Promoting School Competition Through School Choice: A Market Design Approach

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Spread of school choice around the globe

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School authorities take into account preferences of students/parents

Market design researchers have been offering specific mechanisms

Resulting real-life system reforms: Boston, NYC, New Orleans

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Resulting real-life system reforms: Boston, NYC, New Orleans

Their aim=

Assigning students to schools efficiently, fairly, and simply

"If we implement choice among public schools,
we unlock the values of competition.
Schools that compete for students will
make those changes
that allow them to succeed."

from *National Governors' Association Report*

Focus of much of policy debate
on school choice
=How to improve school quality
by promoting competition
(rather than how to assign
students to schools with fixed quality)

Motivation

We introduce several criteria of whether a SC mechanism incentivizes schools to improve their quality

A mechanism

respects improvements of school quality

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respects improvements of school quality

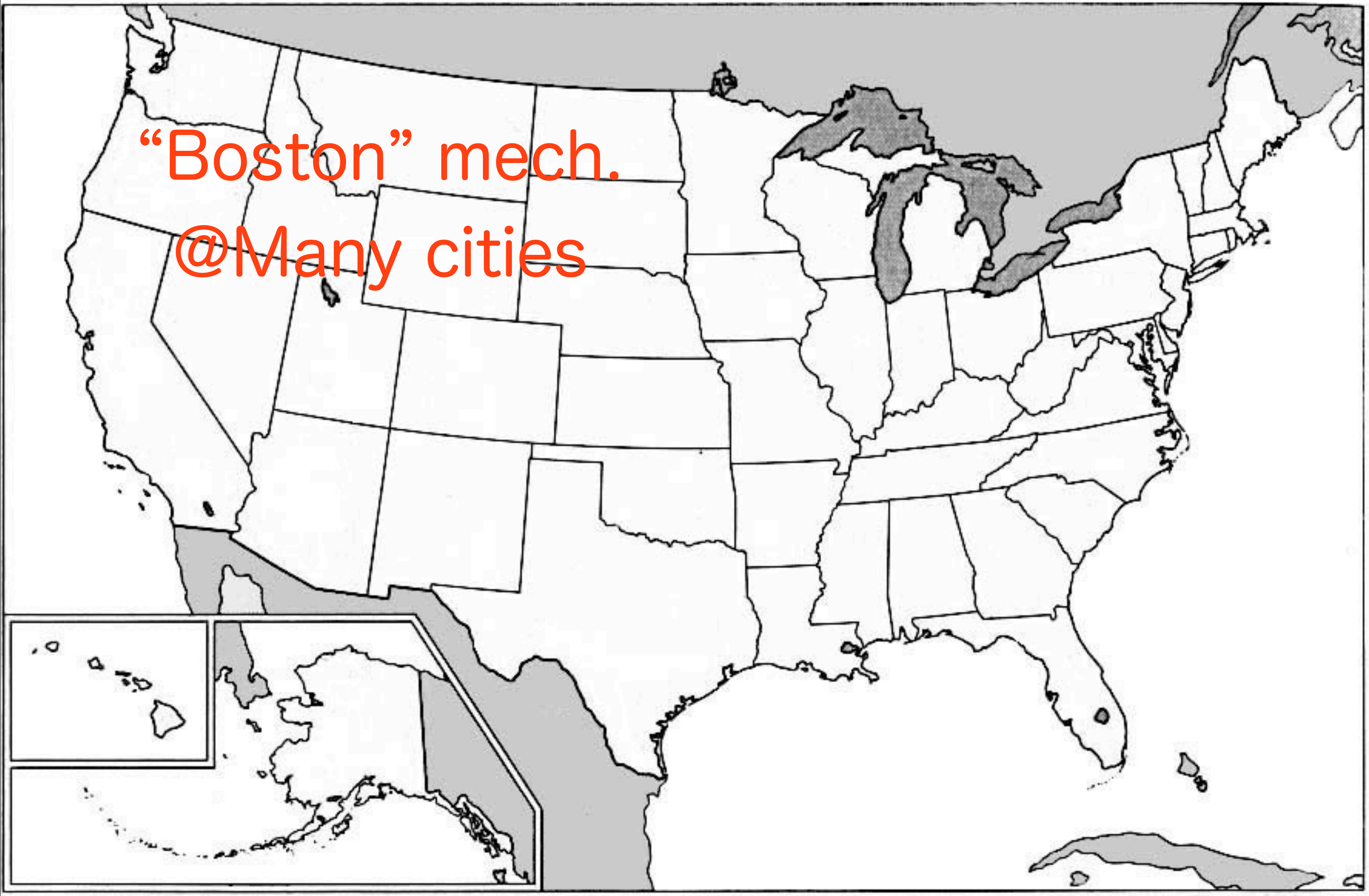
if

when a school improves & thereby becomes more preferred by students, that school becomes weakly better off

Motivation

We introduce several criteria of whether a SC mechanism incentivizes schools to improve their quality &

determine if these criteria are satisfied by focal SC mechanisms.


“Boston” mech.
@Many cities


"Boston" mech.
@Many cities
"Student-Optimal Stable"
@Boston, NYC etc
"Top Trading Cycles" mech.
@New Orleans

Result

For incentivizing schools to improve,

SOSM > Boston > TTC

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Criteria
RI in General Environments
RI for Desirable Students in General Environments
RI in Large Environments
RI for Desirable Students in Large Environments
RI in Terms of Enrollment
RI of Student Quality

Too Many Results

For incentivizing schools to improve,

SOSM > Boston > TTC

Criteria	SOSM	Boston	TTC
RI in General Environments	$\times$	$\times$	$\times$
RI for Desirable Students in General Environments	$\times$	$\times$	$\times$
RI in Large Environments	$\checkmark$	$\times$	$\times$
RI for Desirable Students in Large Environments	$\checkmark$	$\times$	$\times$
RI in Terms of Enrollment	$\checkmark$	$\checkmark$	$\times$
RI of Student Quality	$\checkmark$	$\checkmark$	$\checkmark$

Model

Students	Schools
s_1 with $>_{s_1}$	c_1 with $>_{c_1}$
⋮	⋮
s_m with $>_{s_m}$	c_n with $>_{c_n}$

Each school has a quota &
a preference over sets of students.

A Criterion of Promoting Competition: Respecting Improvements of School Quality

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A student preference profile is an *improvement for school c* over another

A Criterion of Promoting Competition: Respecting Improvements of School Quality

A student preference profile is
an *improvement for school c* over another
if
all students rank c weakly higher
(while keeping rankings of
the other schools unchanged)

A Criterion of Promoting Competition: Respecting Improvements of School Quality

A mechanism

respects improvements of school quality

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any improvement for any school c

makes c weakly better off

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※Balinaki-Sonmez (99): RI of student quality

Stable Mechanisms

e.g. "*Student-Optimal Stable*" Mechanism

=Student-Proposing
Deferred Acceptance Mechanism

SOSM Does Not Respect Improvements

SOSM Does Not Respect Improvements

Schools

$\succ_c : s, \bar{s}, \emptyset,$

$\succ_{\bar{c}} : \bar{s}, s, \emptyset,$

Students

$\succ_s : \bar{c}, c, \emptyset,$

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SOSM Does Not Respect Improvements

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Capacity of c=2

Capacity of $\bar{c}$=1

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Students

$\succ_s: \bar{c}, c, \emptyset,$

$\succ_{\bar{s}}: \bar{c}, c, \emptyset,$

$$\begin{pmatrix} c & \bar{c} \\ s & \bar{s} \end{pmatrix}$$

Capacity of c=2

Capacity of $\bar{c}$=1

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$\downarrow$

$\succ'_{\bar{s}}: c, \bar{c}, \emptyset.$

Improvement for c

$$\begin{pmatrix} c & \bar{c} \\ s & \bar{s} \end{pmatrix}$$

Capacity of c=2

Capacity of $\bar{c}$=1

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Improvement for c

Before

$$\begin{pmatrix} c & \bar{c} \\ s & \bar{s} \end{pmatrix}$$

After

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Capacity of c=2

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Before

$$\begin{pmatrix} c & \bar{c} \\ s & \bar{s} \end{pmatrix}$$

After

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c strictly worse off by improvement

Impossibility for Stable Mechanisms

Proposition

No stable mechanism respects improvements.

