A Contribution to Health Capital Theory

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RAND **LABOR AND POPULATION**

A contribution to health capital theory[*]

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Abstract

I present a theory of the demand for health, health investment and longevity, building on the human capital framework for health and addressing limitations of existing models. I predict a negative correlation between health investment and health, that the health of wealthy and educated individuals declines more slowly and that they live longer, that current health status is a function of the initial level of health and the histories of prior health investments made, that health investment rapidly increases near the end of life and that length of life is finite as a result of limited life-time resources (the budget constraint). I derive a structural relation between health and health investment (e.g., medical care) that is suitable for empirical testing.

Keywords: socioeconomic status, education, health, demand for health, health capital, medical care, life cycle, age, labor, mortality

JEL Codes : D91, I10, I12, J00, J24

1   Introduction

The demand for health is one of the most central topics in Health Economics. The canonical model of the demand for health and health investment (e.g., medical care) arises from Grossman (1972a, 1972b, 2000) and theoretical extensions and competing economic models are still relatively few. In Grossman's human capital framework individuals demand medical care (e.g., invest time and consume medical goods and services) for the consumption benefits (health provides utility) as well as production benefits (healthy individuals have greater earnings) that good health provides. The model provides a conceptual framework for interpretation of the demand for health and medical care in relation to an individual's resource constraints, preferences and consumption needs over the life cycle. Arguably the model has been one of the most important contributions of Economics to the study of health behavior. It has provided insight into a variety of phenomena related to health, medical care, inequality in health, the relationship between health and socioeconomic status, occupational choice, etc (e.g., Cropper, 1977; Muurinen and Le Grand, 1985; Case and Deaton, 2005) and has become the standard (textbook) framework for the economics of the derived demand for medical care.

Yet several authors have identified limitations to the literature spawned by Grossman's seminal 1972 papers[1] (see Grossman, 2000, for a review and rebuttal of some of these limitations). A standard framework for the demand for health, health investment (e.g., medical care) and longevity has to meet the significant challenge of providing insight into a variety of complex phenomena. Ideally it would explain the significant differences observed in the health of socioeconomic status (SES) groups - often called the "SES-health gradient". In the United States, a 60-year-old top-income-quartile male reports to be in similar health as a 20-year-old bottom-income-quartile male (Case and Deaton 2005) and similar patterns hold for other measures of SES, such as education and wealth, and other indicators of health, such as disability and mortality (e.g., Cutler et al. 2010; van Doorslaer et al. 2008). Initially diverging, the disparity in health between low- and high-SES groups appears to narrow after ages 50-60. Yet, Case and Deaton (2005) have argued that health production models are unable to explain differences in the health deterioration rate (not just the level) between socioeconomic groups.

Another stylized fact of the demand for medical care is that healthy individuals do not go to the doctor much: a strong negative correlation is observed between measures of health and measures of health investment. However, Wagstaff (1986a) and Zweifel and Breyer (1997) have pointed to the inability of health production models to predict the observed negative relation between health and the demand for medical care.

Introspection and casual observation further suggests that healthy individuals are those that began life healthy and that have invested in health over the life course. Thus one would expect that health depends on initial conditions (e.g., initial health) and the history of health investments, prices, wages, medical technology and environmental conditions. Yet, Usher (1975) has pointed to the lack of "memory" in model solutions. For example the solution for health typically does not

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[1]Throughout this paper I refer to this literature as the health production literature.

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depend on its initial value or the histories of health investment and biological aging.

Further, Case and Deaton (2005) note that *"...If the rate of biological deterioration is constant, which is perhaps implausible but hardly impossible, ...people will "choose" an infinite life ..."*. This suggests that complete health repair is possible, regardless of the speed of the process (the rate itself does not matter in causing health to decline) and regardless of the budget constraint, and as a result declines in health status are driven, not by the rate of deterioration of the health stock, but by the rate of increase of the rate of deterioration (Case and Deaton, 2005). Thus a necessary condition in health production models is that the biological aging rate increases with age to ensure that life is finite and health declines and to reproduce the observed rapid increase in medical care near the end of life. Case and Deaton (2005) argue, however, that a technology that can effect such complete health repair is implausible.

Last, Ehrlich and Chuma (1990) have pointed out that under the constant returns to scale (CRTS) health production process assumed in the health production literature, the marginal cost of investment is constant, and no interior equilibrium for health investment exists. Ehrlich and Chuma argue that this is a serious limitation that introduces a type of indeterminacy ("bang-bang") problem with respect to optimal investment and health maintenance choices. The importance of this observation appears to have gone relatively unnoticed: contributions to the literature that followed the publication of Ehrlich and Chuma's work in 1990 have continued to assume a health production function with CRTS in health investment.[2] This may have been as a consequence of the following factors: First, Ehrlich and Chuma's finding that health investment is undetermined (under the usual assumption of a CRTS health production process) was incidental to their main contribution of introducing the demand for longevity (or "quantity of life") and the authors did not explore the full implications of a DRTS health production process. Second, Ehrlich and Chuma's argument is brief and technical.[3] This has led Reid (1998) to argue that *"...the authors [Ehrlich and Chuma] fail to substantiate either claim [bang-bang and indeterminacy] ..."*, suggesting there is room for further research into the argument made by Ehrlich and Chuma. Third, there was the incorrect notion that Ehrlich and Chuma had changed the structure of the model substantially and that the alleged indeterminacy of health investment did not apply to the original formulation in discrete time (e.g., Reid, 1998). Last, because of the increased complexity of a health production model that includes endogenous length of life (demand for longevity) Ehrlich and Chuma (1990) had to resort to a particular sensitivity analysis, suitable to optimal control problems (Oniki, 1973),

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[2]E.g., Bolin et al. (2001, 2003); Case and Deaton (2005); Erbsland, Ried and Ulrich (2002); Jacobsen (2000); Leu and Gerfin (1992); Liljas (1998); Nocera and Zweifel (1998); Wagstaff (1986a); Ried (1996, 1998). To the best of my knowledge the only exception is an unpublished working paper by Dustmann and Windmeijer (2000) who take the model by Ehrlich and Chuma (1990) as their point of departure. Bolin et al. (2002a, 2002b) assume that the health investment function is a decreasing function of health. Thus they impose a relationship between health and health investment to ensure that the level of investment in health decreases with the health stock rather than deriving this result from first principles.

[3]It involves a reference to a graph with health investment on one axis and the ratio of two Lagrange multipliers on the other. The authors note that the same results hold in a discrete time setting, using a proof based on the last period preceeding death (see their footnote 4).

in which the directional effect of a parameter change can be investigated. Ehrlich and Chuma's (1990) insightful work is therefore limited to generating directional predictions. This suggested that obtaining insight into the characteristics of a DRTS health production model would require numerical analysis or the kind of sensitivity analysis performed by Ehrlich and Chuma (1990) – while it would not substantially change the nature of the theory. For example, it was thought that introducing DRTS would result in individuals reaching the desired health stock gradually rather than instantaneously (e.g., Grossman, 2000, p. 364) – perhaps not a sufficiently important improvement to warrant the increased level of complexity.

What then is needed to address the above mentioned limitations? I argue that the answer is two-fold: 1) a reinterpretation is needed of the health stock equilibrium condition, one of the most central relations in the health production literature, as determining the optimal level of health investment and not the "optimal" level of the health stock, and 2) one needs to assume DRTS in the health production process as Ehrlich and Chuma (1990) have argued.

In this paper I present a theory of the demand for health, health investment and longevity based on Grossman (1972a, 1972b) and the extended version of this model by Ehrlich and Chuma (1990). In particular, this paper explores in detail the implications of a DRTS health production process. The theory I develop is capable of reproducing the phenomena discussed above and of addressing the above mentioned five limitations.

