The Econometrics of Equilibrium Search

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June 2012

PARTIAL SEARCH MODELS

- Devine, Theresa J. & Kiefer, Nicolas M., 1991. "Empirical Labor Economics: The Search Approach," Oxford University Press
- Christensen, Bent Jesper & Kiefer, Nicholas M., 1991. "The Exact Likelihood Function for an Empirical Job Search Model," Econometric Theory, Cambridge University Press, vol. 7(04), pages 464-486, December.

Data

Labor force survey usually observe workers continuously within a time interval $[t_0, t_1]$ and gather retrospective information at t_0 so that, for unemployed workers at t_0, it is possible to record:

- the elapsed unemployment duration at t_0: τ_0,
- the residual unemployment duration after t_0: τ_1 (note that $\tau_1 \leq t_1 - t_0$),
- the accepted wage w at $t_0 + \tau_1$ if the worker leaves unemployment by the end of the recording period $t_1 - t_0$.

Optimal search strategy

- The optimal strategy when employed is to accept any wage offer strictly greater than the present wage contract.
- The reservation wage is the solution to the equation:

$$\phi = b + \frac{\lambda_0}{\rho} \int_{\phi}^{\overline{w}} \overline{F}(w)dw$$

  where

  - b is UI income,
  - λ_0 is the job offer arrival rate,
  - ρ is the discount rate,
  - F is the wage offer distribution.

Likelihood

- The actual unemployment duration has an exponential distribution with parameter $\lambda_0 \overline{F}(\phi)$ (instantaneous probability of receiving an offer times the probability that it be acceptable).
- The Poisson property implies that τ_0 and τ_1 are independent and exponentially distributed.
- The density of (τ_0, τ_1, w) is therefore equal to:

$$\ell\left(\tau_0, \tau_1, w\right) = \lambda_0 \overline{F}(\phi) \exp\left(-\lambda_0 \overline{F}(\phi) \tau_0\right) \cdot \left[\lambda_0 \overline{F}(\phi) \exp\left(-\lambda_0 \overline{F}(\phi) \tau_1\right) \cdot \frac{f(w)}{\overline{F}(\phi)}\right]^z \cdot \left[\exp\left(-\lambda_0 \overline{F}(\phi) \tau_1\right)\right]^{1-z}$$

where z indicates censoring: $z = 1$ if $\tau_1 < t_1 - t_0$, $z = 0$ if $\tau_1 = t_1 - t_0$.

  - Note that the distribution of wage offers is identified only conditional on $w > \phi$ (with density $\frac{f(w)}{\overline{F}(\phi)}$).

Maximum likelihood

- Parametrize F and maximize likelihood wrt λ_0, F and ϕ subject to the condition that all observed wages are greater than ϕ.
- The reservation wage ϕ is estimated by the minimal accepted job offer.
- Estimate b as

$$\widehat{b} = \widehat{\phi} - \frac{\widehat{\lambda}_0}{\rho} \int_{\widehat{\phi}}^{\overline{w}} \overline{\widehat{F}}(w)dw.$$

- With covariates, use constrained ML.

EQUILIBRIUM SEARCH MODELS

Two types of wage setting mechanisms:

- Wage posting,
- Bargaining (including sequential auctions).

WAGE POSTING MODELS

- Firm make take-it-or-leave it wage offers.
- Nash equilibrium

No search on the job

- Albrecht, J. W., and Axell, B. 1984 . ''An Equilibrium Model of Search Unemployment,'' Journal of Political Economy 92, 824-840.
  - Show that a non degenerate equilibrium wage distribution results from dispersion in unemployed workers' reservation wages (exogenous heterogeneity in UI income, b)
- Estimation: Eckstein, Zvi & Wolpin, Kenneth I, 1990. "Estimating a Market Equilibrium Search Model from Panel Data on Individuals," Econometrica, Econometric Society, vol. 58(4), pages 783-808, July.

On-the-job search

- When workers can search off-the-job and on-the-job, and firms post wages, it can be shown that the equilibrium wage offer distribution has no mass point.
- *Discrete productivity distribution*: Mortensen, D. T. 1990 . ''Equilibrium Wage Distributions: A Synthesis,'' in Panel Data and Labor Market Studies, J. Hartog, G. Ridder, and J. Theeuwes, Eds. , pp. 279-296. New York: North-Holland.
- *Continuous productivity distribution*: Burdett, Kenneth & Mortensen, Dale T, 1998. "Wage Differentials, Employer Size, and Unemployment," International Economic Review, vol. 39(2), pages 257-73, May.

