Marriage with Labour Supply

Jean-Marc Robin
(joint with Nicolas Jacquemet, Paris 1)

Sciences-Po & UCL

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INTRODUCTION

Aim

- Integrate Chiappori's (Ecta, 1988, JPE, 1992) collective model within a search-matching framework derived from Shimer-Smith (Ecta, 2000).
    - Indeed, collective models make the sharing rule depend on matching and market factors (wage ratio, aggregate sex ratio, divorce rules, etc) without providing a formal model.
- Long term aim: evaluate the effects of policies targeted at the household level (WFTC, EITC, ...)
- On the methodological side, this papers adds to Shimer-Smith by studying the identification and the estimation of two-sided search-matching models from cross-sectional data on wages and hours worked.

Why (random) search?

... instead of perfect-information assignment as recent work by Siow and Chiappori (and coauthors).

- More realistic? It takes time to find the right partner.
- Naturally yields mismatch.
- Easier to deal with continuous characteristics.
- Forward looking agents and risk are naturally incorporated.
- ...

Literature

- Bargaining models.
  - Manser, Brown (IER, 80), McElroy, Horney (IER, 81), Becker (1981), Lundberg, Pollak (JPE, 93, JEPersp, 96).
- On non-unitary models of the household see survey by Chiappori, Donni (2009).
- Non-equilibrium search models of match formation.
  - Ermisch (2003), Gould, Paserman (JUrbE, 2003).
- Rich applied, essentially macro literature of search-matching models aiming at explaining time trends (such as declining marriage rate, increasing female college graduation rate) and the role of policy.
  - Aiyagari, Greenwood, Guner (JPED, 2000), Greenwood, Guner, Knowles (AER, 2000) Caucutt, Guner, John Knowles (RED, 2002), Brien, Lillard, Stern (IER, 2006), Chiappori, Weiss (JEEA, 2006, JoLE, 2007), Chiappori and Oreffice (JPED, 2008), etc.

- Perfect-information match formation and intra-family ressources allocation.
    - Choo, Seitz and Siow (2008) and Chiappori, Salanie, Weiss (2010).
- Theory of search and matching in marriage markets.
    - Sattinger (IER, 1995), Lu, McAfee (inbook, 1996), Burdett, Coles (QJE, 1997), **Shimer and Smith (Ecta, 2000).**

THE MODEL

Populations

- Only source of heterogeneity is labour productivity or wages.
- $\ell_m(x), \ell_f(y)$ denote the number of males, females with labour productivity (wage) x, y.
  - $L_m = \int \ell_m(x)\,\mathrm{d}x$ and $L_f = \int \ell_f(y)\,\mathrm{d}y$
- $u_m(x)$ and $u_f(y)$ are the measures of singles of types x and y.
  - $U_m = \int u_m(x)\,\mathrm{d}x$ and $U_f = \int u_f(y)\,\mathrm{d}y$
- $n(x, y)$ is the number of married couples with characteristics (x, y)
  - $N = \iint n(x, y)\,\mathrm{d}x\,\mathrm{d}y$

Meetings, matching, separations

- δ is the (exogenous) divorce rate.
- Only singles search.
- $M(U_m, U_f)$ is the *meeting function* (number of meetings per unit of time)
- $\lambda_m = \frac{M(U_m, U_f)}{U_m}$ and $\lambda_f = \frac{M(U_m, U_f)}{U_f}$ are the meeting rates.
- Not all meetings induce marriage.
- $\alpha(x, y) \in [0, 1]$ is the (endogenous) probability of marriage (*matching* probability).

Steady-state restrictions

- In steady-state equilibrium, the number of divorces per unit of time is equal to the number of marriages:

$$\delta n(x,y) = u_m(x)\,\lambda_m \frac{u_f(y)}{U_f}\alpha(x,y) = \lambda u_m(x)\,u_f(y)\,\alpha(x,y)$$

where $\lambda = \dfrac{M(U_m, U_f)}{U_m U_f}$.

- Marginalisation:

$$\delta \int n(x,y)\,\mathrm{d}y = \delta\left[\ell_m(x) - u_m(x)\right] = \lambda u_m(x) \int u_f(y)\alpha(x,y)\,\mathrm{d}y$$

- Same for $u_f(y)$

Linear preferences

- Individuals draw utility from consumption (c) and leisure (ℓ).
- Indirect utility flow:

$$v_m(x, xT + t) = \frac{xT + t - A_m(x)}{B_m(x)}$$

where T is total time and t is non-labour income.