This paper contributes to this literature as follows. First, I reduce the complexity of a theory with a DRTS health production process (as in Ehrlich and Chuma, 1990) by arguing for a different interpretation of the health stock equilibrium condition, one of the most central relations in the health production literature: this relation determines the optimal level of health investment (not the health stock), conditional on the level of the health stock. The health production literature has thus far not employed the alternative interpretation of the health equilibrium condition and consistently utilizing it allows me to develop the health production literature further than was previously possible. This is because the equilibrium condition for the health stock is of a much simpler form than the condition which is typically utilized to determine the optimal level of health investment. Many of the subsequent contributions this paper makes follow from the alternative interpretation advocated here.

Second, I show that the alternative interpretation allows for an intuitive understanding as to why the assumption of DRTS in the health production function is necessary, or no solution to the optimization problem exists. Essentially, the CRTS process as utilized in the health production literature represents a degenerate case. This is no new result (Ehrlich and Chuma, 1990), but this paper provides more intuitive, less technical and additional arguments as to why health investment is not determined under the assumption of a CRTS health production process. This is important because the implications of the indeterminacy are substantial (e.g., Ehrlich and Chuma, 1990), yet the debate does not appear to have been settled in favor of a DRTS health production process as illustrated by its lack of use in the health production literature.

Third, the alternative interpretation allows for explorations of a stylized representation of the first-order condition which enable an intuitive understanding of the optimal solution for health

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investment. I find that a unique optimal solution for health investment exists (thus addressing the indeterminacy as Ehrlich and Chuma, 1990, have also shown). Given an optimal level for health investment, and because in this interpretation the health stock is determined by the dynamic equation for health, the stock is found to be a function of the histories of past health investments and past biological aging rates, addressing the criticism of Usher (1975). Further, I find that the optimal level of health investment decreases with the user cost of health capital and increases with wealth and with the consumption and production benefit of health. Thus I show that one does not need to resort to numerical analyses to gain insight into the characteristics of the solution. This is important because, arguably, the Grossman model has been successful, in part, because of its ability to guide empirical analyses through the intuition that simple representations provide (e.g., Wagstaff, 1986b; Muurinen and Le Grand, 1985).

Fourth, the alternative interpretation allows developing relations for the effects of variations in SES (wealth, education) and in health on the optimal level of health investment.[4] These relations complement explorations of stylized representations by allowing one to distinguish first- from second- and third-order effects and to explore the mechanisms (pathways) that combine to produce the final directional outcome, again, without the need to resort to numerical analyses. Under plausible assumptions the theory predicts a negative correlation between health and health investment (in cross-section). This is an important new result that addresses the criticism by Wagstaff (1986a) and Zweifel and Breyer (1997). Further, greater wealth, higher earnings over the life cycle and more education and experience are associated with slower health deterioration, addressing the criticism by Case and Deaton (2005).[5]

Fifth, empirical tests of the health production literature have thus far been based on structural and reduced form equations derived under the assumption of a CRTS health production process. Arguably, health capital theory has not yet been properly tested because these structural and reduced form relations suffer from the issue of the indeterminacy of health investment (and essentially represent a degenerate case). Absent an equivalent relation for a DRTS health production process I once more employ the alternative interpretation to derive a structural relation between health and health investment (e.g., medical care) that is suitable for empirical testing. The structural relation contains the CRTS health production process as a special case, thereby allowing empirical tests to verify or reject this common assumption in the health production literature.

Last, I perform numerical simulations to illustrate the properties of the theory. These simulations show that the model is capable of reproducing the rapid increase in health investment near the end of life and that the optimal solution for length of life is finite for a constant biological aging rate, addressing the criticism by Case and Deaton (2005) that health production models are characterized by complete health repair. In sum, I find that the theory can address each of the five limitations discussed above.

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[4]Employing Oniki's (1973) method as in Ehrlich and Chuma (1990) is somewhat comparable to the analysis performed here. Unfortunately, due to space limitations, the detailed analysis underlying the directional predictions by Ehrlich and Chuma (1990) has not been published, but is available on request from the authors.

[5]These results are also obtained by exploring a stylized representation of the first-order condition.

The paper is organized as follows. Section 2 presents the model in discrete time and discusses the characteristics of the first-order conditions. In particular this section offers an alternative interpretation of the first-order conditions. Section 3 explores the properties of a DRTS health production process, in several ways, by: a) exploring a stylized representation of the first-order condition for health investment to gain an intuitive understanding of its properties, b) analyzing the effect of differences in health and socioeconomic status (wealth and education) on the optimal level of health investment and consumption, c) developing structural-form relations for empirical testing of the model and d) presenting numerical simulations of health, health investment, assets and consumption profiles and length of life. Section 4 summarizes and concludes. The Appendix provides detailed derivations and mathematical proofs.

2   The demand for health, health investment and longevity

I start with Grossman's basic formulation (Grossman, 1972a, 1972b, 2000) for the demand for health and health investment (e.g., medical care) in discrete time (see also Wagstaff, 1986a; Wolfe, 1985; Zweifel and Breyer, 1997; Ehrlich and Chuma, 1990).[6] Health is treated as a form of human capital (health capital) and individuals derive both consumption (health provides utility) and production benefits (health increases earnings) from it. The demand for medical care is a derived demand: individuals demand "good health", not the consumption of medical care.

Using discrete time optimal control (e.g., Sydsaeter, Strom and Berck, 2005) the problem can be stated as follows. Individuals maximize the life-time utility function

$$\sum_{t=0}^{T-1} \frac{U(C_t, H_t)}{\prod_{k=1}^{t}(1+\beta_k)},$$

(1)

where individuals live for T (endogenous) periods, β_k is a subjective discount factor and individuals derive utility $U(C_t, H_t)$ from consumption C_t and from health H_t. Time t is measured from the time individuals begin employment. Utility increases with consumption $\partial U_t/\partial C_t > 0$ and with health $\partial U_t/\partial H_t > 0$.

The objective function (1) is maximized subject to the dynamic constraints:

$$H_{t+1} = f(I_t) + (1-d_t)H_t,$$

(2)

$$A_{t+1} = (1+\delta_t)A_t + Y(H_t) - p_{X_t}X_t - p_{m_t}m_t,$$

(3)

the total time budget Ω_t

$$\Omega_t = \tau_{w_t} + \tau_{I_t} + \tau_{C_t} + s(H_t),$$

(4)

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[6]In line with Grossman (1972a; 1972b) and Ehrlich and Chuma (1990) I do not incorporate uncertainty in the health production process. This would unnecessarily complicate the optimization problem and require numerical methods, while it is not needed to explain the stylized facts regarding health behavior discussed in this paper. For a detailed treatment of uncertainty within the Grossman model the reader is referred to Ehrlich (2000), Liljas (1998), and Ehrlich and Yin (2005).

and initial and end conditions: H_0, H_T, A_0 and A_T are given. Individuals live for T periods and die at the end of period $T-1$. Length of life T (Grossman, 1972a, 1972b) is determined by a minimum health level H_{min}. If health falls below this level $H_t \leq H_{min}$ an individual dies ($H_T \equiv H_{min}$).

Health (equation 2) can be improved through investment in health I_t and deteriorates at the biological aging rate d_t. The relation between the input, health investment I_t, and the output, health improvement $f(I_t)$, is governed by the health production function $f(\cdot)$. The health production function $f(\cdot)$ is assumed to obey the law of diminishing marginal returns in health investment. For simplicity of discussion I use the following simple functional form

$$f(I_t) = I_t^{\alpha}, \tag{5}$$

where $0 < \alpha < 1$ (DRTS).[7,8]

Assets A_t (equation 3) provide a return δ_t (the rate of return on capital), increase with income $Y(H_t)$ and decrease with purchases in the market of consumption goods and services X_t and medical goods and services m_t at prices p_{X_t} and p_{m_t}, respectively. Income $Y(H_t)$ is assumed to be increasing in health H_t as healthy individuals are more productive and earn higher wages (Currie and Madrian, 1999; Contoyannis and Rice, 2001).