Impossibility for Stable Mechanisms

<u>Proposition</u>

No stable mechanism respects improvements.

<u>Proof</u>

In the example, verify that the stable matching is unique at each preference profile.

Pareto Efficient Mechanisms (for Students)

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SOSM is not PE, but others are.

(1) "*Boston*" mechanism:

Pareto Efficient Mechanisms (for Students)

SOSM is not PE, but others are.

(1) "*Boston*" mechanism:

- Used in many school districts.
- Recently under attack due to instability & poor incentive property.

Pareto Efficient Mechanisms (for Students)

SOSM is not PE, but others are.

(2) "*Top Trading Cycles*" mech.:

Pareto Efficient Mechanisms (for Students)

SOSM is not PE, but others are.

(2) "*Top Trading Cycles*" mech.:

- Not only PE but also strategyproof.
- Started to be used in New Orleans

Impossibility for PE Mechanisms

Proposition

No PE mechanism respects improvements.

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No PE mechanism respects improvements.

Proof

By a complicated counterexample (explained later if time permits)

When Does a Stable/PE Mechanism Respect Improvements?

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<u>Theorem (Informal)</u>

There is a stable or PE mechanism that respects improvements.

⇕

School preferences are "almost" the same

When Does a Stable/PE Mechanism Respect Improvements?

<u>Theorem (Informal)</u>

There is a stable or PE mechanism that respects improvements.

⇕

School preferences are "almost" the same

Very restrictive

Only uniformly negative results so far...

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What can be said on
a desirable school choice mechanism?

SOSM Does NOT Respect Improvements

Schools

$\succ_c: s, \bar{s}, \emptyset,$

$\succ_{\bar{c}}: \bar{s}, s, \emptyset,$

Students

$\succ_s: \bar{c}, c, \emptyset,$

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↓

$\succ'_{\bar{s}}: c, \bar{c}, \emptyset.$

Improvement for c

Before $\begin{pmatrix} c & \bar{c} \\ s & \bar{s} \end{pmatrix}$ After $\begin{pmatrix} c & \bar{c} \\ \bar{s} & s \end{pmatrix}$

Unnaturally few schools & students

c strictly worse off despite improvement

Respecting Improvements in Large Environments?

School districts usually contain many schools & students.

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School districts usually contain many schools & students.

In such environments, the violation of RI may be rare.

Respecting Improvements in Large Environments?

Consider a model of large environments by Kojima-Pathak (08), where

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Consider a model of large environments by Kojima-Pathak (08), where

(1) size indexed by the # of schools
(students also increase as schools do)
(2) preferences drawn from a prob. dist.

Approximate Respecting Improvements in Large Environments

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$\alpha_c(\varphi)$:= Prob that mech. φ does not RI for school c at realized preferences

Approximate Respecting Improvements in Large Environments

$\alpha_c(\varphi)$:= Prob that mech. φ does not RI for school c at realized preferences

φ *approximately RI in large environments* if $\forall c$,

$$\alpha_c(\varphi) \to 0$$

(as the # of school $\to \infty$).

Theorem

Any stable mechanism (e.g. SOSM) approximately respects improvements in large environments.

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The Boston or TTC mechanism does NOT approximately RI even in large environments.

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Proof Sketch (0/3)

Violation of RI

=Worse off by an improvement

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Violation of RI

=Worse off by an improvement

=Better off by a disimprovement

Proof Sketch (0/3)

Violation of RI

=Worse off by an improvement

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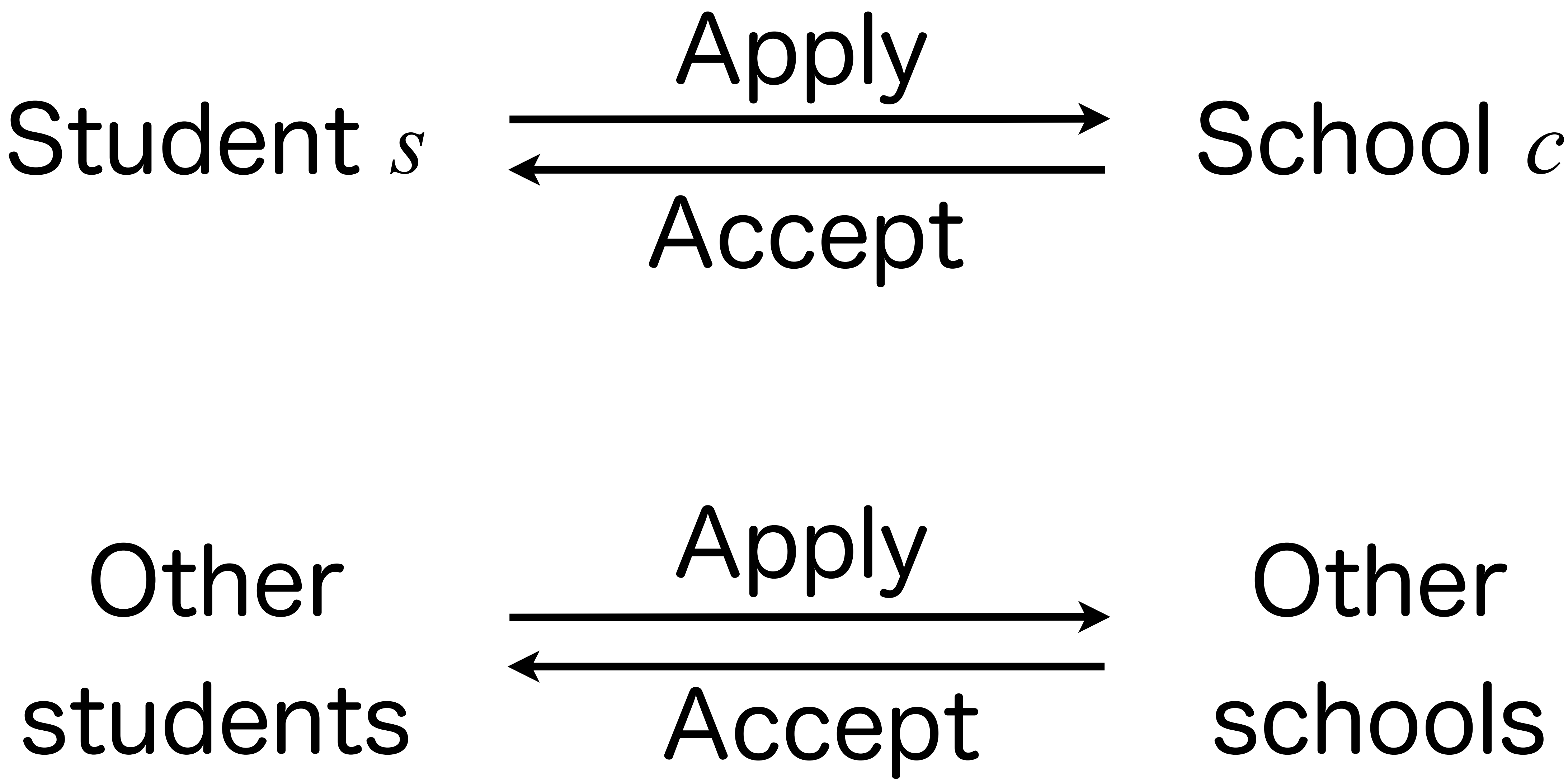
Why such a situation may occur?