- Leisure follow by Roy's identity as

$$\ell_m(x, xT + t) = A'_m(x) + b'_m(x)[xT + t - A_m(x)]$$

where $b_m(x) = \log B_m(x)$ and b'_m denotes derivative.

Time Use for Married Individuals

- Marriage allows individuals to benefit from economies of scale and task specialisation.
- Home production is $H(p_m, p_f, x, y) + z$, a function of
  - time spent in home production by both spouses, p_m, p_f,
  - productivity x, y,
  - a source of noise, z, drawn at the first meeting from a zero-mean distribution denoted G. It aims at capturing all other dimensions of mutual attractiveness but labor market productivity.
- Optimal time use $p_m^1(x, y), p_f^1(x, y)$ solve

$$C(x, y) = \max_{p_m, p_f} \{H(p_m, p_f, x, y) - xp_m - yp_f\}.$$

Individual surpluses

- $W_m(x)$ be the value of singlehood (to be derived later).
- $W_m(v, x)$ is the value of a marriage yielding flow utility v to a male x.
- Option value equation:

$$rW_m(v, x) = v + \delta [W_m(x) - W_m(v, x)]$$

  where r is discount rate.
- Individual surplus:

$$S_m(v, x) = W_m(v, x) - W_m(x) = \frac{v - rW_m(x)}{r + \delta}$$

- Similar definitions for females

Bargaining

- Spouses split home production,

$$t_m + t_f = C(x, y) + z,$$

by Nash bargaining.

- Transfers t_m and t_f solve

$$\max_{t_m, t_f} S_m \left(v_m(x, xT + t_m), x \right)^{\beta} S_f \left(v_f(y, yT + t_f), y \right)^{1-\beta}$$

subject to condition

$$t_m + t_f \leq C(x, y) + z.$$

Transfers

- Define $s_m(x)$ and $s_f(y)$ such as the continuation values:

$$rW_m = \frac{xT + s_m - A_m}{B_m} \quad \text{and} \quad rW_f = \frac{yT + s_f - A_f}{B_f}$$

- The solution for transfers is:

$$t_m(x,y,z) = s_m(x) + \beta[C(x,y) + z - s_m(x) - s_f(y)]$$
$$t_f(x,y,z) = s_f(y) + (1-\beta)[C(x,y) + z - s_m(x) - s_f(y)]$$

Matching

- Singles x and y decide to match if the overall surplus is positive, i.e.

$$s(x,y$$

The values for singles

- The value of being single, for males, solves the option value equation:

$$
\begin{aligned}
rW_m(x) &= v_m(x, xT + C_m(x)) \\
&\quad + \lambda \quad \max\{S_m(v_m(x, xT + t_m(x, y, z)), x), 0\} \,\mathrm{d}G(z)\, u_f(y)\,\mathrm{d}y
\end{aligned}
$$

  where $C_m(x) = \max_{p_m}\{H_m(p_m, x) - xp_m\}$ is home production for single men.

- Equivalently,

$$
\begin{aligned}
s_m(x) &= -(xT - A_m(x)) + B_m rW_m(x) \\
&= \frac{\lambda\beta}{r+\delta} \quad \max\{z + C(x, y) - s_m(x) - s_f(y), 0\} \,\mathrm{d}G(z)\, u_f(y)\,\mathrm{d}y
\end{aligned}
$$

- A symmetric expression can be derived for females.

Equilibrium

The equilibrium is a fixed point (u_m, u_f, s_m, s_f) of the following system of four functional equations:

$$u_m(x) = \frac{\ell_m(x)}{1 + \frac{\lambda}{\delta} \int u_f(y)\alpha(x,y)\,\mathrm{d}y} \quad \text{and} \quad u_f(y) = \frac{\ell_f(y)}{1 + \frac{\lambda}{\delta} \int u_m(x)\alpha(x,y)\,\mathrm{d}x}$$

$$s_m(x) = \frac{C_m(x) + \frac{\lambda\beta}{r+\delta} \int \max\{z + C(x,y) - s_f(y), s_m(x)\}\,\mathrm{d}G(z)\,u_f(y)\,\mathrm{d}y}{1 + \frac{\lambda\beta}{r+\delta} U_f}$$

$$s_f(y) = \frac{C_f(y) + \frac{\lambda(1-\beta)}{r+\delta} \iint \max\{z + C(x,y) - s_m(x), s_f(y)\}\,\mathrm{d}G(z)\,u_m(x)\,\mathrm{d}x}{1 + \frac{\lambda(1-\beta)}{r+\delta} U_m}$$