Goods and services X_t purchased in the market and own time inputs τ_{C_t} are used in the production of consumption C_t. Similarly medical goods and services m_t and own time inputs τ_{I_t} are used in the production of health investment I_t. The efficiencies of production are assumed to be a function of the consumer's stock of knowledge E (an individual's human capital exclusive of health capital [e.g., education]) as the more educated may be more efficient at investing in health (see, e.g., Grossman 2000):

$$I_t = I[m_t, \tau_{I_t}; E], \tag{6}$$
$$C_t = C[X_t, \tau_{C_t}; E]. \tag{7}$$

The total time available in any period Ω_t (equation 4) is the sum of all possible uses τ_{w_t} (work), τ_{I_t} (health investment), τ_{C_t} (consumption) and $s(H_t)$ (sick time; a decreasing function of health). In this formulation one can interpret τ_{C_t}, the own-time input into consumption C_t as representing leisure.[9]

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[7]For $\alpha = 1$ we have Grossman's original formulation of a linear health production process.

[8]Mathematically, equation (5) is equivalent to the assumption made by Ehrlich and Chuma (1990) of a dual cost-of-investment function with decreasing returns to scale (their equation 5) and a linear health production process ($\alpha = 1$ in equation 5 in this paper). Conceptually, however, there is an important distinction. In principle one could imagine a scenario where the investment function I_t has constant or even increasing returns to scale in its inputs of health investment goods / services m_t and own time τ_{I_t}, but where the ultimate health improvement (through the health production process) has diminishing returns to scale in its inputs m_t and τ_{I_t} as assumed in equation (5; this paper). Arguably, it is not the process of health investment but the process of health production (the ultimate effect on health) that is expected to exhibit decreasing returns to scale.

[9]Because consumption consists of time inputs and purchases of goods/services in the market one can conceive leisure as a form of consumption consisting entirely or mostly of time inputs. Leisure, similar to consumption, provides utility and its cost consists of the price of goods/services utilized and the opportunity cost of time.

Income $Y(H_t)$ is taken to be a function of the wage rate w_t times the amount of time spent working τ_{w_t},

$$Y(H_t) = w_t \left[\Omega_t - \tau_{I_t} - \tau_{C_t} - s(H_t) \right]. \tag{8}$$

Thus, we have the following optimal control problem: the objective function (1) is maximized with respect to the control functions X_t, τ_{C_t}, m_t and τ_{I_t} and subject to the constraints (2, 3 and 4). The Hamiltonian of this problem is:

$$\mathfrak{I}_t \;=\; \frac{U(C_t, H_t)}{\prod_{k=1}^{t}(1+\beta_k)} + q_t^H H_{t+1} + q_t^A A_{t+1}, \quad t = 0, \ldots T-1 \tag{9}$$

where q_t^H is the adjoint variable associated with the dynamic equation (2) for the state variable health H_t and q_t^A is the adjoint variable associated with the dynamic equation (3) for the state variable assets A_t.[10]

The optimal control problem presented so far is formulated for a fixed length of life T (see, e.g., Seierstad and Sydsaeter, 1977, 1987; Kirk, 1970; see also section 3.4.1). To allow for differential mortality one needs to introduce an additional condition to the optimal control problem to optimize over all possible lengths of life T (Ehrlich and Chuma, 1990). One way to achieve this is by first solving the optimal control problem conditional on length of life T (i.e., for a fixed exogenous T), inserting the optimal solutions for consumption C_t^* and health H_t^* (denoted by $*$) into the "indirect utility function"

$$V_T \equiv \sum_{t=0}^{T-1} \frac{U(C_t^*, H_t^*)}{\prod_{k=1}^{t}(1+\beta_k)}, \tag{10}$$

and maximizing V_T with respect to T.[11]

2.1 First-order conditions

Maximization of (9) with respect to the control functions m_t and τ_{I_t} leads to the first-order condition for health investment I_t

$$\frac{\pi_{I_t}}{\prod_{k=1}^{t}(1+\delta_k)} \;=\; -\sum_{i=1}^{t} \left[\frac{\partial U(C_i, H_i)/\partial H_i}{q_0^A \prod_{j=1}^{i}(1+\beta_j)} + \frac{\partial Y(H_i)/\partial H_i}{\prod_{j=1}^{i}(1+\delta_j)} \right] \frac{1}{\prod_{k=i}^{t}(1-d_k)}$$

$$+ \; \frac{\pi_{I_0}}{\prod_{k=1}^{t}(1-d_k)}, \tag{11}$$

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[10]For a CRTS health production function ($f(I_t) \propto I_t$) as employed in the health production literature we have to explicitly impose that health investment is non negative, $I_t \geq 0$ (see Galama and Kapteyn 2009). This can be done by introducing an additional multiplier q_t^I in the Hamiltonian (equation 9) associated with the condition that health investment is non negative, $I_t \geq 0$. This is not necessary for a DRTS health production function, where diminishing marginal benefits and choice of suitable functional forms ensure that the optimal solution for health investment I_t is non negative.

[11]This is mathematically equivalent to the condition utilized by Ehrlich and Chuma (1990) (in continuous time) that the Hamiltonian equal zero at the end of life $\mathfrak{I}_T = 0$ (transversality condition).

where π_{I_t} is the marginal cost of health investment I_t

$$\pi_{I_t} \equiv \frac{p_{m_t} I_t^{1-\alpha}}{\alpha[\partial I_t / \partial m_t]} = \frac{w_t I_t^{1-\alpha}}{\alpha[\partial I_t / \partial \tau_{I_t}]}, \tag{12}$$

and the Lagrange multiplier q_0^A is the shadow price of wealth (see, e.g, Case and Deaton, 2005).

An alternative expression is obtained by using the final period $T-1$ as point of reference

$$\frac{\pi_{I_t}}{\prod_{k=1}^{t}(1+\delta_k)} = \sum_{i=t+1}^{T-1} \left[\frac{\partial U(C_i, H_i)/\partial H_i}{q_0^A \prod_{j=1}^{i}(1+\beta_j)} + \frac{\partial Y(H_i)/\partial H_i}{\prod_{j=1}^{i}(1+\delta_j)} \right] \prod_{k=t+1}^{i-1}(1-d_k)$$

$$+ \frac{\pi_{I_{T-1}} \prod_{k=t+1}^{T-1}(1-d_k)}{\prod_{k=1}^{T}(1+\delta_k)}. \tag{13}$$

Using either the expression (11) or (13) for the first-order condition for health investment and taking the difference between period t and $t-1$ we obtain the following expression

$$(1-d_t)\pi_{I_t} = \pi_{I_{t-1}}(1+\delta_t) - \left[\frac{\partial U(C_t, H_t)/\partial H_t \prod_{j=1}^{t}(1+\delta_j)}{q_0^A \prod_{j=1}^{t}(1+\beta_j)} + \frac{\partial Y(H_t)}{\partial H_t} \right], \tag{14}$$

or

$$\frac{\partial U(C_t, H_t)}{\partial H_t} = q_0^A \left(\sigma_{H_t} - \varphi_{H_t} \right) \frac{\prod_{j=1}^{t}(1+\beta_j)}{\prod_{j=1}^{t}(1+\delta_j)}, \tag{15}$$

where σ_{H_t} is the user cost of health capital at the margin

$$\sigma_{H_t} \equiv \pi_{I_t} \left[(d_t + \delta_t) - \frac{\Delta \pi_{I_t}}{\pi_{I_t}}(1+\delta_t) \right], \tag{16}$$

φ_{H_t} is the marginal production benefit of health

$$\varphi_{H_t} \equiv \frac{\partial Y(H_t)}{\partial H_t}, \tag{17}$$

and $\Delta \pi_{I_t} \equiv \pi_{I_t} - \pi_{I_{t-1}}$.