Theory

- Steady state: $\left[\delta+\lambda_1\overline{F}(w)\right](1-u)\,G(w)=\lambda_0 uF(w)$, where
  - δ is match destruction rate (exogenous),
  - λ_1 is on-the-job offer arrival rate,
  - G is cross-section wage distribution.
- The steady-state flow equation establishes a bijection between F and G.
- Profit maximization: $w^*(p)=\arg\max_{w\geq\phi}(p-w)\,\ell(w)$, subject to

$$\ell(w)=\frac{L}{N}\frac{dG(w)}{dF(w)}=\frac{L}{N}\frac{\delta(\delta+\lambda_1)}{\left[\delta+\lambda_1\overline{F}(w)\right]^2},$$

and

$$\phi=b+(\lambda_1-\lambda_0)\int_\phi^{\overline{w}}\frac{\overline{F}(w)\,dw}{\rho+\delta+\lambda_1\overline{F}(w)}.$$

- Market equilibrium: $F(w^*(p))=\Gamma(p)$ (the exogenous distribution of productivity).

Homogeneous agents

- H. Bunzel & B. J. Christensen & P. Jensen & N. M. Kiefer & L. Korsholm & L. Muus & G. R. Neumann & M. Rosholm, 2001. "Specification and Estimation of Equilibrium Search Models," Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 4(1), pages 90-126, January.
- Mixed stategy:

$$(p-w)\,\ell(w) = (p-\phi)\,\ell(\phi) \Leftrightarrow \delta + \lambda_1 \overline{F}\,(w) = (\delta + \lambda_1)\sqrt{\frac{p-w}{p-\phi}}$$

- Same ML procedure as partial equilibrium search with one additional constraint.

Homogeneous agents, segmented markets

- Gerard J. van den Berg & Geert Ridder, 1998. "An Empirical Equilibrium Search Model of the Labor Market," Econometrica, vol. 66(5), pages 1183-1222, September.
- Mix BM homogeneous model by allowing p to vary across separate islands.

Discrete productivity distribution

- Bowlus, Audra J & Kiefer, Nicholas M & Neumann, George R, 1995. "Estimation of Equilibrium Wage Distributions with Heterogeneity," Journal of Applied Econometrics, John Wiley & Sons, Ltd., vol. 10(S), pages S119-31, Suppl. De.
- Bowlus, Audra J & Kiefer, Nicholas M & Neumann, George R, 2001. "Equilibrium Search Models and the Transition from School to Work," International Economic Review, vol. 42(2), pages 317-43, May.
- Estimate Mortensen's (1990) model by ML.
  - Likelihood not smooth due to discreteness.
  - Global optimization algorithm required, spec. simulated annealing.

Continuous productivity disribution 1

- Bontemps, Christian & Robin, Jean-Marc & van den Berg, Gerard J, 2000. "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation," International Economic Review, vol. 41(2), pages 305-58, May.
- Bontemps, Christian & Robin, Jean-Marc & Van den Berg, Gerard J, 1999. "An Empirical Equilibrium Job Search Model with Search on the Job and Heterogeneous Workers and Firms," International Economic Review, vol. 40(4), pages 1039-74, November.
- Labor force survey data on unemployment duration and accepted wages, and employment wages and mobility.

Continuous productivity disribution 2

Two-stage ML estimation

1. First, estimate G and g using a non parametric estimator (a kernel estimator for example).
2. Second, replace $\overline{F}$ and f in likelihood using steady-state flow equation, and maximize the likelihood.
3. Use the FOC of the profit maximization programme to estimate a productivity value for each observed wage:

$$-1+\frac{2\kappa\left(p-w\right)}{1+\kappa\overline{F}\left(w\right)}f\left(w\right)=0\Leftrightarrow p-w=\frac{1+\kappa\overline{F}\left(w\right)}{2\kappa f\left(w\right)}$$

Continuous productivity disribution 2

Extensions

- Endogenous search intensity: Bent Jesper Christensen & Rasmus Lentz & Dale T. Mortensen & George R. Neumann & Axel Werwatz, 2005. "On-the-Job Search and the Wage Distribution," Journal of Labor Economics, University of Chicago Press, vol. 23(1), pages 31-58, January.
- Productivity shocks and trends: Gadi Barlevy, 2008. "Identification of Search Models using Record Statistics," Review of Economic Studies, Wiley Blackwell, vol. 75(1), pages 29-64, 01.
  - Feasible assuming piece-rate contracts (wage proportional to productivity; equilibrium distribution of piece rates).