with

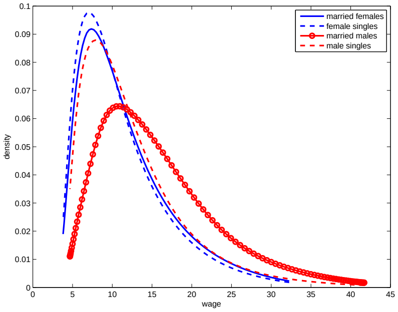
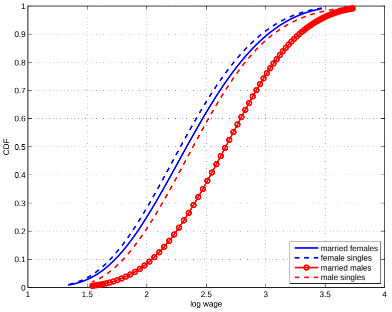
$$U_m = \int u_m(x)\,\mathrm{d}x, \quad U_f = \int u_f(y)\,\mathrm{d}y, \quad \lambda = \frac{M(U_m, U_f)}{U_m U_f}$$

$$\alpha(x,y) = 1 - G\left(s_m(x) + s_f(y) - C(x,y)\right)$$

DATA AND DESCRIPTIVE STATISTICS

Wage distributions

Marginals

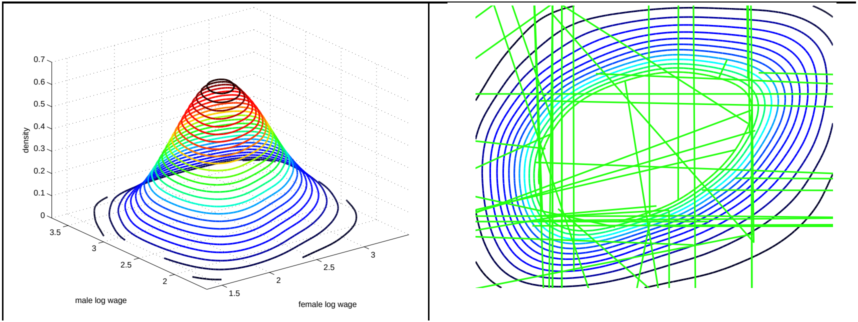
(a) Density	(b) CDF
	

Wage distributions

Joint distribution of (x, y) amongst married couples

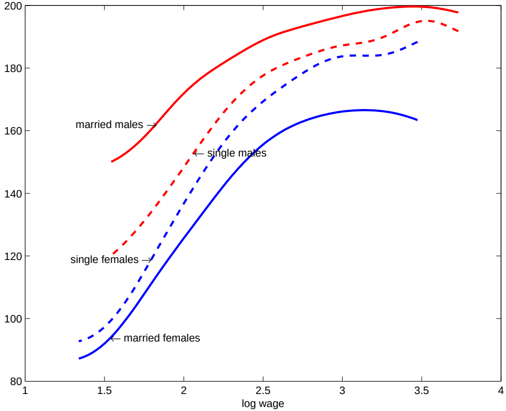
- 25% correlation!