Maximization of (9) with respect to the control functions X_t and τ_{C_t} leads to the first-order condition for consumption C_t

$$\frac{\partial U(C_t, H_t)}{\partial C_t} = q_0^A \pi_{C_t} \frac{\prod_{j=1}^{t}(1+\beta_j)}{\prod_{j=1}^{t}(1+\delta_j)}, \tag{18}$$

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where π_{C_t} is the marginal cost of consumption C_t

$$\pi_{C_t} \equiv \frac{p_{X_t}}{\partial C_t / \partial X_t} = \frac{w_t}{\partial C_t / \partial \tau_{C_t}}. \tag{19}$$

The first-order condition (11) (or the alternative forms 13 and 15) determines the optimal solution for the control function health investment I_t. The first-order condition (18) determines the optimal solution for the control function consumption C_t.[12] The solutions for the state functions health H_t and assets A_t then follow from the dynamic equations (2) and (3). Length of life T is determined by maximizing the indirect utility function V_T (see 10) with respect to T.

2.2 An alternative interpretation of the first-order condition

One of the most central relations in the health production literature is the first-order condition (15). This relation equates the marginal consumption benefit of health $\partial U_t / \partial H_t$ to the user cost of health capital σ_{H_t} and the marginal production benefit of health φ_{H_t}, and is interpreted as an equilibrium condition for the health stock H_t. It is equivalent to, e.g., equation (11) in Grossman (2000) and equation (13) in Ehrlich and Chuma (1990).[13] An alternative interpretation of relation (15) is, however, that it determines the optimal level of health investment I_t. My argument is as follows.

First, the first-order condition (15) is the result of maximization of the optimal control problem with respect to investment in health and hence, first and foremost, it determines the optimal level of health investment I_t. Optimal control theory distinguishes between control functions and state functions. Control functions are determined by the first-order conditions and state functions by the dynamic equations (e.g., Seierstad and Sydsaeter, 1977, 1987; Kirk, 1970). The first-order condition (15) is thus naturally associated with the control function health investment I_t and the state function health H_t is determined by the dynamic equation (2).[14]

Second, in the health production literature the optimal solution for health investment I_t is assumed to be determined by the first-order condition (11) (or the alternative form 13). It is equivalent to, e.g., equation (9) in Grossman (2000) and equation (8) in Ehrlich and Chuma

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[12]Because the first-order condition for health investment goods / services m_t and the first-order condition for own time inputs τ_{I_t} are identical (see Appendix section A) one can consider a single control function I_t (health investment) instead of two control functions m_t and τ_{I_t}. The same is true for consumption C_t. Because of this property, the optimization problem is reduced to two control functions I_t and C_t (instead of four) and two state functions H_t and A_t.

[13]Notational differences with respect to Grossman (2000) are: $q_0^A \to \lambda$, $\pi_{I_t} \to \pi_t$, $\partial U_t / \partial H_t \to [\partial U / \partial h_t][\partial h_t / \partial H_t] = U h_t G_t$ (where h_t is healthy time, a function of health H_t), $\varphi_{H_t} \to W_t G_t$, $\delta_t \to r$, $d_t \to \delta_t$, $\beta_t \to 0$, and $T \to n$. Notational differences with respect to Ehrlich and Chuma (1990), apart from using discrete rather than continuous time, are: $q_0^A \to \lambda_A(0)$, $\pi_{I_t} \to g(t)$, $\partial U_t / \partial H_t \to [\partial U(t) / \partial h(t)][\partial h(t) / \partial H(t)] = U_h(t)\varphi'(H(t))$ (where $h(t)$ is healthy time), $\varphi_{H_t} \to w\varphi'(H(t))$, $\delta_t \to r$, $d_t \to \delta(t)$ and $\beta_t \to \rho$.

[14]Analogously, the first-order condition (18) is associated with the control variable consumption C_t and the dynamic equation (3) is associated with the state function assets A_t.

(1990).[15] However, it can be shown that the first-order conditions (11) and (15) are mathematically equivalent

$$(11) \Leftrightarrow (15), \tag{20}$$

proof of which is provided in the Appendix (section B). Thus if equation (11) is the first-order condition for health investment I_t (the interpretation in the health production literature) then equation (15) is too (and vice versa).

From a purely mathematical standpoint one could conceive the condition (15) as determining the level of the health stock because a direct relation exists between health H_t and health investment I_t, namely the dynamic equation (2). Optimizing with respect to health investment entails optimizing with respect to health. Thus, in principle, one ought to be able to reconcile both interpretations. However, the health production literature assumes CRTS in the health production process.[16] In section 3.1.2 I show that under this particular assumption the level of health investment is not determined, i.e. that it represents a special degenerate case. As a result, both approaches cannot be reconciled in this particular case.

In the remainder of this paper I will use relations (11) and (15) as being equivalent. Both conditions determine the optimal level of health investment I_t, conditional on the level of the health stock H_t.

3   A DRTS health production process

In this section I explore the properties of a health production process in several ways. In section 3.1 I discuss a stylized representation of the first-order condition for health investment to gain an intuitive understanding of its properties. In particular I contrast the characteristics of the solution for health investment under a DRTS health production process with that of a CRTS process. In this section I also provide additional arguments for the claim made by Ehrlich and Chuma (1990) that DRTS in the health production process are necessary to guarantee the existence of a solution to the optimization problem.[17] In section 3.2 I explore the effect of differences in health and socioeconomic status (wealth and education) on the optimal level of health investment

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[15]One important difference between the results derived by Grossman (equation 9 in Grossman, 2000) and those derived here is the absense in Grossman's derivations of the reference point π_{I_0} in equation (11) or the reference point $\pi_{I_{T-1}}$ in equation (13). Using optimal control techniques I find these reference points to be required in a discrete time formulation (see equations 11 and 13). This is also true for a continuous time formulation. To the best of my knowledge this observation has not been made before. It has important implications for the model's interpretation as the begin or end point references allows one to ensure that the solution is consistent with the begin and end conditions for health and assets: H_0, H_T, A_0 and A_T.

[16]I.e., $f(I_t) = I_t^\alpha$ with $\alpha = 1$ (equations 2 and 5) and a Cobb-Douglas (CRTS) relation between investment in medical care I_t and its inputs own time and goods/services purchased in the market.

[17]Providing further corroboration of their claim is important because the implications are substantial and the debate does not appear to have been settled in favor of a DRTS health production process as illustrated by its lack of use in the health production literature.

and consumption. In section 3.3 I derive structural form relations for empirical testing of the model. Last, in section 3.4 I perform numerical simulations of health, health investment, assets and consumption profiles and length of life.

In the following I assume diminishing marginal utilities of consumption $\partial^2 U_t / \partial C_t^2 < 0$ and of health $\partial^2 U_t / \partial H_t < 0$, and diminishing marginal production benefit of health $\partial \varphi_{H_t} / \partial H_t = \partial^2 Y_t / \partial H_t^2 < 0$. In addition I make the usual assumption of a Cobb-Douglas CRTS relation between the inputs goods/services purchased in the market and own-time and the outputs investment in curative care I_t and consumption C_t. As a result we have $\pi_{I_t} \propto I_t^{1-\alpha}$ and $\partial \pi_{C_t} / \partial C_t = 0$ (see equations 81 and 84 in Appendix section D).

3.1 Stylized representation

In this section I contrast the properties of a DRTS health production process[18] (section 3.1.1) with those of a CRTS health production process[19] (section 3.1.2).

3.1.1 Decreasing returns to scale

Figure 1 provides a stylized representation of the first-order condition for health investment I_t (15): it graphs the marginal benefit and marginal cost of health as a function of health investment I_t (left-hand side) and as a function of health H_t (right-hand side).[20]

Consider the left-hand figure first. The optimal level of health investment I_t is determined by equating the consumption benefit of health $\partial U_t / \partial H_t$ with the cost of maintaining the health stock $q_0^A(\sigma_{H_t} - \varphi_{H_t})$ (here and in the remainder of the discussion in this section I ommit for convenience of notation the term $\prod_{j=1}^t (1 + \beta_j)[\prod_{j=1}^t (1 + \delta_j)]^{-1}$).