BARGAINING AND SEARCH-MATCHING

Precursors

- Eckstein, Zvi & Wolpin, Kenneth I, 1995. "Duration to First Job and the Return to Schooling: Estimates from a Search-Matching Model," Review of Economic Studies, Wiley Blackwell, vol. 62(2), pages 263-86, April.
- Except for this seminal paper, there has been few estimations of dynamic bargaining models on micro data.
- (Very little search-matching in that paper.)

Sequential auctions 1

- Fabien Postel-Vinay & Jean-Marc Robin, 2002. "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," Econometrica, vol. 70(6), pages 2295-2350, November.
- Special bargaining mechanism where firms pick the lowest wage in the bargaining set set:
  - Unemployed workers are paid their reservation wage.
  - Poached employees receive the outcome of Bertrand competition between the incumbent employer and the poacher.

Sequential auctions 2

- Match productivity: xy, where x denotes worker ability and y is firm labor productivity.
- UI earnings also proportional to ability: bx.
- Zero value of a vacancy.
- Let $U(x)$ be the unemployment value and $W(w, x, y)$ the value of wage contract w.
- Wage contract for unemployed workers solves $W(w, x, y) = U(x)$.
- Wage contract for an worker being auctioned by two firms $y < y'$:

  $$W(w, x, y') = Q(x, y).$$

  This is the wage that delivers in firm y' the value of match (x, y), i.e. $Q(x, y) = W(xy, x, y)$.
- Solution (linear utility): $w = x \cdot \left(y - \frac{\lambda_1}{\rho+\delta} \int_y^{y'} \overline{F}(z)\, dz \right).$

Sequential auctions 3

Steady-state earnings distribution

In steady state (assuming log utility):

$$\ln w \overset{d}{=} \ln x + \ln \phi(1, z, y) = \ln x + \ln z - \frac{\lambda_1}{\rho + \delta} \cdot \int_z^y \overline{F}(t) \frac{dt}{t}$$

with

1. $x \sim H$ (exogenous)
2. $y \sim \ell(y) = \frac{L}{N} \frac{\delta(\delta+\lambda_1)}{\left[\delta+\lambda_1 \overline{F}(y)\right]^2} f(y)$, independently of ε: **no sorting**.
3. $z|y \sim$ dist. with cdf $\left(\frac{\delta + \lambda_1 \overline{F}(y)}{\delta + \lambda_1 \overline{F}(z)}\right)^2$ on $\{b\} \cup [\underline{y}, y]$

Sequential auctions 2

Data and estimation

- French matched employer-employee data.
- y is measured by mean log-wage per firm:

$$\begin{aligned} \ln \widehat{y} &\equiv E(\ln w|y) \\ &= \ln y - \frac{\delta}{\rho+\delta}\left[\delta+\lambda_1\overline{F}(y)\right]^2 \cdot \int_b^y \frac{\rho+\delta+(1-\sigma)\lambda_1\overline{F}(t)}{\left[\delta+\lambda_1\overline{F}(t)\right]^2}\frac{dt}{t} \end{aligned}$$

  Can be inverted (y given $\widehat{y}$) analytically!
- Then nonparametric deconvolution for x.
- Findings:
  - Individual ability differences explain about 50% of the log wage variance for managers and engineers, 20% for workers with lower executive functions, about 15% for technicians and technical supervisors and virtually nothing for the other unskilled categories.
  - Once the person effect has been removed, firm effects and search frictions explain approximately identical parts of the residual variance.