(a) 3-D plot	(b) Projection on the (x, y) plane



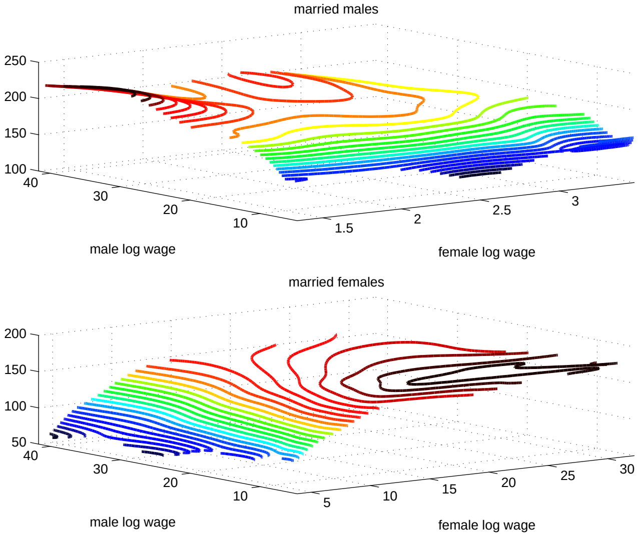
Hours

Nonparametric estimates of mean hours given own wages



Hours

Nonparametric estimates of mean hours given both spouses' wages



IDENTIFICATION

Matching probability

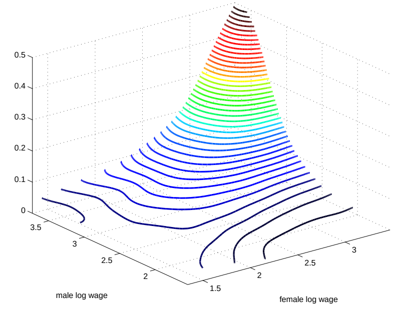
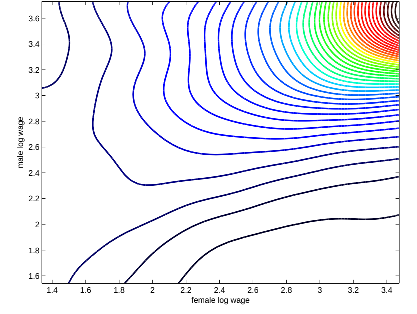
- The equilibrium flow condition implies that

$$\alpha(x,y) = \frac{\delta}{\lambda} \frac{n(x,y)}{u_m(x)u_f(y)}.$$

- The matching probability is identified by comparing the distribution of types among married couples to what it should be in absence of sorting.
- We display an estimate for the following calibration of δ and λ:
  - The divorce rate is set to 8% annual, which is consistent to a median marriage duration of about 8 years (Census, 2005).
  - The meeting rate is not identified in absence of data on datings. We arbitrarily calibrate it so that the meeting rate would be twice a year for single men ($\lambda_m = 1/6$).

Estimation

- Exponential growth.
- Steeper along x than y (harder to see but married men earn more!).

(a) 3-D plot	(b) Flat projection
	

Mean transfers

- Let σ denote the std of z and G_0 the distribution of z/σ.
- Then,

$$\frac{s(x,y)}{\sigma} = -G_0^{-1}(1-\alpha(x,y))$$

$$\frac{s_m(x)}{\beta\sigma} = \frac{\lambda}{r+\delta} \int \Big[\mathbb{E}_{G_0} \max\left\{\frac{s(x,y)}{\sigma} + \frac{z}{\sigma}, 0\right\}\Big] u_f(y)\,\mathrm{d}y \Big]$$

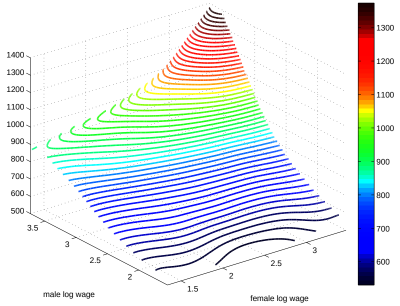
$$\frac{\overline{t}_m(x,y)}{\beta\sigma} = \frac{s_m(x)}{\beta\sigma} + \mathbb{E}_{G_0}\left[\frac{s(x,y)}{\sigma} + \frac{z}{\sigma} \,\middle|\, \frac{s(x,y)}{\sigma} + \frac{z}{\sigma} > 0\right]$$

- Hence, *mean transfers for married couples,* $\frac{\overline{t}_m(x,y)}{\beta\sigma}$ *and* $\frac{\overline{t}_f(x,y)}{(1-\beta)\sigma}$, *are identified from steady-state wage distributions given* λ *and* G_0.

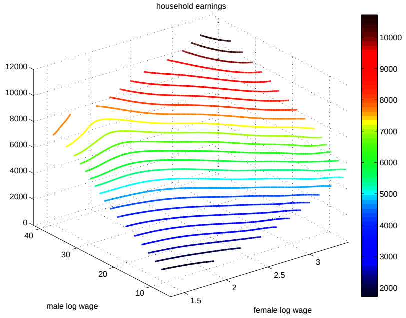
Household production ($\overline{t}_m + \overline{t}_f$)

- We set σ equal to 1000 (the order of magnitude of monthly earnings), the bargaining power β equal to 1/2, and G_0 is specified standard normal.