Utility is derived from health H_t and consumption C_t but not from health investment I_t (the demand for medical care is a derived demand). Further, the evolution of the health stock H_t is determined by the dynamic equation (2) which can be written (using 5) as

$$H_t = H_0 \prod_{j=0}^{t-1}(1 - d_j) + \sum_{j=0}^{t-1} I_j^\alpha \prod_{i=j+1}^{t-1}(1 - d_i). \tag{21}$$

In other words, health H_t is a function of past health investment I_s but not of current health investment I_t ($s < t$). Thus the consumption benefit of health $\partial U_t / \partial H_t$ is independent of the level of health investment I_t: this is shown as the horizontal solid line labeled $\partial U_t / \partial H_t$.

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[18]$0 < \alpha < 1$ and a Cobb-Douglas health investment process I_t.

[19]$\alpha = 1$ and a Cobb-Douglas health investment process I_t.

[20]While in principle one can derive predictions for the level of health investment I_t from the left-hand figure without the need to resort to the right-hand figure, it is useful to consider the right-hand figure in order to illustrate the effect of differences in the health stock H_t on the optimal level of health investment (see section 3.2) and to make comparisons with the usual interpretation of this relation as determining the "optimal" health stock (rather than optimal investment; see section 3.1.2).

The cost of maintaining the health stock is a function of the shadow price of wealth q_0^{A},[21] the user cost of health capital σ_{H_t}, the production benefit of health φ_{H_t}, and an exponential factor that varies with age t depending on the difference between the time preference rate β_t and the rate of return on capital δ_t. The marginal cost of health investment π_{I_t} and hence the user cost of health capital σ_{H_t} is increasing in the level of investment in health I_t ($\pi_{I_t} \propto I_t^{1-\alpha}$; see equation 81 in Appendix section D). The marginal production benefit of health φ_{H_t} is not a function of the level of health investment I_t. As a result, the cost of maintaining the health stock is upward sloping in the level of health investment (labeled $q_0^{A}(\sigma_{H_t} - \varphi_{H_t})$). The intersection of the two curves determines the optimal level of health investment (dotted vertical line labeled I_t).
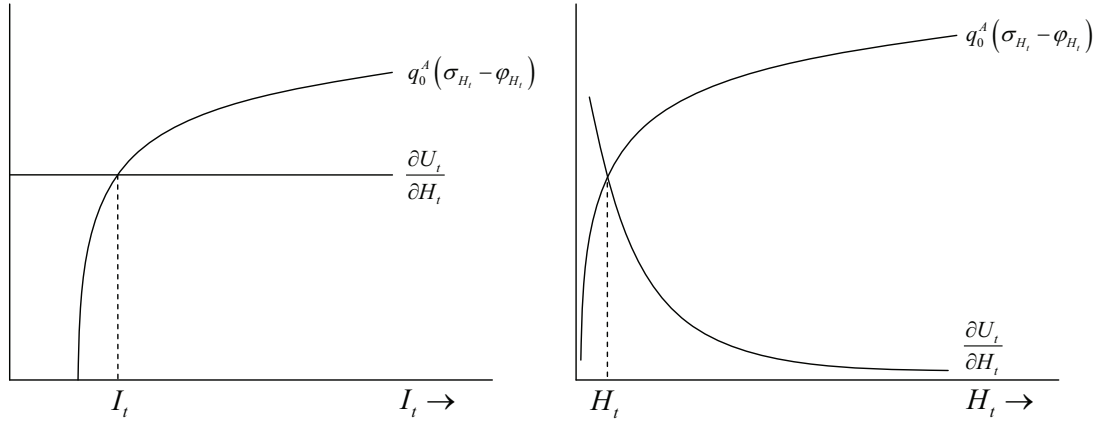

Figure 1: *Marginal benefit versus marginal cost of health for a DRTS health production process. In labeling the curves I have omitted the term $\prod_{j=1}^{t}(1 + \beta_j)[\prod_{j=1}^{t}(1 + \delta_j)]^{-1}$.*

Now consider the right-hand side of Figure 1. The marginal consumption benefit of health $\partial U_t / \partial H_t$ is downward sloping (convex) in health (curve labeled $\partial U_t / \partial H_t$) and the cost of maintaining the health stock $q_0^{A}(\sigma_{H_t} - \varphi_{H_t})$ is upward sloping (concave) in health (curve labeled $q_0^{A}(\sigma_{H_t} - \varphi_{H_t})$) due to the diminishing marginal production benefit of health φ_{H_t}. Since health is a stock its level is given (dotted vertical line labeled H_t) and provides a constraint: the two curves have to intersect at this level H_t. It is possible for the two curves to intersect at H_t through endogenous health investment I_t. A higher(/lower) level of health investment I_t increases(/decreases) (ceteris paribus) the marginal cost of health investment and hence the user cost of health capital. As a result the cost of maintaining the health stock (curve labeled $q_0^{A}(\sigma_{H_t} - \varphi_{H_t})$) shifts upward(/downward) while the marginal benefit of health (curve labeled $\partial U_t / \partial H_t$) remains stationary (it is not a function of the level of health investment).

The level of the marginal consumption benefit of health (labeled $\partial U_t / \partial H_t$ on the left-hand

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[21]q_0^{A} is decreasing in initial assets and life-time earnings. See, e.g., Wagstaff (1986a).

side of Figure 1) for which the health stock is at H_t (draw a horizontal line from the left-hand to the right-hand side of Figure 1) determines the optimal solution for health investment I_t. The optimal level of health investment I_t decreases with the user cost of health capital σ_{H_t} and increases with wealth (lower q_0^A) and with the consumption $\partial U_t / \partial H_t$ and production φ_{H_t} benefit of health. Further, the optimal level of health investment I_t is a direct function of the level of health stock H_t as can be seen from the first-order condition (15) and from its stylized representation in Figure 1 (more on this in the next sections 3.2 and 3.3). Hence, for a DRTS health production process a unique solution for health investment I_t exists for every level of the health stock H_t. This addresses the issue of the indeterminacy of health investment (e.g., Ehrlich and Chuma, 1990).

3.1.2 Constant returns to scale

Figure 2 provides a stylized representation of the first-order condition (15) for health investment for a CRTS health production process, as typically assumed in the health production literature: it graphs the marginal benefit and marginal cost of health as a function of health investment I_t (left-hand side) and as a function of the health stock H_t (right-hand side). In the following I follow the discussion in the previous section 3.1.1 and emphasize the differences with respect to a DRTS health production process.
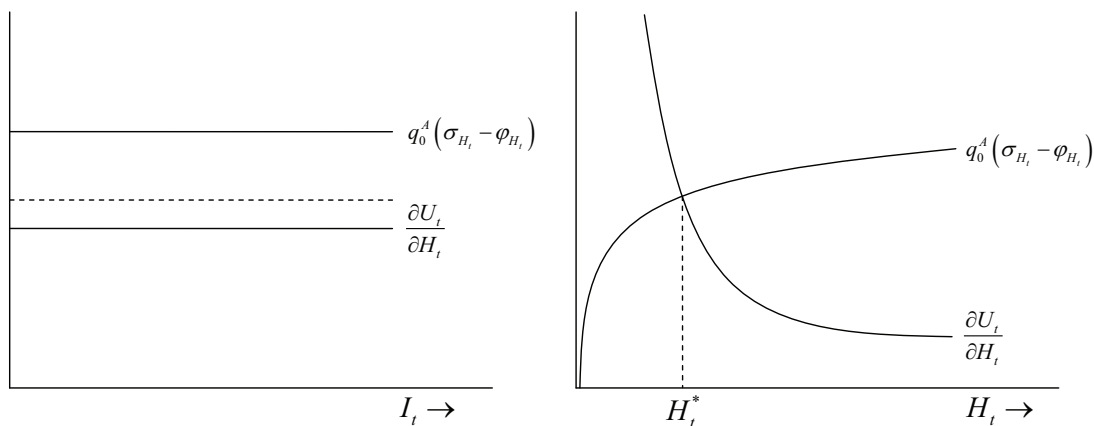
Figure 2: *Marginal benefit versus marginal cost of health for a CRTS health production process. In labeling the curves I have omitted the term* $\prod_{j=1}^{t}(1 + \beta_j)[\prod_{j=1}^{t}(1 + \delta_j)]^{-1}$.