Mixing sequential auctions and Nash bargaining

- Matthew S. Dey & Christopher J. Flinn, 2005. "An Equilibrium Model of Health Insurance Provision and Wage Determination," Econometrica, vol. 73(2), pages 571-627, 03.
- Pierre Cahuc & Fabien Postel-Vinay & Jean-Marc Robin, 2006. "Wage Bargaining with On-the-Job Search: Theory and Evidence," Econometrica, vol. 74(2), pages 323-364, 03.
- Additional Nash bargaining to share the surplus:

$$W(w, x, y') = Q(x, y) + \beta \left[Q(x, y') - Q(x, y) \right]$$

Sequential auctions and productivity dynamics

- Jesper Bager & Francois Fontaine & Fabien Postel-Vinay & Jean-Marc Robin, 2006. "A Feasible Equilibrium Search Model of Individual Wage Dynamics with Experience Accumulation"
- Uses Barlevy's restriction to the set of contracts: piece-rate contracts.
- Simulated GMM.

SORTING

- No sorting follows from knife-edge hypotheses:
  - match value $= xy$
  - UI $= bx$
  - Strong free entry hypothesis: $V(y) = 0$ (value of vacancy)
- More general models should generate sorting.
- Rafael Lopez de Melo, "Sorting in the Labor Market: Theory and Measurement" (builds on Moscarini, Giuseppe, 2001. "Excess Worker Reallocation," Review of Economic Studies, Wiley Blackwell, vol. 68(3), pages 593-612, July)
- Jeremy Lise, Costas Meghir and Jean-Marc Robin, "Matching, Sorting and Wages"
- N. Jacquemet and JM Robin, "Marriage and Labor Supply"

Matching and Search

Robert Shimer

June 5, 2012

BFI/Stevanovich Center conference on Matching Problems

Why Search?

- explicit model of decentralized matching
- realistic in many situations
  - marriage market
  - labor market
  - product market
  - over-the-counter securities markets
- potential resolution to nonexistence issues?

Simple(st?) TU Framework

- based on Shimer-Smith (2000)
- homosexual marriage model
- dynamic, continuous time
- fixed population $(x, i) \in [0, 1]^2$ (type x, "name" i)
- risk-neutral, infinitely-lived, discount rate $r > 0$
- match between (x, i) and (y, j) produces flow output $f(x, y) \equiv f(y, x)$
- normalize utility of unmatched agent to 0
- random search when unmatched, choose whether to match
- matches break up at fixed rate to maintain aggregate steady state

Steady State

- endogenous outcomes:
  - $u(x)$: share of type x who are unmatched
  - $m(x,y) \in [0,1]$: probability x and y match given an opportunity
- exogenous parameters
  - δ: arrival rate of match destruction shock
  - ρ: arrival rate of meeting with someone
    $\rho \int_0^1 u(y)dy$: arrival rate of meeting with unmatched agent
- steady state: $\delta(1-u(x)) = \rho u(x) \int_0^1 m(x,y)u(y)dy$
- meet unmatched agents at rate ρ: $\delta(1-u(x)) = \rho u(x) \frac{\int_0^1 m(x,y)u(y)dy}{\int_0^1 u(y)dy}$

Value Functions

- suppose x gets flow income $w(x|y)$ in a match with y
  - $w(x|y) + w(y|x) = f(x,y)$
- $V(x)$: expected lifetime income of x if unmatched
- $W(x|y)$: expected lifetime income of x if matched with y
- Bellman equations:

$$rV(x) = \rho \int_0^1 m(x,y)(W(x|y) - V(x))u(y)dy$$
$$rW(x|y) = w(x|y) + \delta(V(x) - W(x|y))$$

- equilibrium matching:

$$m(x,y) = \begin{cases} 1 \\ 0 \end{cases} \quad \text{if} \begin{cases} W(x|y) > V(x) \text{ and } W(y|x) > V(y) \\ W(x|y) < V(x) \text{ or } W(y|x) < V(y) \end{cases}$$

Nash Bargaining

- suppose w satisfies $W(x|y) - V(x) = W(y|x) - V(y)$ for all (x, y)
  - bilateral efficiency: $W(x|y) \gtreqless V(x) \Leftrightarrow W(y|x) \gtreqless V(y)$
- it follows that