Proof Sketch (1/3)

Consider the algorithm in SOSM

Proof Sketch (1/3)



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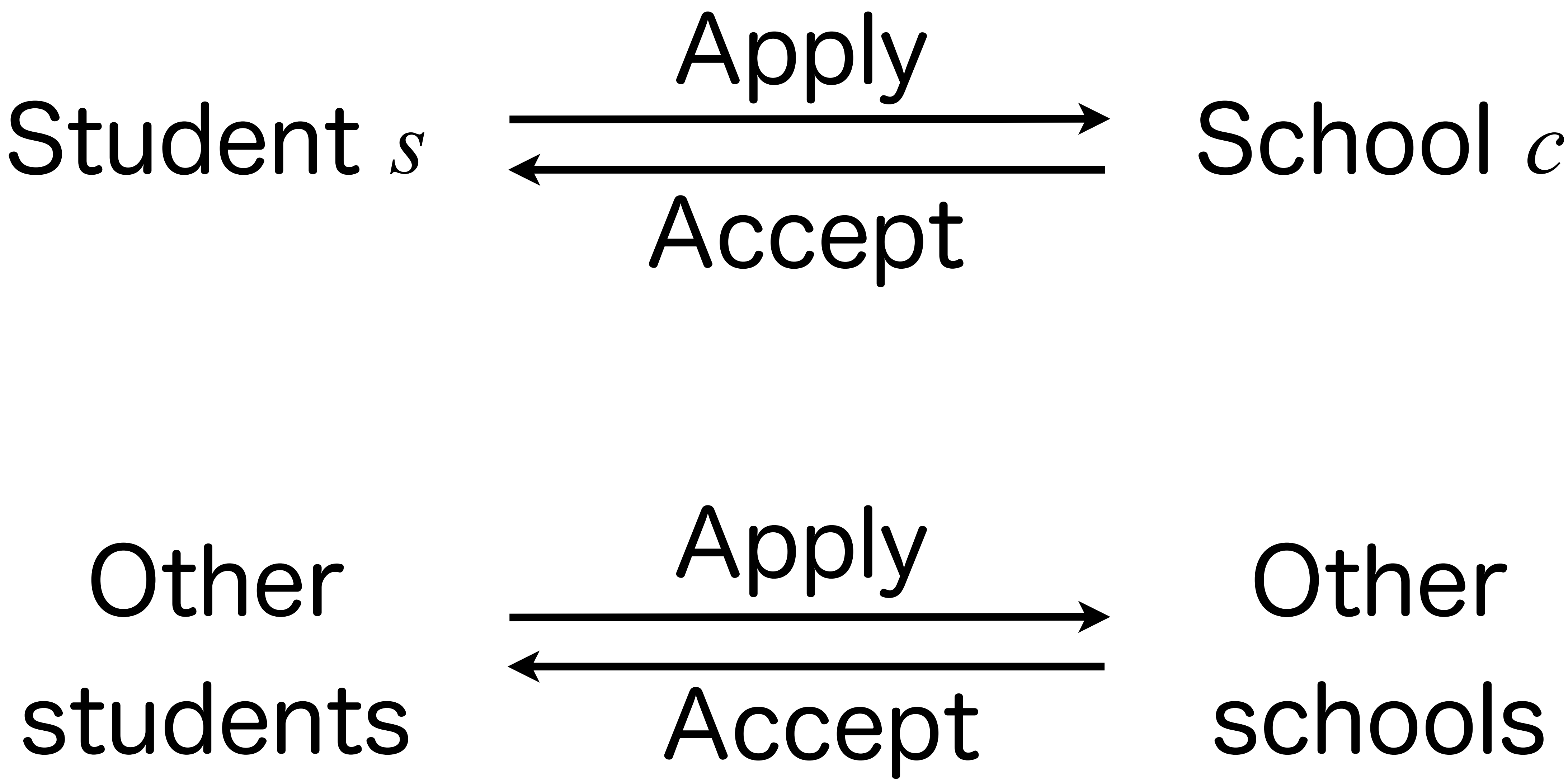
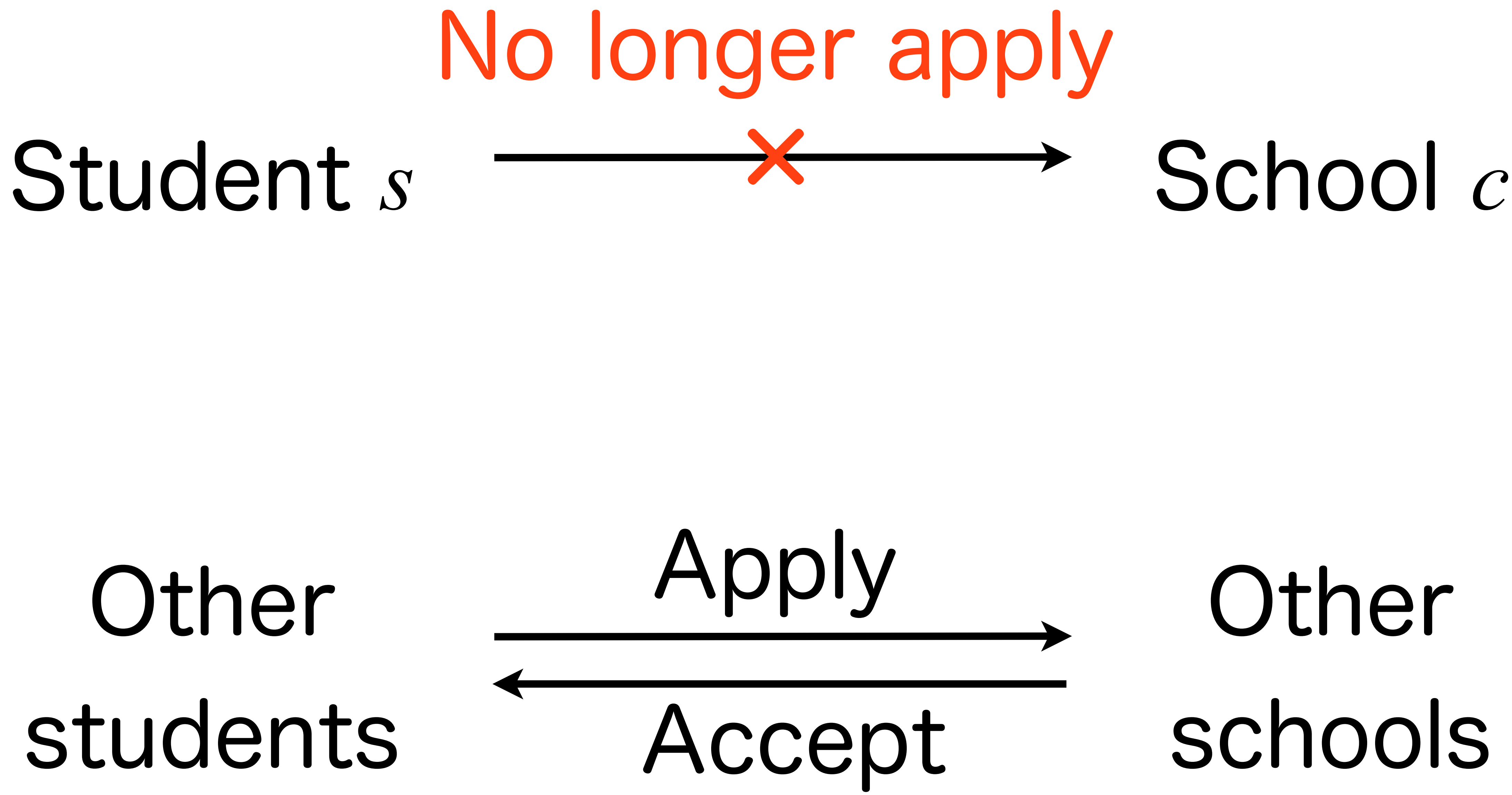


Assume c disimproves for s

Proof Sketch (1/3)