(a) Household production

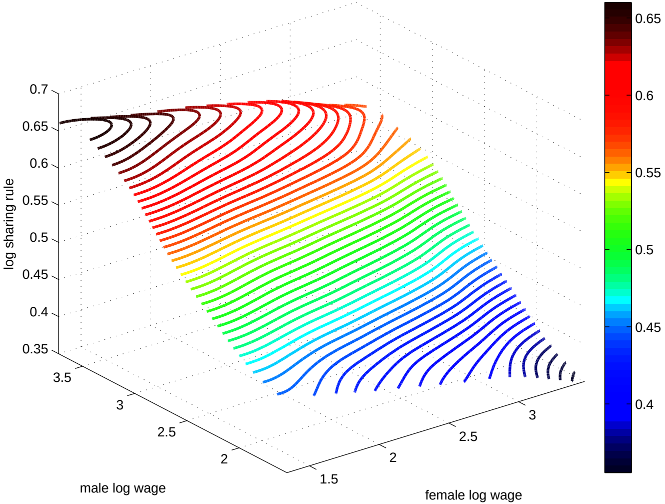


(b) Household earnings

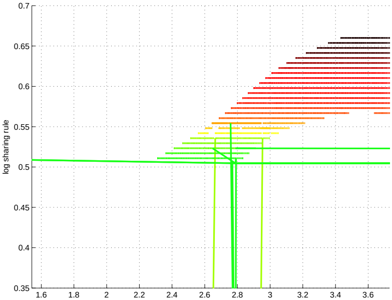


Sharing rule ($\frac{\bar{t}_m}{\bar{t}_m + \bar{t}_f}$)

- Approximately a plane in 3D!
- Steeper along x than along y.



(c) Flat x,z-projection



(d) Flat y,z-projection

Interpretation

- Married men are better paid than singles. They must therefore be more desirable.
- *Models says that male wage increases public good production.*
- Married women earn more only when married to a high-wage male.
- *Model says that female wage increases household production only when matched with a high-wage male.*
- *Model says that men can thus claim a bigger share of the surplus.*

Income effects

- Hours supplied:

$$h_m(x, xT + t) = T - A'_m(x) - b'_m(x)[xT + t - A_m(x)]$$

- Matching hours worked by married males with hours worked by single males on same wages,

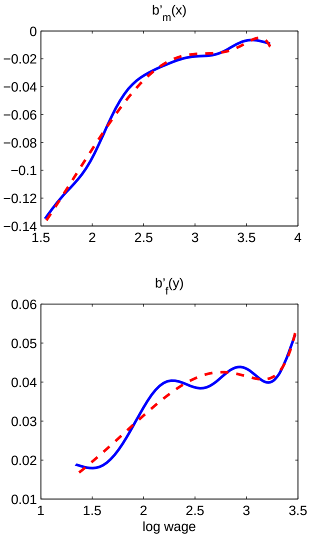
$$h^1_m(x, y, z) - h^0_m(x) = -b'_m(x)t_m(x, y, z),$$

and integrating over z and married couples given (x, y),

$$\Delta_m(x, y) \equiv \mathbb{E}(h^1_m|x, y) - \mathbb{E}(h^0_m|x) = -b'_m(x)\beta\sigma\frac{\overline{t}_m(x, y)}{\beta\sigma}.$$

- Regressing $\Delta_m(x, y)$ on $\frac{\overline{t}_m(x,y)}{\beta\sigma}$ for fixed x yields $b'_m(x)\beta\sigma$.
- With only one private good, it is not possible to to separate b'_m, b'_f from β and σ.

Nonparametric estimates (red = 4th order approx)

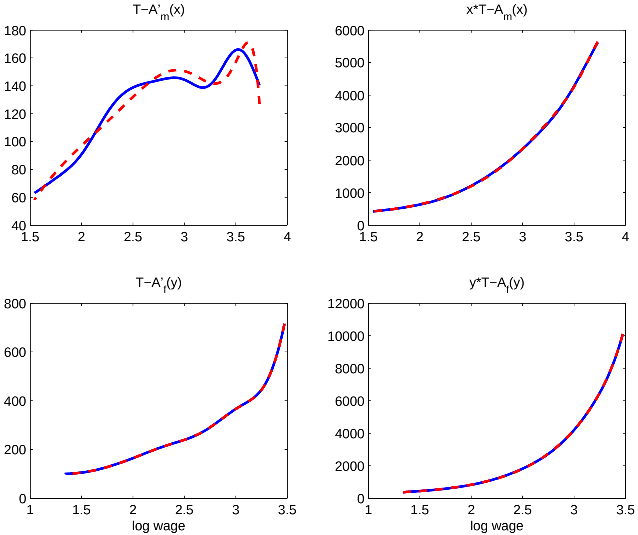


Price effects

- $A_m(x)$ and $A_f(y)$ follow from differential equations:

$$\frac{\mathrm{d}[xT - A_m(x)]}{\mathrm{d}x} - b'_m(x)[xT - A_m(x)] = h^0_m(x),$$
$$\frac{\mathrm{d}[yT - A_f(y)]}{\mathrm{d}y} - b'_f(y)[yT - A_f(y)] = h^0_f(y),$$

  using initial conditions $A_m(0) = A_f(0) = 0$ and $B_m(0) = B_f(0) = 1$.