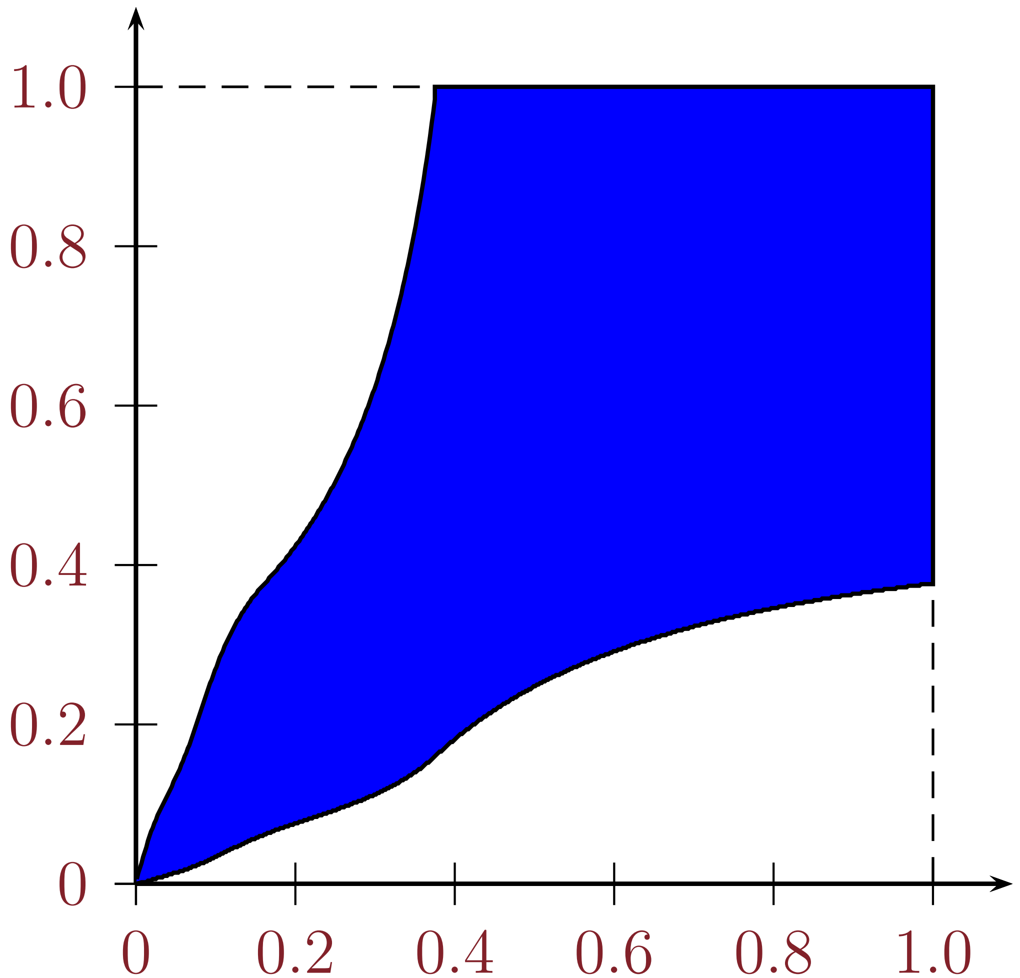
$$rV(x) = \frac{\rho}{2(r+\delta)} \int_0^1 \max\{f(x,y) - rV(x) - rV(y), 0\} u(y) dy$$

and

$$m(x,y) = \begin{cases} 1 & \text{if } f(x,y) \gtrless rV(x) + rV(y) \\ 0 & \end{cases}$$

- note fixed point problem: $m \mapsto u \mapsto V \mapsto m$
  - existence? Manea: "Bargaining in Dynamic Markets..."
  - uniqueness? easy to construct counterexamples