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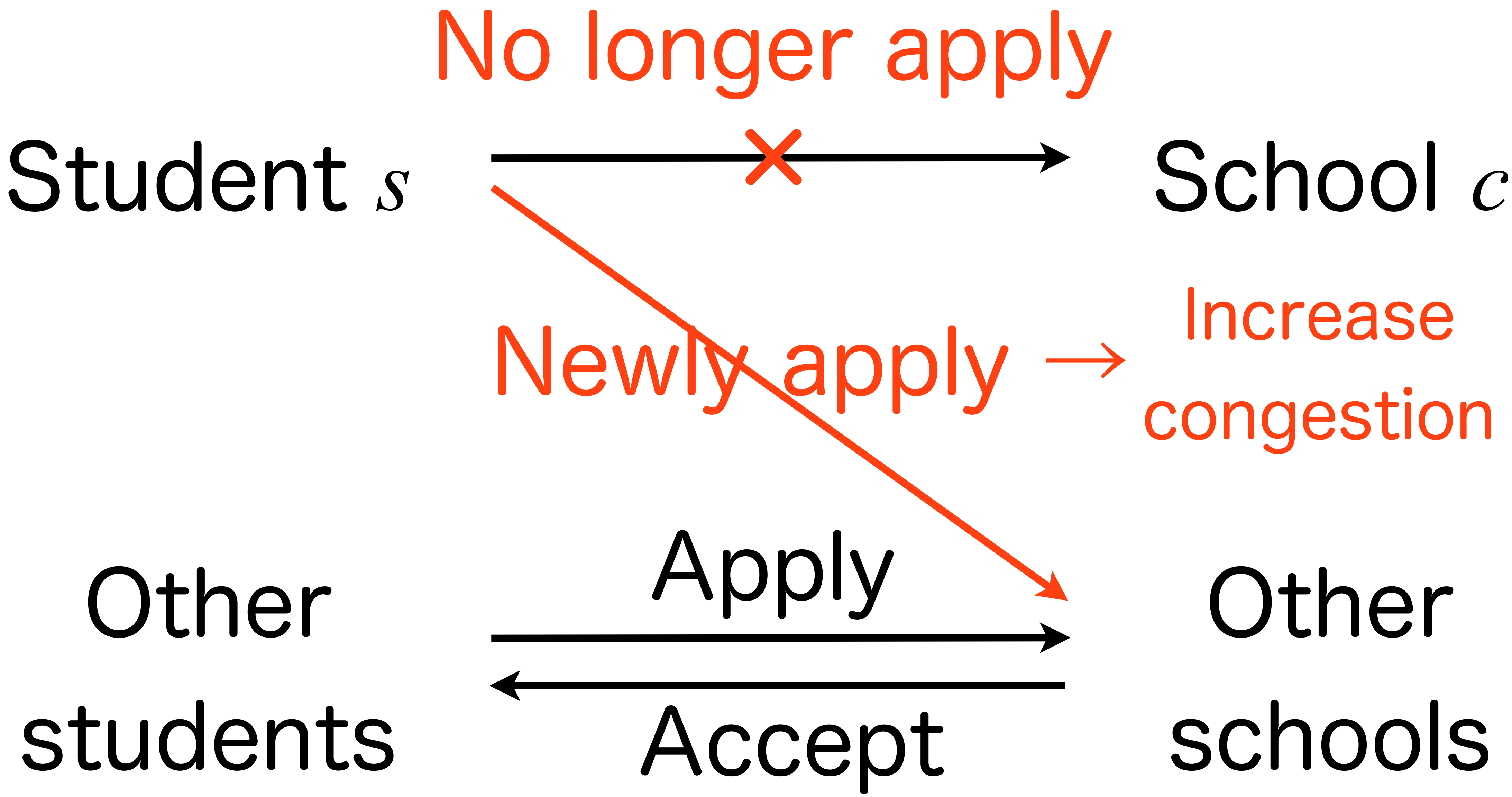

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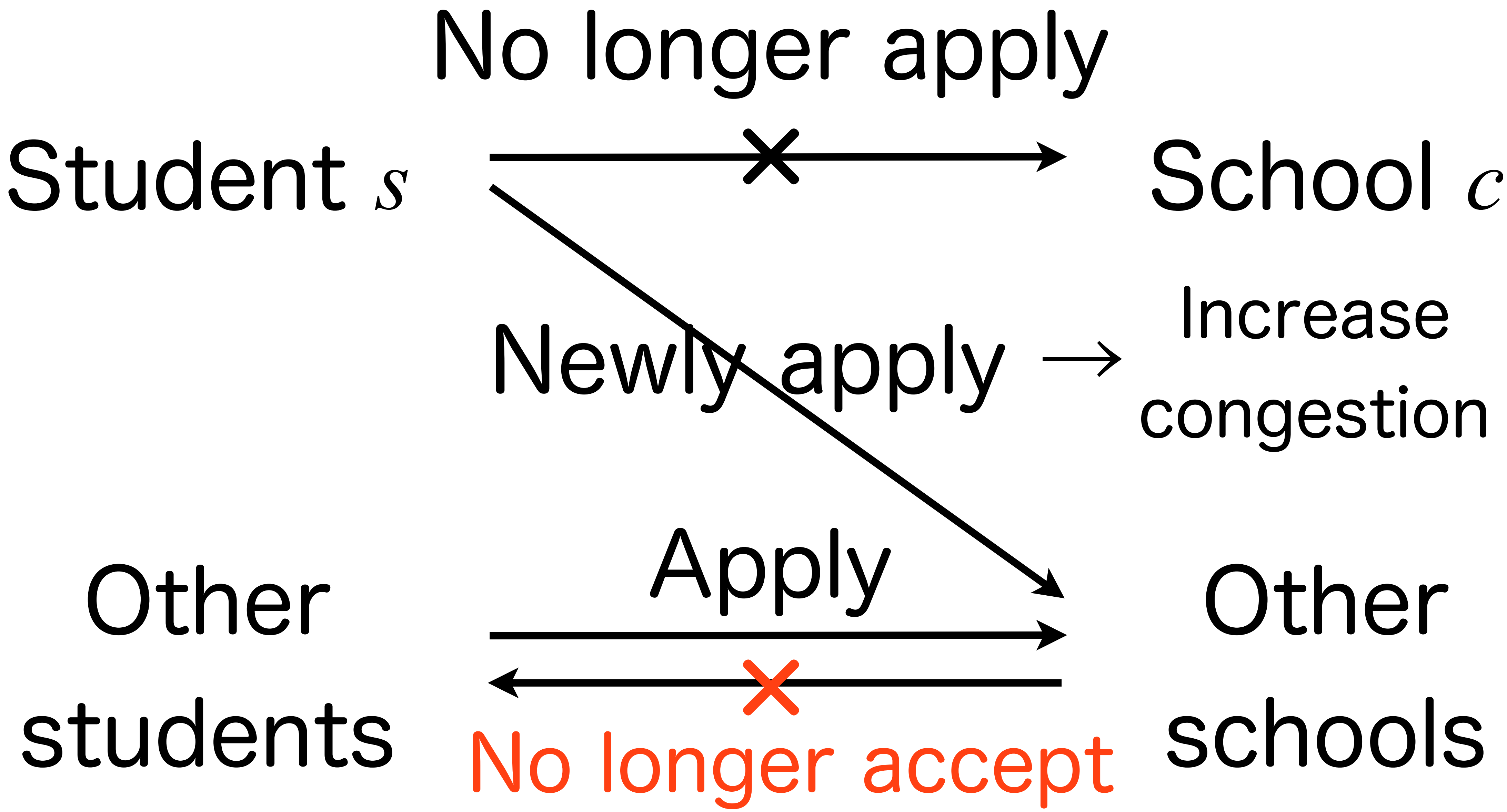