T−A'm(x)
x*T−Am(x)
T−A'f(y)
y*T−Af(y)
180
160
140
120
100
80
60
40
1.5
2
2.5
3
3.5
4
6000
5000
4000
3000
2000
1000
0
800
600
400
200
1
12000
10000
8000
log wage

Identification of G_0

- Making use of

$$h_m^1(x,y) - h_m^0(x) = -b_m'(x) t_m(x,y,z),$$

we have that

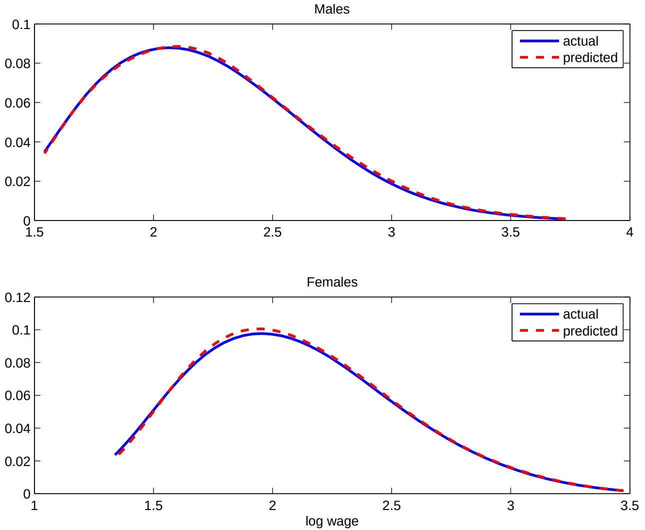
$$\begin{aligned} \frac{h_m^1(x,y) - h_m^0(x)}{-b_m'(x)\beta\sigma} &= \frac{s_m(x)}{\beta\sigma} + \frac{s(x,y)}{\sigma} + \frac{z}{\sigma} \\ &= \frac{\bar{t}_m(x,y)}{\beta\sigma} + \frac{z}{\sigma} - \mathbb{E}\left(\frac{z}{\sigma} \middle| x, y, \frac{z}{\sigma} > -\frac{s(x,y)}{\sigma} \right), \end{aligned}$$

- So one could design a nonparametric strategy to identify G_0 but it is likely to be imprecise. That is why we preferred to set $z \sim N(0, \sigma^2)$.

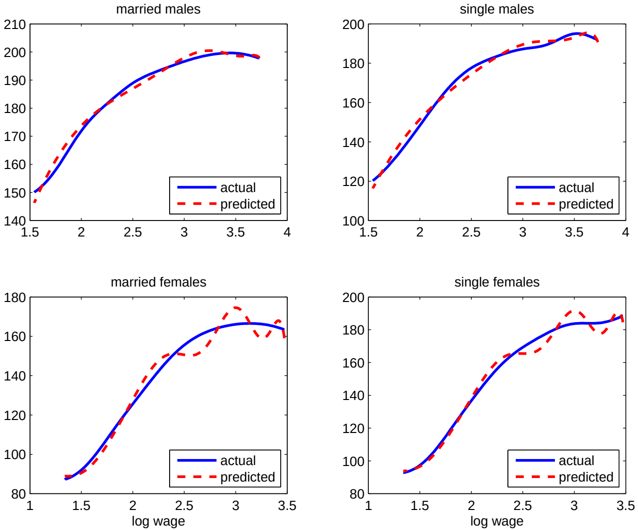
Goodness of fit of small-order polynomial approximation

- Take estimated structural estimates and solve for the equilibrium.

Wage densities among singles



Mean hours


married males
single males
married females
single females
actual
predicted
log wage
210
200
190
180
170
160
150
140
120
100
80
1
1.5
2
2.5
3
3.5
4

Conclusion

- We generalise the matching model of marriage by accounting for labour supply decisions.
- On the agenda:
  - Simulate social programs;
- Open avenues:
  - Incorporate recent extensions of the collective model: participation to the labor market, choice of children, etc
  - Heterogeneous divorce rates
  - Endogenous divorce via "on-the-marriage search"
  - Multidimensional matching