Consider the left-hand side first. Unlike the DRTS process, for a CRTS process the marginal cost of health investment π_{I_t}, and hence the user cost of health capital σ_{H_t}, is independent of the level of health investment I_t ($\pi_{I_t} \propto I_t^{1-\alpha}$ = constant for $\alpha = 1$; see equation 81 in Appendix section D). Thus, not only the marginal utility of health $\partial U_t / \partial H_t$ but also the net marginal cost is independent of the level of health investment I_t: this is shown as the horizontal solid lines labeled

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$q_0^A(\sigma_{H_t} - \varphi_{H_t})$ and $\partial U_t / \partial H_t$.

Because individuals cannot adjust their health instantaneously, the level of the health stock H_t at age t is given and provides a constraint for the optimization problem at age t. Generally the constraint provided by H_t will result in different values for the marginal benefit and marginal cost of health: this is depicted by the two horizontal lines having distinct levels (they do not overlap). The intersection of the two solid curves would determine the optimal level of health investment I_t but only in the peculiar case that both lines exactly overlap does such an optimal solution exist. Thus for most values of the health stock no solution for health investment I_t exists.

Now consider the right-hand side of Figure 2. The consumption benefit of health $\partial U_t / \partial H_t$ is downward sloping to represent diminishing marginal utility in health. The cost of maintaining the health stock $q_0^A(\sigma_{H_t} - \varphi_{H_t})$ is upward sloping to represent diminishing marginal production benefits of health φ_{H_t}. As the graph shows, a unique level of health H_t^* exists (dashed vertical line) for which the consumption benefit of health equals the cost of maintaining the health stock. The health production literature assumes this unique solution H_t^* describes the "optimal" health path. Turning again to the left-hand side of Figure 2, note that for this particular value of the health stock H_t^* the consumption benefit of health $\partial U_t / \partial H_t$ and the cost of maintaining the health stock $q_0^A(\sigma_{H_t} - \varphi_{H_t})$ overlap (they both lie on the dashed horizontal line). Thus a solution for the level of investment in health I_t exists, but any non negative value can be allowed: once more the optimal level of investment in health I_t is not determined.

In order to illustrate that this result does not depend on the equivalence of the first-order conditions (11) and (15) I show next that this result also holds for (11), the relation that is utilized in the health production literature as determining the optimal level of health investment. The first-order condition for health investment (11) equates the current marginal monetary cost of investment in health π_{I_t} (left-hand side; LHS) with a function of the current and all *past* values of the marginal utility of health $\partial U_s / \partial H_s$ and the marginal production benefit of health φ_{H_s} ($0 \leq s \leq t$) (right-hand side; RHS). The LHS of (11) is not a function of health investment as the marginal monetary cost of health investment π_{I_t} is independent of the level of investment for a CRTS health production process. The RHS of (11) is also not a function of current investment I_t because the marginal utility of health $\partial U_s / \partial H_s$ and the marginal production benefit of health φ_{H_s} are functions of the health stock H_s ($0 \leq s \leq t$) which in turn is a function of past but not current health investment I_s ($s < t$; see equation 21). Thus the first-order condition for health investment (11) is not a function of health investment I_t and the level of health investment is not determined.

Ehrlich and Chuma (1990) have reached the same conclusion on the basis of a technical argument. From equation (59) or (60) it follows that the marginal monetary cost of health investment π_{I_t} is the ratio of two Lagrange multipliers

$$\pi_{I_t} = \frac{q_t^H}{q_t^A}. \tag{22}$$

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The right-hand side of (22) is not a function of health investment I_t by definition.[22,23] For a CRTS health production proces π_{I_t} is also not a function of health investment I_t and hence the level of health investment is not determined by the first-order conditon for health investment.

3.2 Variation in health and socioeconomic status

In this section I explore the effects of differences in health and socioeconomic status. I employ the first-order condition (15) to explore the effects of differences in initial assets (section 3.2.1) and in initial health (section 3.2.2) on the level of health investment I_t.

3.2.1 Variation in initial assets

Consider two optimal life time trajectories, different only (ceteris paribus) in their initial level of assets, A_0, and, $A_0 + \Delta A_0$, and the resulting difference in the two optimal life time trajectories

$$
\begin{aligned}
q_0^A &\rightarrow q_0^A + \Delta q_{0,A}^A \\
C_t &\rightarrow C_t + \Delta C_{t,A} \\
I_t &\rightarrow I_t + \Delta I_{t,A} \\
H_t &\rightarrow H_t + \Delta H_{t,A},
\end{aligned}
\tag{23}
$$

where $\Delta q_{0,A}^A$, $\Delta C_{t,A}$, $\Delta I_{t,A}$ and $\Delta H_{t,A}$ denote associated shifts in the shadow price of wealth q_0^A and in the optimal solutions for consumption C_t, health investment I_t and health H_t at each age t. A higher capital endowment lowers the shadow price of wealth (i.e., negative $\Delta q_{0,A}^A$). This in turn affects the level of consumption C_t and health investment I_t over the life cycle. Gradually differences in health investment I_t lead to differences in health H_t.

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[22]As Isaac Ehrlich pointed out to me in a private communication, the co-state variables (Lagrangian multipliers) cannot be a function of the flow of investment because they measure the value of the *stocks* of health capital and monetary wealth, which are not affected by the *flows* of investment in health and earnings, respectively, although they shift with time in current values. The mathematical proof is part of Pontryagin optimal control theory and the maximum principle.

[23]Grossman (2000) has questioned the argument by Ehrlich and Chuma (1990) noting (in a discrete time setting) that the first-order condition for health investment (13) equates the current marginal monetary cost of investment in health π_{I_t} (LHS) with a function of all *future* values of the marginal utility of health $\partial U_s/\partial H_s$ and the marginal production benefit of health φ_{H_s} ($t < s \leq T - 1$) (RHS). These in turn are functions of health and health is a function of all *past* values of health investment I_s ($0 \leq s < t$; see equation 21). Thus the RHS of the first-order condition for health investment (13) is a function of current health investment I_t (and, in fact, all future and all past values as well) and hence a solution for health investment I_t ought to exist. This apparent discrepancy can be reconciled by noting that implicit in the first-order condition for health investment (13) is the use of the final period $t = T - 1$ as the point of reference, while the relation (21) for the health stock uses the initial period $t = 0$ as the point of reference. Consistently using the initial period $t = 0$ as the point of reference, i.e., using the form (11) instead of (13) for the first-order condition for health investment, one finds that the RHS of (11) is not a function of current investment as the health stock is a function of past but not current health investment I_s ($s < t$). Likewise, consistently using the final period $t = T$ as the point of reference, i.e., using the alternative expression $H_t = H_T/[\prod_{i=t}^{T-1}(1 - d_i)] - \sum_{j=t}^{T-1} I_j^\alpha/[\prod_{i=t}^{j}(1 - d_i)]$ and comparing this with the first-order condition (13) one finds that the first-order condition is independent of current health investment I_t.