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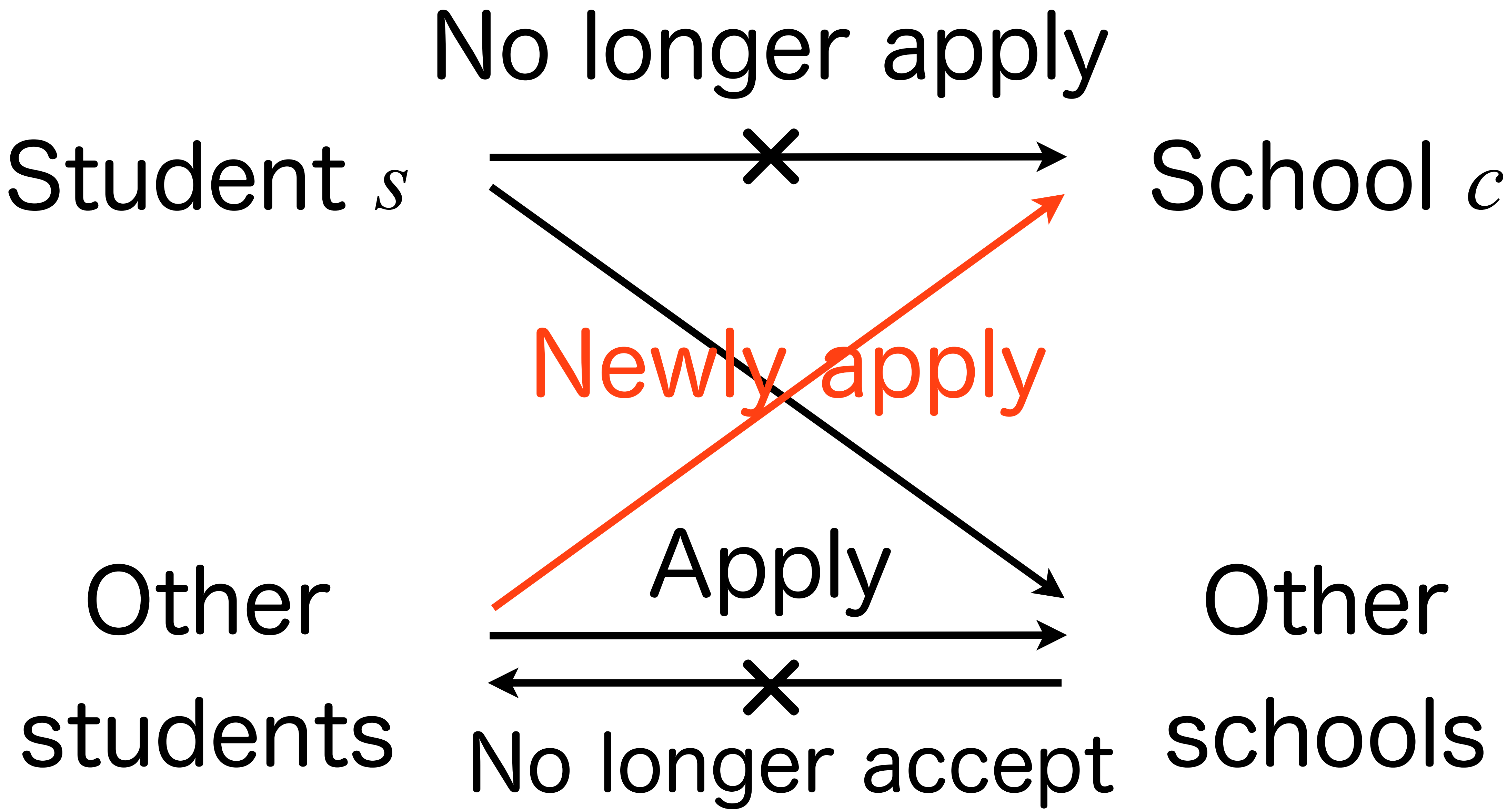

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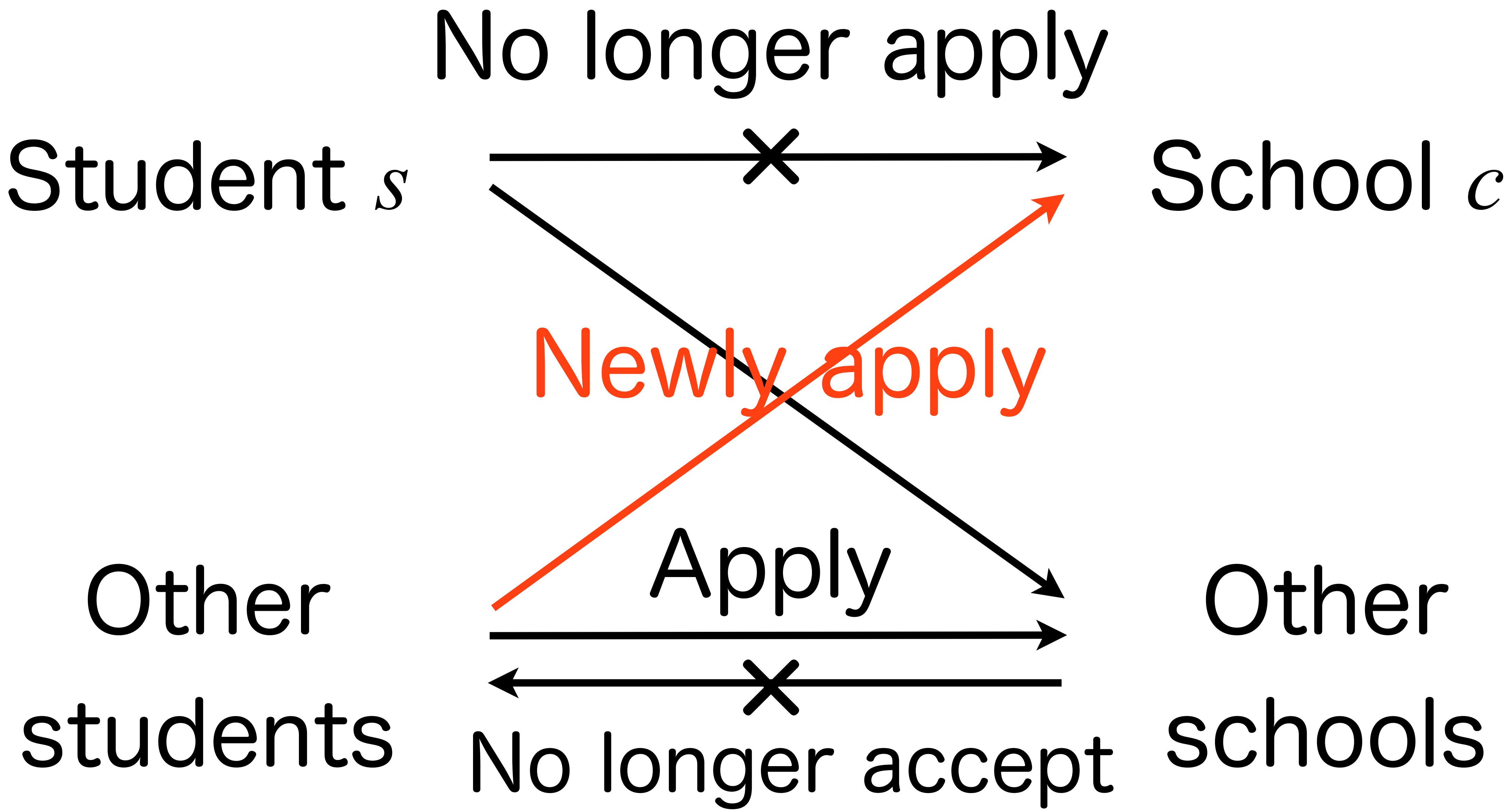
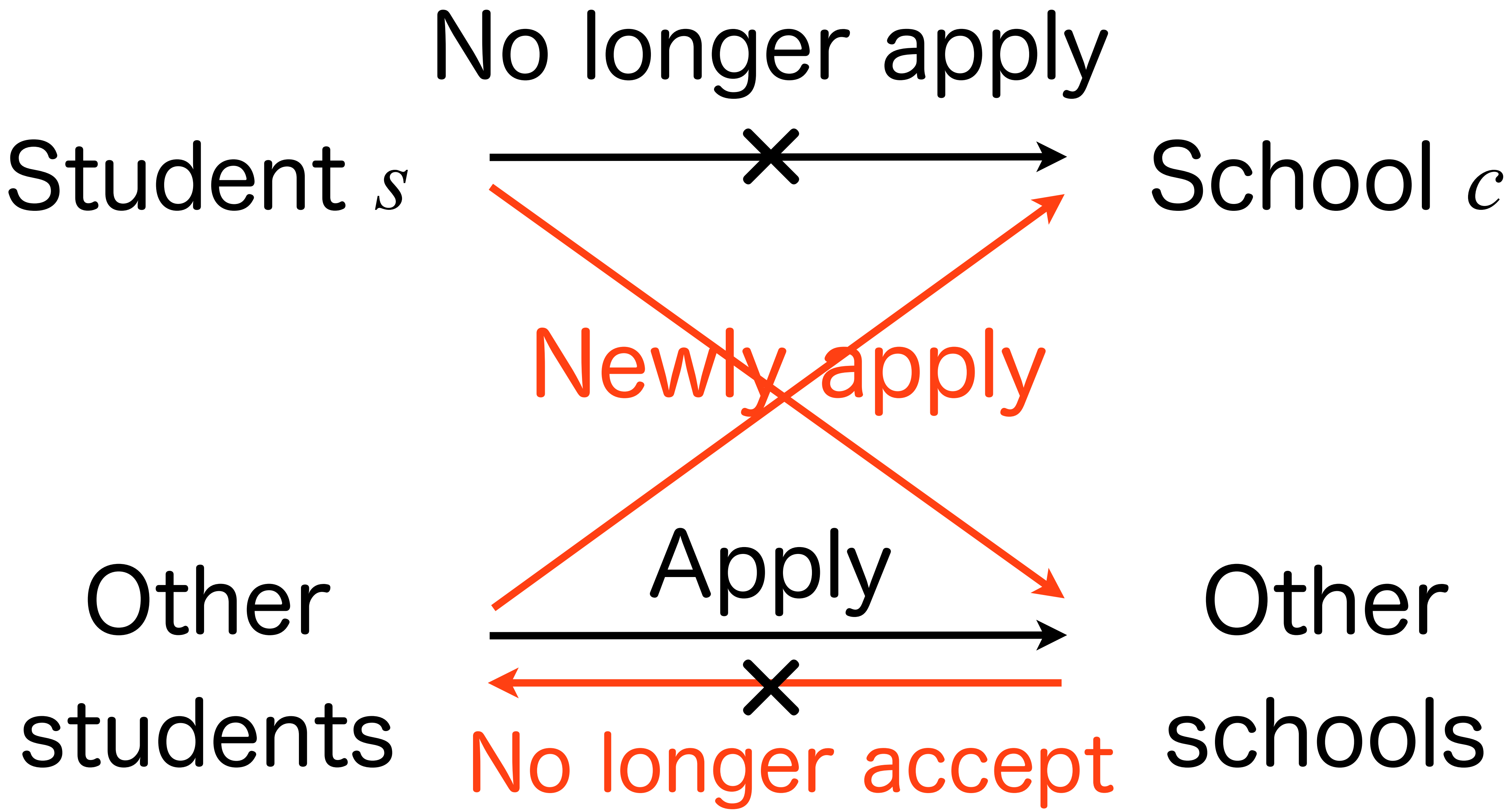


