

- The data used for estimation is drawn from the NLSY79 -1979-2002
- Individuals aged 14-21 in 1979
- White Males from the core sample only
- Individuals included following the end of education.
- Drop those who say they are out of the labor force (mainly disabled) and those who have served in the military.
- Subdivide in three education groups (less than high school, High school, College)
- The key advantage of the NLSY is that we can observe job changes and wage changes.
- We deterend wages based on CPS data.


## "Identification"

- Ideally the model should be estimated on matched data - and we have started this with a simplified version
- Here however the main results will come from one sided data.
- A key question is what features of the data help identify the model and in particular complementarity and sorting
- The main drivers are:
- the mean and variance of wage growth across jobs
- The duration of a job match at different wage levels
- The productivity shock process is driven by fluctuations of wages within job (and in particular pay cuts) and by the job-to-job transitions
- Given our functional form parameters the model is heavily overidentified.


## Estimation

- To estimate the model we use method of moments combined with MCMC
- The key idea derives from Chernozhukov and Hong (2003).
- Suppose estimation is to be based on maximising a criterion function $f(y \mid \theta)$
- Assume some prior (we will take the diffuse one) $\pi(\theta)$.
- Then a "posterior" can be written as $g(\theta \mid y) \propto \exp (f(y \mid \theta)) \pi(\theta)$


## Estimation

- Chernozhukov and Hong show that:
- that the average of the draws from the "posterior" converges to an estimator that is asymptotically equivalent to maximising the original criterion
- That the suitable quantiles of the sample of draws estimates the confidence interval of the parameters
- To obtain draws from the posterior we can use MCMC combined with the Metropolis-Hastings algorithm (see Chibb, 2001)


## Estimation

- As a Markov transition process we use a random walk $\theta_{n}=\theta_{n-1}+\eta_{n}$ where $\eta_{n}$ is drawn from a normal.
- The parameters are constrained to lie in the acceptable parameter space (e.g. to be positive or in the unit interval)


## Estimation

- The MCMC sample is constructed as follows:
- Keep $\theta_{n}$ with probability $\min \left\{1, \frac{g\left(\theta_{n} \mid y\right)}{g\left(\theta_{n-1} \mid y\right)}\right\}$.
- Otherwise the new draw is set equal to $\theta_{n-1}$
- The variance of $\eta_{n}$ is reset after a block of $m$ iterations to $2 \operatorname{Var}\left(\theta^{n}\right)$ following Cassella and Roberts. This gives approximately $50 \%$ acceptance probability
- The basis of the procedure is a quadratic distance criterion of several moments weighted by their sample variance.


## Measurement error

- We use the monthly records of earnings from the NLSY
- These are likely to be measured with error ad the error will be correlated across records.
- We this assume an $\operatorname{AR}(1)$ process for within year measurement error
- We further assume that this error process is independent across years


## The Production Function

We parameterize the production function to be CES

$$
f(x, y)=\left(\exp \left(f_{1}+f_{2} \Phi^{-1}(x)\right)^{f_{4}}+\exp \left(f_{1}+f_{3} \Phi^{-1}(y)\right)^{f_{4}}\right)^{1 / f_{4}}
$$

- x and y and parameterized as uniform
- $f_{2}$ and $f_{3}$ capture the st. dev. of worker and firm heterogeneity
- $f_{1}$ is TFP and is identified form the variance of wages.


## Some Parameter Estimates

## Education

Dropout High Sch Some Coll College

| $s_{1}$ search - employed $\left(s_{0}=1\right)$ | 1.58 | 0.22 | 0.23 | 0.31 |
| :---: | :---: | :---: | :---: | :---: |
| $\beta$ bargaining power | 0.22 | 0.07 | 0.03 | 0.21 |
| $\xi$ Job Destruction rate | 0.027 | 0.0002 | 0.003 | 0.0014 |
| $\delta$ Arrival rate of shocks | 0.039 | 0.12 | 0.1 | 0.03 |
| Elasticity of Subst | $\infty$ | 0.53 | 0.64 | 0.56 |
| $f_{2}$ s.d. of $x$ | 0.54 | 1.65 | 1.85 | 1.45 |
| $f_{3}$ s.d. of $y$ | 2.83 | 1.16 | 1.21 | 1.23 |
| $\rho$ (measurement error) | 0.85 | 0.96 | 0.96 | 0.40 |
| $\sigma$ (measurement error) | 0.096 | 0.05 | 0.05 | 0.012 |