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Using a first-order Taylor expansion of the first-order conditions for health investment (15) and for consumption (18) and eliminating $\Delta C_{t,A}$ we have (for details see Appendix section C)

$$
\frac{\left.\frac{\partial \pi_I}{\partial I}\right|_{I_t,H_t}(d_t+\delta_t)}{\left.\sigma_H\right|_{I_t,H_t}-\left.\varphi_H\right|_{H_t}}\Delta I_{t,A}
$$

$$
+\left\{\frac{\left.\frac{\partial \pi_I}{\partial H}\right|_{I_t,H_t}(d_t+\delta_t)-\left.\frac{\partial \varphi_H}{\partial H}\right|_{H_t}}{\left.\sigma_H\right|_{I_t,H_t}-\left.\varphi_H\right|_{H_t}}-\frac{\left.\frac{\partial^2 U}{\partial H^2}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial H}\right|_{C_t,H_t}}-\frac{\frac{\left.\frac{\partial^2 U}{\partial C \partial H}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial H}\right|_{C_t,H_t}}\left(\frac{\left.\frac{\partial^2 U}{\partial C \partial H}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial C}\right|_{C_t,H_t}}-\frac{\left.\frac{\partial \pi_C}{\partial H}\right|_{C_t,H_t}}{\left.\pi_C\right|_{C_t,H_t}}\right)}{\frac{\left.\frac{\partial \pi_C}{\partial C}\right|_{C_t,H_t}}{\left.\pi_C\right|_{C_t,H_t}}-\frac{\left.\frac{\partial^2 U}{\partial C^2}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial C}\right|_{C_t,H_t}}}\right\}\Delta H_{t,A}
$$

$$
=-\left(\frac{\frac{\left.\frac{\partial^2 U}{\partial C \partial H}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial H}\right|_{C_t,H_t}}}{\frac{\left.\frac{\partial \pi_C}{\partial C}\right|_{C_t,H_t}}{\left.\pi_C\right|_{C_t,H_t}}-\frac{\left.\frac{\partial^2 U}{\partial C^2}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial C}\right|_{C_t,H_t}}}+1\right)\frac{\Delta q_{0,A}^A}{q_0^A}. \tag{24}
$$

The relation (24) describes the change in the level of health investment at age t of a trajectory with initial assets $A_0 + \Delta A_0$ compared to a trajectory with initial assets A_0. On the RHS the coefficient of the relative change in the shadow price of wealth $\Delta q_{0,A}^A/q_0^A$ consists of a first-order (direct) effect of a change in wealth (the factor 1) and a second-order (indirect) effect operating through the effect that a corresponding change in consumption has on the level of health investment (the remaining term). Assuming the first-order effect dominates, the term on the RHS is positive because an increase in assets (positive ΔA_0) decreases the shadow price of wealth (negative $\Delta q_{0,A}^A$).[24]

On the LHS we have a term in $\Delta I_{t,A}$ and one in $\Delta H_{t,A}$. The coefficient of the term in $\Delta I_{t,A}$ equals the relative change in the cost of maintaining the health stock $\left.\sigma_H\right|_{I_t,H_t}-\left.\varphi_H\right|_{H_t}$ resulting from variation in the level of health investment I_t.[25] The marginal cost of health investment π_{I_t} increases with the level of health investment for a DRTS health production process. As a result the coefficient of the term in $\Delta I_{t,A}$ is positive.

Consider the initial period $t = 0$. Because health at $t = 0$ is given by the initial condition H_0 we have $\Delta H_{0,A} = 0$ (differences in health between the trajectories with initial assets $A_0 + \Delta A_0$ and

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[24]The second-order term on the RHS equals the relative change in the marginal utility of health $\left.\partial U/\partial H\right|_{C_t,H_t}$ resulting from variation in consumption C_t (numerator) divided by the relative change in the marginal cost of consumption $\left.\pi_C\right|_{C_t,H_t}$ minus the marginal benefit (utility) of consumption $\left.\partial U/\partial C\right|_{C_t,H_t}$, resulting from variation in consumption C_t (denominator). For the usual assumptions of a Cobb-Douglas consumption process and diminishing marginal utility of consumption we have $\left.\partial\pi_C/\partial C\right|_{C_t,H_t} = 0$ and $\left.\partial^2 U/\partial C^2\right|_{C_t,H_t} < 0$. In this case the sign of the second-order term on the RHS depends on whether consumption and health are complements $\left.\partial^2 U/\partial C\partial H\right|_{C_t,H_t} > 0$ or substitutes $\left.\partial^2 U/\partial C\partial H\right|_{C_t,H_t} < 0$ in utility. Research by Finkelstein, Luttmer and Notowidigdo (2008) suggests that the marginal utility of consumption declines as health deteriorates, i.e. that $\left.\partial^2 U/\partial C\partial H\right|_{C_t,H_t} > 0$, in which case the second-order term is also positive.

[25]Note that $\left.\partial\sigma_H/\partial I\right|_{I_t,H_t} - \left.\partial\varphi_H/\partial I\right|_{H_t} = \left.\partial\pi_I/\partial I\right|_{I_t,H_t}(d_t+\delta_t) - \left.\partial\Delta\pi_I/\partial I\right|_{I_t,H_t}(1+\delta_t) \sim \left.\partial\pi_I/\partial I\right|_{I_t,H_t}(d_t+\delta_t)$.

with A_0 occur at later ages). Because $\Delta H_{0,A} = 0$ an increase in assets (which lowers the shadow price of wealth, i.e., negative $\Delta q_{0,A}^A$) increases the level of initial health investment, i.e. positive $\Delta I_{0,A}$ (see equation 24).

A simple graph helps to illustrate this result. Figure 3 shows a stylized representation of the first-order condition for inital health investment I_0 (15) as a function of I_0. A higher initial endowment of capital (positive ΔA_0) lowers the shadow price of wealth (negative $\Delta q_{0,A}^A$), thus shifting the net cost of maintaining the health stock downward (curve labeled $(q_A^0 + \Delta q_{0,A}^A)(\sigma_H - \varphi_H)|_{I_0 + \Delta I_{0,A}, H_0}$; first-order effect). A lower shadow price of wealth also increases the initial level of consumption C_0,[26] potentially affecting the marginal utility of health (second-order effect). If consumption and health are complements $\partial^2 U / \partial C \partial H|_{C_t, H_t} > 0$ in utility, the marginal utility of health shifts upward (curve labeled $\partial U / \partial H|_{C_0 + \Delta C_{0,A}, H_0}$). The net result is a higher level of initial health investment $I_0 + \Delta I_{0,A}$.

A higher initial endowment of capital (positive ΔA_0) initially induces individuals to invest more in health. As a result their health deteriorates slower. This addresses the criticism of Case and Deaton (2005) that health production models do not predict differences in the effective health deterioration rate with wealth.
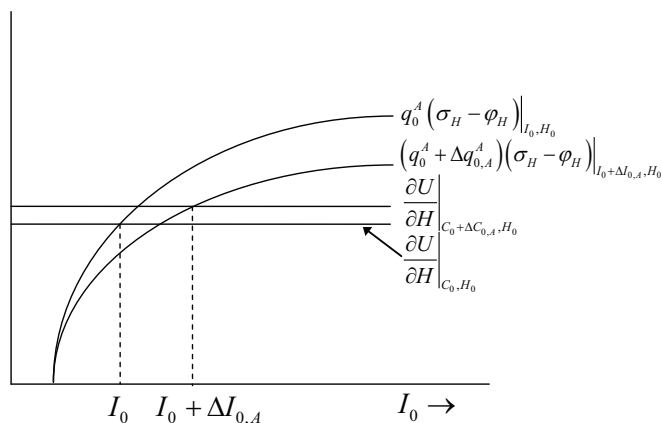

Figure 3: *Differences in initial assets: Marginal consumption $\partial U / \partial H$ and marginal production benefit φ_H of health versus the user cost of health capital at the margin σ_H as a function of initial health investment I_0.*

Now consider the next period ($t = 1$). Because of higher health investment $\Delta I_{0,A}$ in the initial period ($t = 0$) health will be higher in the next period $\Delta H_{1,A} > 0$ ($t = 1$). If the level of health

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[26]See equation (77) and note once more that for the usual assumptions of a Cobb-Douglas consumption process and diminishing marginal utility of consumption we have $\partial \pi_C / \partial C|_{C_t, H_t} = 0$ and $\partial^2 U / \partial C^2|_{C_t, H_t} < 0$. Further, $\partial U / \partial C|_{C_t, H_t} > 0$ and, for $t = 0$, $\Delta H_{0,A} = 0$.

investment remains higher in subsequent periods, both health trajectories will start to deviate, i.e. $\Delta H_{t,A}$ would grow over time. How would this affect the level of health investment?