Others may be more desirable than s for c.

Proof Sketch (1/3)



But such chains are rare in the large market

Proof Sketch (2/3)

Key observation:

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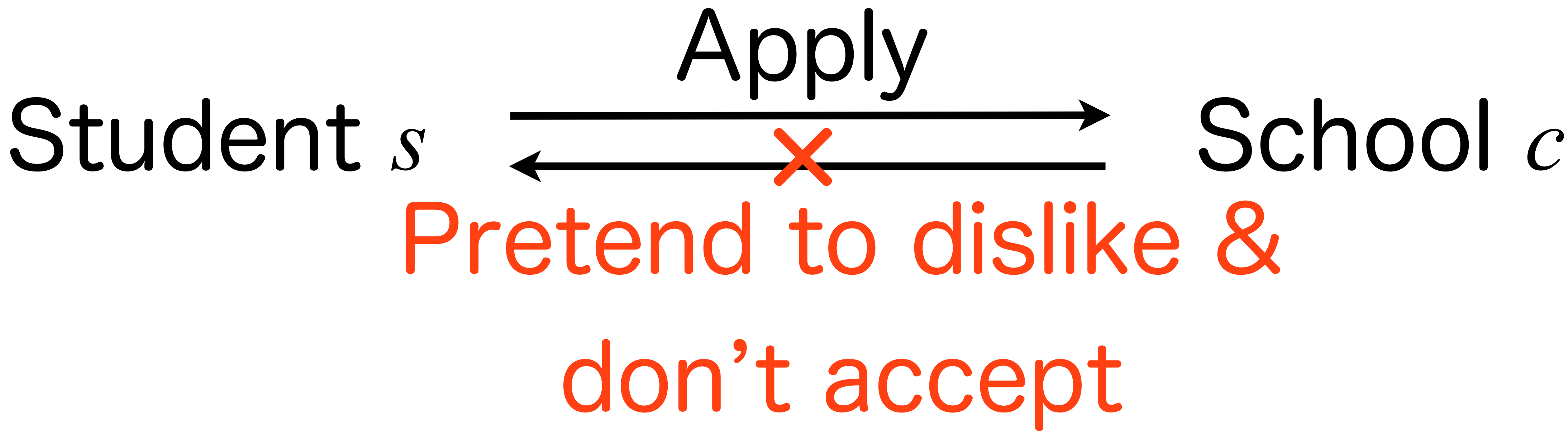
Such a benefit (if any) can be replicated by the following behavior of the disimproving school.

Proof Sketch (2/3)



Proof Sketch (2/3)

Apply

Student s → School c

←✕

Pretend to dislike &

don't accept

Violation of RI for a school

≅

Profitable preference manipulation

by that school

Proof Sketch (2/3)

Lemma

Take any stable mechanism.

Proof Sketch (2/3)

<u>Lemma</u>

Take any stable mechanism.
If it does not RI for a school
at a preference profile,
then it is not optimal for that school to
report its true preference
at that preference profile.

Proof Sketch (3/3)

Take the contraposition:

For any stable mechanism,

Strategy-proofness for schools $\rightarrow$ RI.

Proof Sketch (3/3)

Take the contraposition:

For any stable mechanism,

Strategy-proofness for schools → RI.

<u>Lemma</u> (K-P(08)+Pathak-Sonmez(11))

Any stable mechanism is

approximately strategy-proof for schools.

Theorem

The Boston mechanism does NOT approximately respect improvements even in large environments.

Definition of Boston Mechanism

Similar to SOSM, but all matches at each step of the algorithm are final.

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Step t (≥ 1):

Each student who has not been matched to any school at Step t-1 applies for next preferred school (if any)

Definition of Boston Mechanism

Step $t\ (\geq 1)$ (Continued):

Each school considers these students

~~*and students who are kept from Step t-1 together.*~~

It accepts most preferred students up to its quota & rejects everyone else

Definition of Boston Mechanism

Step $t\ (\geq 1)$ (Continued):

Each school considers these students

~~*and students who are kept from Step $t-1$ together.*~~

It accepts most preferred students up to its quota & rejects everyone else

※Students accepted at a step will never be rejected in any later step

Boston Does NOT Respect Improvements: Intuition

In the Boston mechanism, students applying in earlier steps are favored (regardless of school preferences).

Boston Does NOT Respect Improvements: Intuition

In the Boston mechanism, students applying in earlier steps are favored (regardless of school preferences).

So it may be bad news for a school if an undesirable student prefers it more & applies earlier.

Theorem

The TTC mechanism does NOT approximately respect improvements even in large environments.

TTC Does NOT Respect Improvements: Intuition

An undesirable student for a school
can be matched with that school
if he could trade priorities with
a more desirable student for that school.

TTC Does NOT Respect Improvements: Intuition

An undesirable student for a school can be matched with that school if he could trade priorities with a more desirable student for that school.

So an undesirable student pointing to a school earlier may be bad news for that school.

Policy Implication

	SOSM	Boston	TTC
RI in Large Environments	✓	×	×

For incentivizing schools to improve,

SOSM is better than the others.

Policy Implication

	SOSM	Boston	TTC
RI in Large Environments	✓	×	×

For incentivizing schools to improve, SOSM is better than the others.

Robust to changes in the criterion of respecting improvements?

Alternative Criteria of Promoting Competition

Alternative Criteria of Promoting Competition

(1) Respecting improvements when schools care only about enrollment

(2) RI when schools try to improve to attract only “desirable” students

Similar results as in the case with RI

Avenues for Future Research

Empirical test of the different effects of the different mech.s on school quality?

Quantification of them by simulations?

Comparison with
other forms of schools choice?
e.g. Charter schools, vouchers

General Message

Market design needs to consider
how different mechanisms induce
different long-term behavior of agents.

Additional Slides

Model

Students	Schools
s_1	c_1
$\vdots$	$\vdots$
s_m	c_n

Model

Students	Schools
s_1 with $>_{s_1}$	c_1
$\vdots$	$\vdots$
s_m with $>_{s_m}$	c_n

Each student has a strict preference over schools & being unmatched ($\emptyset$).