## Fitting the Dynamics of Earnings

|  | Less than high school |  | High school |  | College |  |
| :--- | ---: | :---: | ---: | :---: | :---: | :---: |
|  | Data | Model | Data | Model | Data | Model |
| $h_{E U}$ | 0.026 | 0.028 | 0.017 | 0.017 | 0.007 | $0.006^{*}$ |
| $h_{U E}$ | 0.121 | 0.114 | 0.134 | 0.128 | 0.133 | 0.125 |
| $h_{\Delta J}$ | 0.054 | $0.066^{*}$ | 0.042 | $0.045^{*}$ | 0.022 | 0.023 |
| $\overline{\Delta w}_{\mid E U E}$ | -0.280 | $-0.104^{*}$ | -0.219 | $-0.046^{*}$ | -0.387 | $-0.074^{*}$ |
| $\overline{\Delta w}_{\mid E E}$ | 0.028 | 0.032 | 0.016 | $0.029^{*}$ | 0.022 | $0.031^{*}$ |
| $\overline{\Delta w}_{\mid \Delta J}$ | 0.086 | $0.060^{*}$ | 0.060 | 0.059 | 0.098 | 0.100 |

## Fitting the Dynamics of Earnings

Variance of wages and wage growth

|  | Less than high school |  | High school |  | College |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Data |  | Model |  | Data | Model |
|  | Data | Model |  |  |  |  |
| $\sigma_{\Delta w_{\mid E U E}}^{2}$ | 0.081 | $0.166^{*}$ | 0.065 | $0.127^{*}$ | 0.111 | 0.158 |
| $\sigma_{\Delta w_{\mid E E}}^{2}$ | 0.045 | 0.040 | 0.037 | $0.033^{*}$ | 0.028 | 0.026 |
| $\sigma_{\Delta w_{\mid \Delta J}}^{2}$ | 0.051 | $0.083^{*}$ | 0.074 | 0.072 | 0.090 | 0.087 |
| $\sigma_{w_{1}}^{2}$ | 0.285 | 0.349 | 0.237 | 0.255 | 0.269 | 0.321 |
| $\sigma_{w_{11}}^{2}$ | 0.272 | 0.283 | 0.245 | 0.303 | 0.332 | 0.357 |
| $\sigma_{w_{21}}^{2}$ | 0.273 | 0.314 | 0.245 | 0.316 | 0.340 | 0.400 |
| $\sigma_{\Delta w}^{2}$ | 0.055 | $0.101^{*}$ | 0.037 | $0.080^{*}$ | 0.039 | $0.071^{*}$ |
| $\sigma_{\Delta w, \Delta w_{-1}}$ | -0.015 | -0.013 | -0.009 | -0.010 | -0.001 | -0.003 |
| $\sigma_{\Delta w, \Delta w_{-2}}$ | -0.007 | $-0.018^{*}$ | -0.007 | $-0.013^{*}$ | -0.008 | $-0.012^{*}$ |

## The Matching sets and sorting


(a) Less Than Highschool


(b) Highschool

## The Matching sets and sorting




(d) 261lege

## Search Externalities and Low Value Matches

- In this economy some very low value matches are inefficient
- This is because they delay higher value matches to occur and lead higher value firms to remain vacant for too long.
- We can define the planners problem, subject to the search frictions and estimate the extent to which this externality causes welfare loss.
- The planner maximises total output including home production net of vacancy costs

$$
\begin{gathered}
\max _{h}{ }^{S P}, u^{S P}, V S P \\
\left.+\int_{x} b(x) u^{S P}(x) d x-c V^{S P}\right\}
\end{gathered}
$$

subject to the flow constraints implied by the frictions.

## Policy Simulations and search Frictions

## Less than High School

- Here we present the welfare implications of various policies when designed to be optimal.

|  | Steady State <br> Output | Market <br> Production | Home <br> Production | Emp <br> Rate |
| :--- | :---: | :---: | :---: | :---: |
|  | Less Than High School |  |  |  |
| Decentralized | 100.00 | 86.62 | 16.06 | 69.78 |
| Planner | 102.03 | 82.24 | 20.61 | 65.46 |
| Frictionless | 104.90 | 101.27 | 3.63 | 90.00 |
| Min Wage | 100.01 | 86.64 | 16.04 | 69.71 |
| Exp Rating | 100.02 | 86.67 | 16.01 | 69.91 |

## Policy Simulations and search Frictions

## College Graduates

|  | Steady State <br> Output | Market <br> Production | Home <br> Production | Emp <br> Rate |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | College Graduate |  |  |  |  |
| Decentralized | 100.00 | 100.68 | 4.01 | 90.40 |  |
| Planner | 113.54 | 102.77 | 11.14 | 76.51 |  |
| Frictionless | 120.83 | 120.83 | 0.00 | 100.00 |  |
| Min Wage | 100.09 | 100.42 | 4.40 | 89.13 |  |
| Exp. Rating | 102.10 | 102.61 | 3.97 | 90.47 |  |

## Evidence from matched data in Sweden

- In a further paper by Lamadon, Lise, Meghir and Robin and is entitled Matching, Sorting, Productivity and wages
- We have set up with Lisa Johnsson (Stockholm University) a new matched employer employee data set
- This includes almost all firms and workers in Sweden.
- However we have limited it to the Stockholm area for tractability. We have excluded financial firms
- We observe individual earnings and work histories.
- At the firm level we observe output, employment, capital, investment etc.