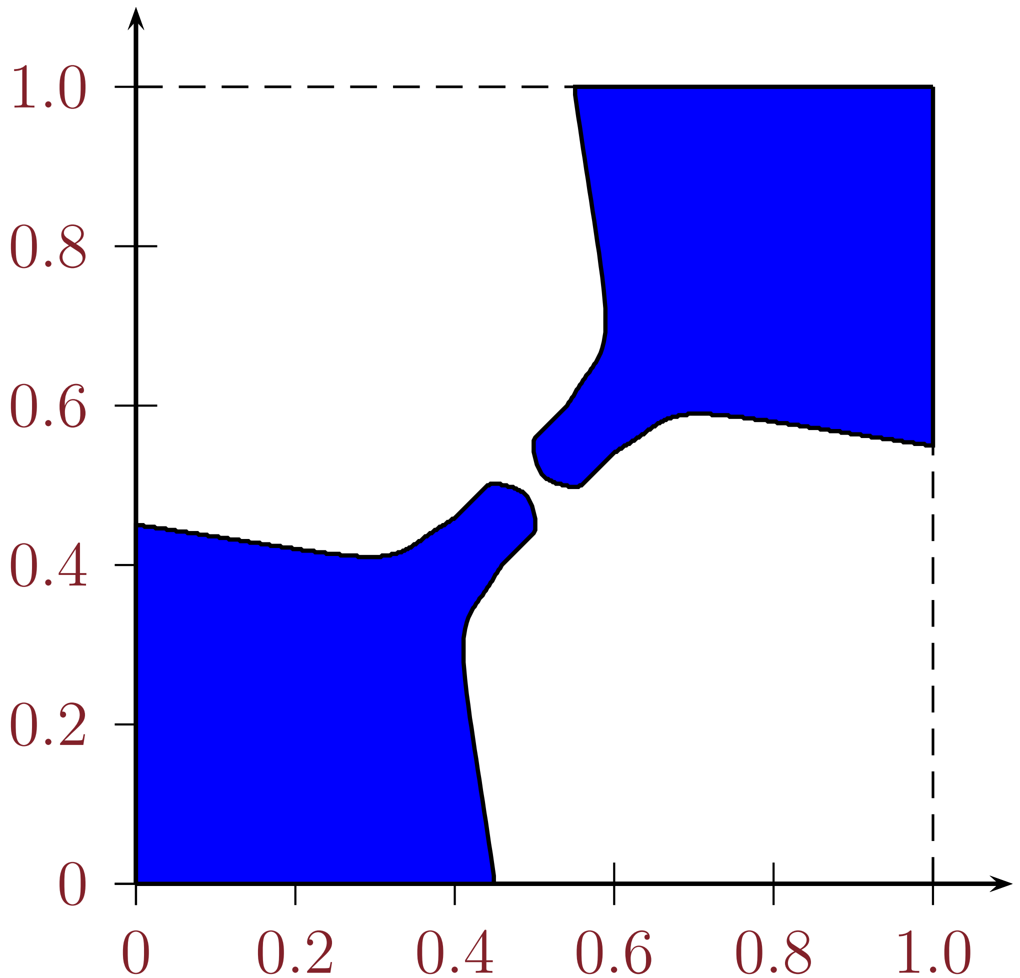
Positively Assortative Matching



$f(x, y) = xy$

$\rho = 100r$, $\delta = r$

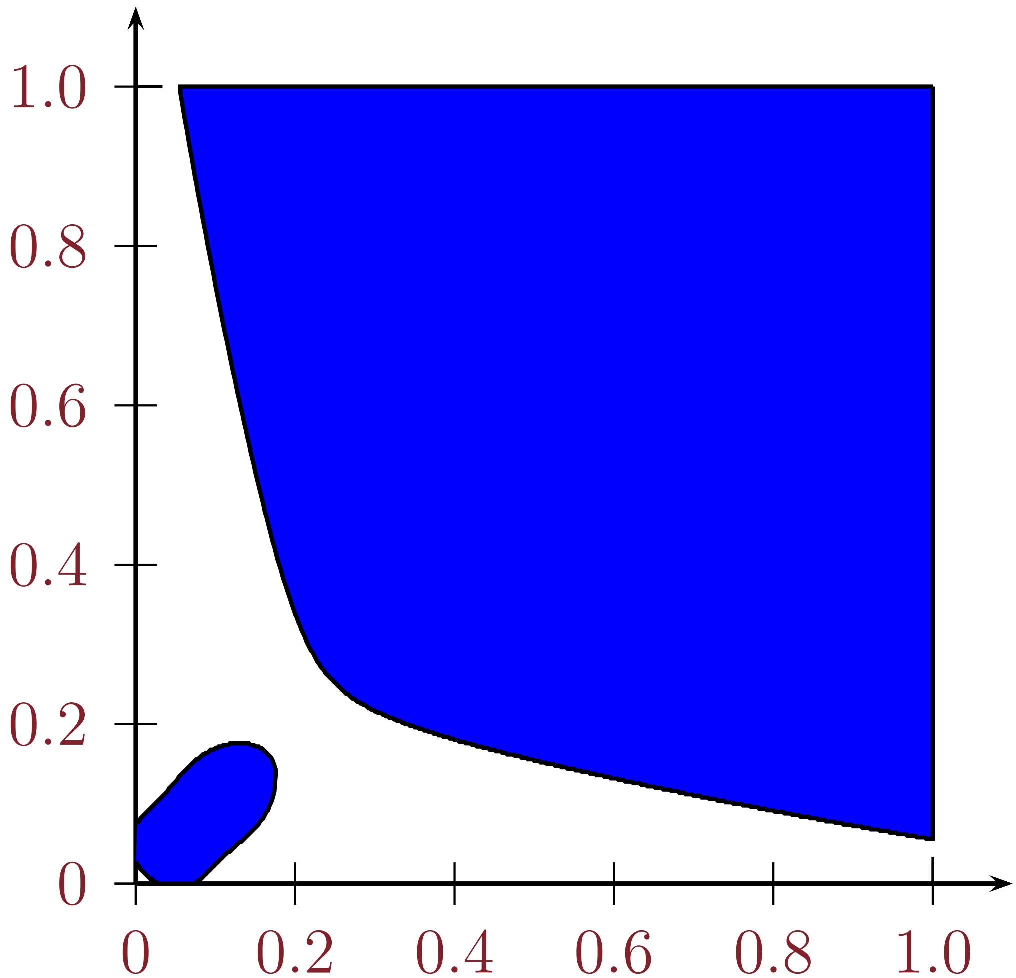
Supermodularity but not PAM



$f(x, y) = (x + y - 1)^2$

$\rho = 100r$, $\delta = r$

Supermodularity but not PAM



$$f(x, y) = (x + y)^2$$

$$\rho = 35r,\ \delta = r$$

Alternative Assumptions

- idiosyncratic component to match
  - output $f(x, y) + g(i, j)$,
  - g is i.i.d. across (i, j), continuous CDF with full support
  - now $m(x, y) \in (0, 1)$ for all (x, y)
  - realistic and may simplify existence proofs
- relax steady state assumption
  - equilibrium with arbitrary initial conditions
  - self-fulfilling cycles
- output-maximization
  - equilibrium is bilaterally efficient
  - but it does not solve any obvious planning problem

Alternative Assumptions

- cost of search rather than discounting
  - supermodularity $\Rightarrow$ PAM (Atakan 2006)
- matches last forever
  - exogenous inflow of new searchers
  - exogenous “clones” of new matches (eliminates multiplicity)
  - nonstationary dynamics
- non-random search
  - easier to find more desirable partners
  - no general agreement on how to parameterize this

Alternative Assumptions

- search by matched agents
  - endogenous instability is new source of multiplicity
  - breach penalties ensure efficient breakups, eliminate instability
- two populations (men and women): not much changes
- multilateral matching (e.g. supply chains):
  - not much research on this (exception: Moen and Yashiv)
  - obviously need to allow for search by matched agents
  - endogenous instability will again naturally arise

Nontransferable Utility

- utility that x gets when matched with y is $h(y)$, increasing
- Bellman equations:

$$rV(x) = \rho \int_0^1 m(x,y)(W(x|y) - V(x))u(y)dy$$
$$rW(x|y) = h(y) + \delta(V(x) - W(x|y))$$

- matching in “classes”: Burdett-Coles (1997) and others
  - classes $[x_1, 1]$, $[x_2, x_1]$, $[x_3, x_2]$, . . .
  - any y acceptable to the set $[0, x_j]$ accepts the set $[x_{j+1}, 1]$

  $\Rightarrow$ if $x \in [x_{j+1}, x_j]$, $m(x,y) = 1$ if and only if $y \in [x_{j+1}, x_j]$
- all the same issues arise in this framework
- can also consider all the same alternative assumptions