Model

Students

s_1 with $>_{s_1}$

⋮

s_m with $>_{s_m}$

Schools

c_1 with $>_{c_1}$

⋮

c_n with $>_{c_m}$

A *matching* μ assigns each s to (at most) one school μ_s.

Model

Students	Schools
s_1 with $>_{s_1}$	c_1 with $>_{c_1}$
$\vdots$	$\vdots$
s_m with $>_{s_m}$	c_n with $>_{c_n}$

In other words, μ assigns each c to a set of students μ_c within quotas.

Model

Students	Schools
s_1 with $>_{s_1}$	c_1 with $>_{c_1}$
$\vdots$	$\vdots$
s_m with $>_{s_m}$	c_n with $>_{c_n}$

A *mechanism* assigns a matching to each (reported) preference profile.

Stable Mechanisms

A matching μ is *individually rational*

if $\forall$ student s, $\mu_s \geq_s \varnothing$.

Stable Mechanisms

A matching μ is *individually rational*

if $\forall$ student s, $\mu_s \geq_s \varnothing$.

A matching is *stable*

if it is IR and $\nexists$ (s, c) such that

- $c >_s \mu_s$ and
- (1) $|\mu_c| < q_c$ or (2) $\exists s' \in \mu_c$ with $s >_c s'$.

e.g. "*Student-Optimal Stable*" Mechanism

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Start at matching where none is matched.

Step t (≥ 1):

Each student who has not been matched to any school at Step t-1 applies for next preferred school (if any)

e.g. "*Student-Optimal Stable*" Mechanism

Step t (≥ 1) (Continued):

Each school considers these students

and students who are kept from Step t-1 together.

It keeps most preferred students

up to its quota & rejects everyone else

e.g. "*Student-Optimal Stable*" Mechanism

Step $t\ (\geq 1)$ (Continued):

Each school considers these students *and students who are kept from Step t-1 together.*

It keeps most preferred students up to its quota & rejects everyone else

※Students kept at a step may be rejected in a later step

e.g. "*Student-Optimal Stable*" Mechanism

The algorithm stops at a step where no rejection occurs, producing a matching.

e.g. "*Student-Optimal Stable*" Mechanism

The algorithm stops at a step where no rejection occurs, producing a matching.

<u>Fact</u>

SOSM outputs a stable matching & is strategy-proof for students.

NO Pareto Efficient Mechanism Promotes Competition: Proof

Schools

$\succ_c : \bar{s}, s, \emptyset,$

$\succ_{\bar{c}} : s, \bar{s}, \emptyset,$

Students

$\succ_s : \bar{c}, \emptyset,$

$\succ_{\bar{s}} : c, \emptyset.$

↓

$\succ'_{\bar{s}} : \bar{c}, c, \emptyset.$

Improvement for $\overline{c}$

Before

$$\begin{pmatrix} c & \bar{c} \\ \bar{s} & s \end{pmatrix}$$

After cannot change

Capacity of both schools 1

NO Pareto Efficient Mechanism Promotes Competition: Proof

Schools

$\succ_c : \bar{s}, s, \emptyset,$

$\succ_{\bar{c}} : s, \bar{s}, \emptyset,$

Students

$\succ_s : \bar{c}, \emptyset,$

$\succ'_{\bar{s}} : \bar{c}, c, \emptyset.$

↓

$\succ''_s : c, \bar{c}, \emptyset.$

Improvement for c

Before

$$\begin{pmatrix} c & \bar{c} \\ \bar{s} & s \end{pmatrix}$$

After

$$\begin{pmatrix} c & \bar{c} \\ s & \bar{s} \end{pmatrix}$$

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NO Pareto Efficient Mechanism Promotes Competition: Proof

Schools

$\succ_c : \bar{s}, s, \emptyset,$

$\succ_{\bar{c}} : s, \bar{s}, \emptyset,$

Students

$\succ_s : \bar{c}, \emptyset,$

$\succ'_{\bar{s}} : \bar{c}, c, \emptyset.$

$\downarrow$

$\succ''_s : c, \bar{c}, \emptyset.$

Improvement for c

Before

$$\begin{pmatrix} c & \bar{c} \\ \bar{s} & s \end{pmatrix}$$

After

$$\begin{pmatrix} c & \bar{c} \\ s & \bar{s} \end{pmatrix}$$

c strictly worse off after improvement

When Does a Stable/PE Mechanism Promote Competition?

A school preference profile is

virtually homogeneous

if

all schools rank students

in exactly the same way

except top $\min_c q_c$ students

When Does a Stable Mechanism Promote Competition?

Theorem

There is a stable mechanism that respects improvements.

⟺

Every school's capacity is 1 or school preferences are VH

When Does a PE Mechanism Promote Competition?

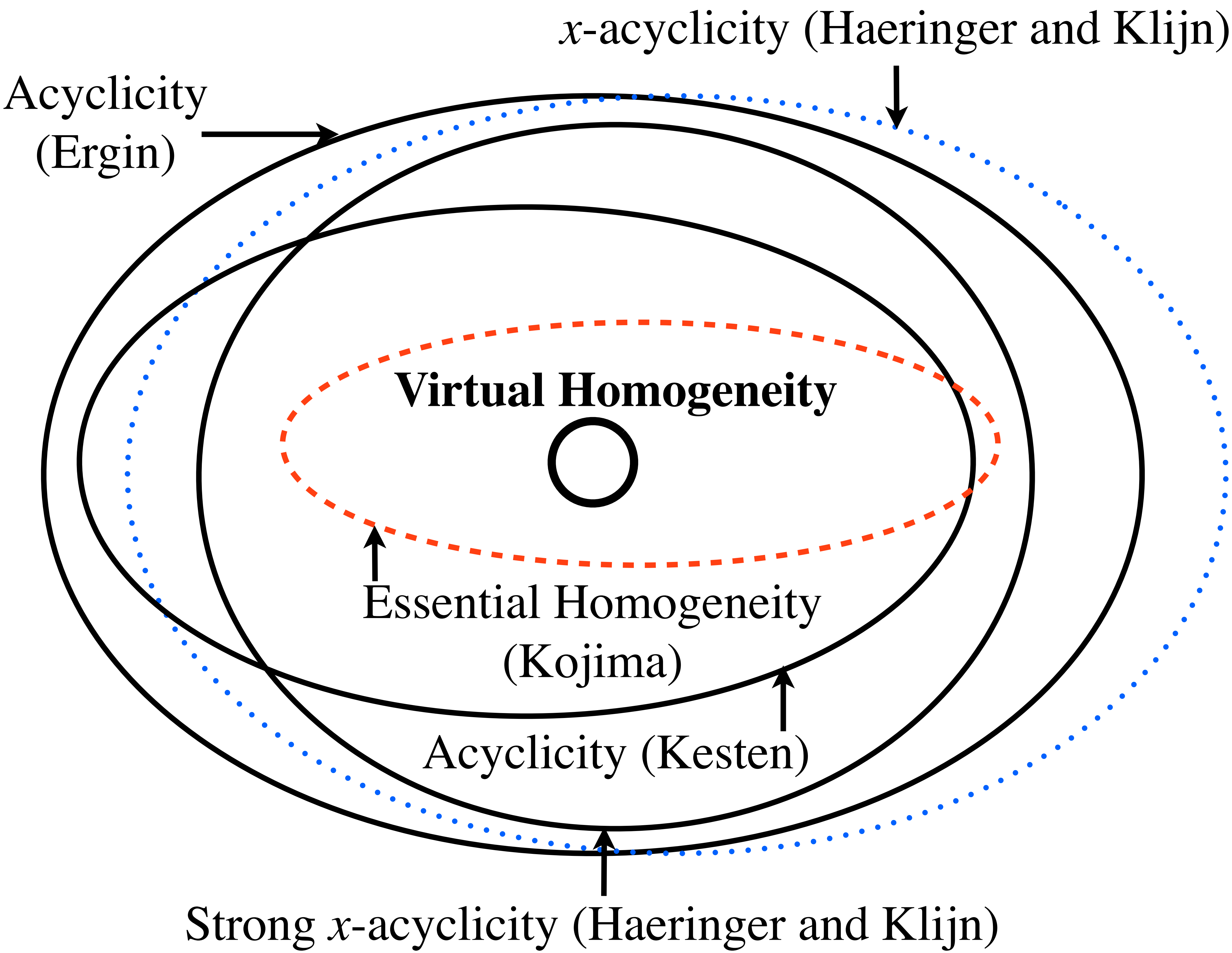
<u>Theorem</u>

There is a PE mechanism that respects improvements.