## Data

1. Register-based labor market statistics (RAMS)

- Workers: all employments during the year - start and end month, worker status, gross wage
- Firms: business sector, institutional sector, ownership control, type of legal entity

2. Longitudinal integration database for health insurance and labour market studies (LISA)

- Workers: age, education, sex, county of residence, marital status, number and ages of children, total earnings

3. Structural business statistics

- Firms: value added, turnover, net profit/loss, wage costs, fixed assets, investments


## Data

## 1. Firms:

- Institutional sectors: private non-financial corporations; central government quasi-corporations
- Business sectors: mining and quarrying; manufacturing; electricity, gas and water supply; construction; wholesale and retail trade; hotels and restaurants; transport, storage and communication; real estate, leasing and business activities
- Types of legal entity: limited partnerships; limited companies other than banking and insurance companies; economic associations; entities of central government


## 2. Workers:

- Age 16-64
- Exclude sailors, farmers and self-employed
- Exclude employments with earnings below one base amount
- Include only the main employment per individual and year


## Data

Autocovariance structure of earnings

|  | High School |  | High School |  | Some |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Dropouts |  | Graduates |  | College |  |
| Order | Men | Women | Men | Women | Men | Women |
| 0 | 0.1179 | 0.1266 | 0.1246 | 0.1713 | 0.1381 | 0.2172 |
| 1 | -0.0326 | -0.0311 | -0.0366 | -0.0447 | -0.0363 | -0.0609 |
| 2 | -0.0035 | -0.0057 | -0.0037 | -0.0124 | -0.0031 | -0.0139 |
| 3 | -0.0010 | -0.0010 | -0.0011 | -0.0003 | -0.0012 | 0.0023 |
| 4 | -0.0005 | -0.0007 | -0.0005 | -0.0012 | -0.0007 | -0.0032 |
| 5 | -0.0005 | -0.0007 | -0.0004 | -0.0014 | -0.0003 | -0.0019 |
| 6 | -0.0004 | -0.0003 | -0.0003 | -0.0007 | -0.0002 | -0.0009 |
| 7 | -0.0002 | -0.0005 | -0.0004 | -0.0004 | -0.0001 | 0.0004 |
| 8 | -0.0003 | -0.0004 | -0.0001 | -0.0004 | 0.0000 | -0.0006 |
| 9 | -0.0002 | -0.0002 | -0.0002 | -0.0002 | -0.0004 | -0.0004 |
| 10 | 0.0000 | -0.0001 | -0.0001 | 0.0000 | 0.0001 | 0.0006 |
| 11 | -0.0001 | -0.0002 | 0.0000 | -0.0002 | -0.0001 | 0.0000 |
| 12 | 0.0001 | 0.0000 | -0.0001 | 0.0001 | -0.0004 | 0.0002 |
| 13 | -0.0001 | 0.0001 | -0.0001 | -0.0003 | -0.0003 | -0.0006 |

## Data

Sorting in the data
$y$ earning conditional on job rank, education, age group and sex


## Data

Heterogeneity in the returns to education

Educ gap in log earnings per gender, age and fir, type


## Data

Gender gap in log earnings per educ, age and fir, type


## Data

firm type quantile vs worker life-best-wage rank


## Data

Sorting - Distribution of workers by firm type
worker type quantile vs firm rank


## SWEDEN: Estimation - Moments

|  | data | model |
| :---: | :---: | :---: |
| main |  |  |
| $E(\log (w))$ | $\begin{array}{r} -4.28 \\ (0.00111) \end{array}$ | -4.17 |
| $\operatorname{var}(\log (w))$ | $\begin{array}{r} 0.298 \\ (0.00113) \end{array}$ | 0.278 |
| $E(y)$ | $\begin{array}{r} 0.0476 \\ (0.000271) \end{array}$ | 0.0574 |
| $\operatorname{var}(y)$ | $\begin{gathered} 0.0129 \\ (0.00968) \end{gathered}$ | 0.00174 |
| $E(w / y)$ | $\begin{array}{r} 0.482 \\ (0.00656) \end{array}$ | 0.422 |
| $\operatorname{cor}(\operatorname{rank}(w), \operatorname{rank}(y))$ | $\begin{array}{r} 0.381 \\ (0.00167) \end{array}$ | 0.488 |
| transitions |  |  |
| $\operatorname{Pr}(E 2 U)$ | $\begin{array}{r} 0.0328 \\ (6.46 \mathrm{e}-05) \end{array}$ | 0.0356 - |
| $\operatorname{Pr}(E E)$ | $\begin{array}{r} 0.938 \\ (0.00014) \end{array}$ | 0.95 |
| $\operatorname{Pr}(J 2 J)$ | $\begin{array}{r} 0.0182 \\ (4.29 \mathrm{e}-05) \end{array}$ | 0.0144 |
| $\operatorname{Pr}($ U2E) | $\begin{array}{r} 0.109 \\ (0.000376) \end{array}$ | 0.0636 |
| $\operatorname{cor}_{J 2 J}\left(\operatorname{rank}\left(y_{t}\right), \operatorname{rank}\left(y_{t+1}\right)\right)$ | $\begin{array}{r} 0.197 \\ (0.00548) \end{array}$ | 0.368 |
| $\operatorname{Pr}_{J 2 J}(\Delta \operatorname{rank}(y) \geq 0)$ | $\begin{array}{r} 0.539 \\ (0.00139) \end{array}$ | 0.763 |
| $E_{J 2 J}(\Delta \operatorname{rank}(y) \mid \Delta \operatorname{rank}(y)<0)$ | $\begin{gathered} -0.0271 \\ (0.000718) \end{gathered}$ | -0.0291 |
| $E_{J 2 J}(\Delta \operatorname{rank}(y) \mid \Delta \operatorname{rank}(y) \geq 0)$ | $\begin{array}{r} 0.0313 \\ (0.000458) \end{array}$ | 0.0363 |

## SWEDEN: Estimation - Parameters

|  | params |
| ---: | ---: |
| $b$ | 0.0608 |
| beta | $(0.0106)$ |
|  | 0.259 |
| $f a$ | $(0.0141)$ |
|  | 0.28 |
| $f$ mx | $(0.0321)$ |
|  | 0.215 |
| $f$ my | $1.0219)$ |
|  | $(0.0441)$ |
| frho | 0.151 |
|  | $(0.0099)$ |
| firmMass | 1.02 |
|  | $(0.15)$ |
| s0 | 1.35 |
|  | $(0.0347)$ |
| s1 | 0.464 |
|  | $(0.016)$ |
| sep | 0.0104 |
|  | $(0.000671)$ |
| sigma | 0.104 |
|  | $(0.0093)$ |

SWEDEN: Equilibrium Matching Distribution


## Equilibrium Surplus Function

## Surplus Function



## Linking with Observables

- It is interesting to examine the link between aspects of the model and observable (possibly endogenous) characteristics $z$, such as education
- For this purpose it is useful to construct the joint distribution of
- We can solve the integral equation

$$
g(w, y)=\int h(w, y \mid z) f(z) d z
$$

- This is achieved by first approximating the known distribution $f(z)$ by a Sieve for some basis functions $\left(\psi_{j}(z)\right)$

$$
f(z)=\sum_{j} \alpha_{j} \psi_{j}(z)
$$

- We can now rewrite the integral equation as

$$
g(w, y) \approx \sum_{k} \alpha_{k} \int h(w, y \mid z) \psi_{j}(z) d z
$$

- We then evaluate at specific pairs of $w_{i}, y_{i}$ to give us a system


## SWEDEN: Unobservable for gender and education groups

- We use wages and proportions in each firm type and the model to estimates the distribution of unobserved productivity of each observable groups



## SWEDEN: Unobservable for gender and education groups



## Model Simulations - Wage variance decomposition

|  | female |  |  | male |  |  | total |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | high | low | medium | high | low | medium |  |
| $\operatorname{Var}[\log (w)-E(\log (w) \mid x, y)]$ | 0.35 | 0.51 | 0.45 | 0.32 | 0.44 | 0.42 | 0.4 |
| $\operatorname{Var}[E(\log (w) \mid x, y)-E(\log (w) \mid x)]$ | 0.57 | 0.39 | 0.44 | 0.63 | 0.47 | 0.49 | 0.5 |
| $\operatorname{Var}[E(\log (w) \mid x)-E(\log (w))]$ | 0.09 | 0.1 | 0.11 | 0.05 | 0.08 | 0.09 | 0.1 |
| total | 0.34 | 0.33 | 0.34 | 0.33 | 0.33 | 0.33 | 0.34 |

## Conclusions

- We provide a rich framework for labor market policy analysis
- We let the data determine how important the frictions are and the extent to which corrective policy action is called for
- We find little scope for employment protection or minimum wages where this is usually targeted, i.e. the lower skill workers
- We also find that mismatch is very important among higher education workers
- Also unemployment for them is entirely due to search frictions