$\Updownarrow$

School preferences are virtually homogeneous

Relationship between VH & “acyclicity”



Large Market Model

- A **random market** is a tuple $\tilde{\Gamma} = (C, S, k, \mathcal{D})$, where
  - k is a positive integer and
  - $\mathcal{D}$ is a pair $(\mathcal{D}_C, \mathcal{D}_S)$ of probability distributions.
- Each random market induces a market by randomly generating preferences of students.
  - $\mathcal{D}_S = (p_c)_{c \in C}$ is a probability distribution on C.
  - Preferences of each student s are drawn independently without replacement using probability distribution $\mathcal{D}_S$ to form the preference list of students of length k.
- The preference distribution of schools is completely general: $\mathcal{D}_C$ may be any distribution (or even degenerate).

Large Market Model

Definition

A sequence of random markets $(\tilde{\Gamma}^n)_{n \in \mathbb{N}}$ is **regular** if there exist positive integers k, $\tilde{q}$ and $\hat{q}$ such that

1. $k^n \leq k$ for all n,
2. $q_c \leq \hat{q}$ for all n and $c \in C^n$,
3. $|S^n| \leq \tilde{q}n$ for all n, and
4. for all n and $c \in C^n$, every $s \in S^n$ is acceptable to c at any realization of preferences for c at $\mathcal{D}_{C^n}$.

- We also impose the condition that the market is **sufficiently thick**, i.e. that there are no ‘super-popular’ schools.
- For example, if $\frac{p_c}{p_{\bar{c}}} \leq T$ for some $T \in \mathbb{R}$ for all $c, \bar{c} \in C$, the market is sufficiently thick.

Definition of TTC Mechanism

Step t: Each student $s \in S$ points to her most preferred school (if any); students who do not point at any school are assigned to $\emptyset$. Each school $c \in C$ points to its most preferred student. As there are a finite number of schools and students, there exists at least one cycle, i.e. a sequence of distinct schools and students $(s_1, c_1, s_2, c_2, \ldots, s_K, c_K)$ such that student s_1 points at school c_1, school c_1 points to student s_2, student s_2 points to school c_2, ..., student s_K points to school c_K, and, finally, school c_K points to student s_1. Every student s_k $(k = 1, \ldots, K)$ is assigned to the school she is pointing at.

ill Behavior of TTC: Example

Students	Schools
$\succ_{s_1}: c_3, c_1, \emptyset,$	$\succ_{c_1}: s_1, s_2, s_3, s_4, \emptyset,$
$\succ_{s_2}: c_2, c_1, \emptyset,$	$\succ_{c_2}: s_1, s_2, \ldots, \emptyset$
$\succ_{s_3}: c_3, c_1, \emptyset,$	$\succ_{c_3}: s_4, s_3, s_2, s_1, \emptyset$
$\succ_{s_4}: c_2, c_4, \emptyset,$	$\succ_{c_4}: s_4, \ldots, \emptyset.$

ill Behavior of TTC: Example

Students	Schools
$\succ_{s_1}: c_3, c_1, \emptyset,$	$\succ_{c_1}: s_1, s_2, s_3, s_4, \emptyset,$
$\succ_{s_2}: c_2, c_1, \emptyset,$	$\succ_{c_2}: s_1, s_2, \ldots, \emptyset$
$\succ_{s_3}: c_3, c_1, \emptyset,$	$\succ_{c_3}: s_4, s_3, s_2, s_1, \emptyset$
$\succ_{s_4}: c_2, c_4, \emptyset,$	$\succ_{c_4}: s_4, \ldots, \emptyset.$

Capacity of c_1=2

Capacity of every other school=1

ill Behavior of TTC: Example

Improvement for c_1

Students Schools

$$\succ'_{s_1}: c_1, c_3, \emptyset, \qquad \succ_{c_1}: s_1, s_2, s_3, s_4, \emptyset,$$

$$\succ_{s_2}: c_2, c_1, \emptyset, \qquad \succ_{c_2}: s_1, s_2, \ldots, \emptyset$$

$$\succ_{s_3}: c_3, c_1, \emptyset, \qquad \succ_{c_3}: s_4, s_3, s_2, s_1, \emptyset$$

$$\succ_{s_4}: c_2, c_4, \emptyset, \qquad \succ_{c_4}: s_4, \ldots, \emptyset.$$

Capacity of c_1=2

Capacity of every other school=1

ill Behavior of TTC: Example

Improvement of c_1 for desirable s_1

$$\succ'_{s_1}: c_1, c_3, \emptyset \qquad \succ_{c_1}: s_1, s_2, s_3, s_4, \emptyset,$$

$$\succ_{s_2}: c_2, c_1, \emptyset, \qquad \succ_{c_2}: s_1, s_2, \ldots, \emptyset$$

$$\succ_{s_3}: c_3, c_1, \emptyset, \qquad \succ_{c_3}: s_4, s_3, s_2, s_1, \emptyset$$

$$\succ_{s_4}: c_2, c_4, \emptyset, \qquad \succ_{c_4}: s_4, \ldots, \emptyset.$$

Capacity of c_1=2

Capacity of every other school=1

Alternative Criteria of Promoting Competition

(1) Respecting improvements when schools care only about enrollment

School preferences are often just "priorities" set by law.

Alternative Criteria of Promoting Competition

(1) Respecting improvements when schools care only about enrollment

- Schools with too few enrollment often closed.
- Budgets often determined based on enrollment.

A mechanism

respects improvements in terms of enrollment

if any improvement for any school c weakly increases c's enrollment

※No logical relationship between original RI & RI in terms of enrollment

Theorem

	SOSM	Boston	TTC
RI in Terms of Enrollment	✓	✓	×

By the previous example.

ill Behavior of TTC: Example

Students

$\succ_{s_1}: c_3, c_1, \emptyset,$

$\succ_{s_2}: c_2, c_1, \emptyset,$

$\succ_{s_3}: c_3, c_1, \emptyset,$

$\succ_{s_4}: c_2, c_4, \emptyset,$

Schools

$\succ_{c_1}: s_1, s_2, s_3, s_4, \emptyset,$

$\succ_{c_2}: s_1, s_2, \ldots, \emptyset$

$\succ_{c_3}: s_4, s_3, s_2, s_1, \emptyset$

$\succ_{c_4}: s_4, \ldots, \emptyset.$

Capacity of c_1=2

Capacity of every other school=1

ill Behavior of TTC: Example

Improvement for c_1

Students	Schools
$\succ'_{s_1}: c_1, c_3, \emptyset,$	$\succ_{c_1}: s_1, s_2, s_3, s_4, \emptyset,$
$\succ_{s_2}: c_2, c_1, \emptyset,$	$\succ_{c_2}: s_1, s_2, \ldots, \emptyset$
$\succ_{s_3}: c_3, c_1, \emptyset,$	$\succ_{c_3}: s_4, s_3, s_2, s_1, \emptyset$
$\succ_{s_4}: c_2, c_4, \emptyset,$	$\succ_{c_4}: s_4, \ldots, \emptyset.$

Capacity of c_1=2

Capacity of every other school=1

Alternative Criteria of Promoting Competition

(1) Respecting improvements when schools care only about enrollment

(2) RI when schools try to improve to attract only "desirable" students

Alternative Criterion

A mechanism

respecting improvements for desirable students

if it respects improvements

in preferences of "desirable" students

=Weakly preferred to some student

to whom a school is originally matched

(before improvements occur)

※RI→RI for DS

Alternative Criterion

Theorem

	SOSM	Boston	TTC
RI for Desirable Students in General Environments	✓	×	×
RI for Desirable Students in Large Environments	✓	×	×

Same result

as in the case with original RI

	SOSM	Boston	TTC
RI in General Markets	×	×	×
RI by Desirable Students in General Markets	×	×	×
RI in Large Markets	✓	×	×
RI for Desirable Students in Large Markets	✓	×	×
RI in Terms of Enrollment	✓	✓	×