The coefficient of $\Delta H_{t,A}$ consists of a first-order effect (the first and second terms) and a second-order effect (the third term). The first term is equal to the relative change in the cost of maintaining the health stock $\sigma_H|_{I_t,H_t} - \varphi_H|_{H_t}$ resulting from variation in health H_t. The marginal cost of health investment $\pi_I|_{I_t,H_t}$ increases with the wage rate (opportunity cost of investing in health and not working) which potentially increases with health (healthy individuals are more productive), i.e. $\partial \pi_I/\partial H|_{I_t,H_t} > 0$. Diminishing marginal benefits of health imply $\partial \varphi_H/\partial H|_{H_t} < 0$. Thus the first term is positive. The second term equals the relative change in the marginal consumption benefit (utility) of health $\partial U/\partial H|_{C_t,H_t}$ resulting from variation in health H_t. The second term is also positive for the usual assumption of diminishing marginal utility of health $\partial^2 U/\partial H^2|_{C_t,H_t} < 0$. Thus both first-order terms are positive.[27] As a result, the difference in the demand for health investment becomes smaller (smaller $\Delta I_{t,A}$) as the deviation in health between the trajectories with initial assets $A_0 + \Delta A_0$ and with A_0 grows (growing $\Delta H_{t,A}$; see equation 24). Greater health reduces the demand for health investment (see also the discussions in sections 3.2.2 and 3.3). At some age the difference between the level of health investment could vanish ($\Delta I_{t,A} \sim 0$) and the effective health deterioration rate $H_{t+1} - H_t$ converge between the trajectory with initial assets $A_0 + \Delta A_0$ and with A_0.[28] Despite this convergence, given similar initial endowed health H_0 and an initial period of higher levels of health investment, individuals with greater endowed wealth remain healthier.

Other indicators of socioeconomic status such as life-time earnings and education behave qualitatively similar to endowed wealth (initial assets). The exploration of the effect of variations in these measures on health investment and health is outside the scope of this paper (but see section 3.3 and Galama and van Kippersluis [2010] for a discussion of the role of life-time earnings and education). The effect of greater earnings over the life cycle on health differs from the effect of greater endowed wealth in that the "wealth" effect is moderated by the higher opportunity cost of time. The effect of education on health is similar to that of greater earnings over the life cycle, but

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[27]The third term, describing a second-order effect, contains the same expression as the second-order term in the coefficient of the relative change in the shadow price of wealth $\Delta q_{0,A}^A/q_0^A$ (which, following the earlier discussion in section 3.2.1, is plausible positive) multiplied by the relative change in the marginal utility of health minus the relative change in the marginal cost of consumption in response to a variation in health: $(\partial^2 U/\partial C \partial H|_{C_t,H_t}/(\partial U/\partial H)|_{C_t,H_t} - (\partial \pi_C/\partial H)|_{C_t,H_t}/\pi_C|_{C_t,H_t}$. The marginal cost of consumption $\pi_C|_{C_t,H_t}$ increases with the wage rate (opportunity cost of devoting own time to consumption and not working) which potentially increases with health (healthy individuals are more productive). If consumption and health are strong complements in utility $(\partial^2 U/\partial C \partial H)|_{C_t,H_t}/(\partial U/\partial H)|_{C_t,H_t} > (\partial \pi_C/\partial H|_{C_t,H_t})/\pi_C|_{C_t,H_t}$ the third term is positive and results in an elevated level of health investment (compared to a situation where there is weak complementarity or substitutability in utility) in response to a higher health stock (positive $\Delta H_{t,A}$).

[28]Note that if at some age the difference in health investment $\Delta I_{t,A}$ becomes negative, i.e., an individual with greater endowed wealth ($\Delta A_0 > 0$) would spend less on health ($\Delta I_{t,A} < 0$), the health difference in the next period $t + 1$ is reduced (smaller $\Delta H_{t+1,A}$), which leads to a less negative or positive difference in the level of health investment $\Delta I_{t+1,A}$, suggesting a process of gradual convergence in the effective rate of health deterioration $H_{t+1} - H_t$ (where we have a relatively constant $\Delta H_{t,A}$ and small $\Delta I_{t,A}$).

with the additional effect of increasing the efficiency of health investment.

3.2.2 Variation in initial health

Consider two optimal life time trajectories, different only (ceteris paribus) in their initial level of health, H_0, and, $H_0 + \Delta H_0$, and the resulting difference in initial ($t = 0$) health investment I_0

$$
\begin{aligned}
I_0 &\rightarrow I_0 + \Delta I_{0,H} \\
C_0 &\rightarrow C_0 + \Delta C_{0,H} \\
q_0^A &\rightarrow q_0^A + \Delta q_{0,H}^A,
\end{aligned}
\tag{25}
$$

where $\Delta I_{0,H}$, $\Delta C_{0,H}$ and $\Delta q_{0,H}^A$ denote associated shifts in the optimal solution for initial health investment I_0, initial consumption C_0 and in the shadow price of wealth q_0^A.

Using a first-order Taylor expansion of the first-order conditions for health investment (15) and for consumption (18), eliminating $\Delta C_{0,H}$, and (in order to simplify the discussion) omitting second-order effects, we have

$$
\begin{aligned}
& \frac{\left.\frac{\partial \pi_I}{\partial I}\right|_{I_0, H_0} (d_0 + \delta_0)}{\left.\sigma_H\right|_{I_0, H_0} - \left.\varphi_H\right|_{H_0}} \Delta I_{0,H} \\
&= -\left\{ \frac{\left.\frac{\partial \pi_I}{\partial H}\right|_{I_0, H_0} (d_0 + \delta_0) - \left.\frac{\partial \varphi_H}{\partial H}\right|_{H_0}}{\left.\sigma_H\right|_{I_0, H_0} - \left.\varphi_H\right|_{H_0}} - \frac{\left.\frac{\partial^2 U}{\partial H^2}\right|_{C_0, H_0}}{\left.\frac{\partial U}{\partial H}\right|_{C_0, H_0}} \right\} \Delta H_0,
\end{aligned}
\tag{26}
$$

(this relation can be obtained by considering 24 for $t = 0$ and labeling the variations with H instead of A).[29] The relation (26) describes the change in the initial level of health investment I_0 of a trajectory with initial health $H_0 + \Delta H_0$ compared to a trajectory with initial health H_0. Under the usual assumptions the first-order relation between health and health investment is negative.[30]

A simple graph helps to illustrate this result. Figure 4 shows a stylized representation of the first-order condition for the inital level of health investment I_0 (15) as a function of I_0. A higher initial endowment of health (positive ΔH_0) lowers the marginal production benefit of health φ_{H_t} thus shifting the net cost of maintaining the health stock upward (curve labeled

---

[29]A higher level of initial health (positive ΔH_0) enables a higher level of earnings (the production benefit of health), thereby raising life-time earnings and lowering the shadow price of wealth (negative $\Delta q_{0,H}^A$). A higher level of health would thus increase the level of health investment through its effect on wealth. This wealth effect is however a second order-effect in the sense that it operates through the effect of health on wealth, and therefore omitted from (26).

[30]The marginal cost of health investment increases with health investment for a DRTS health production process (i.e., $(\partial \pi_I/\partial I)|_{I_0, H_0} > 0$) and with the wage rate (opportunity cost of investing in health and not working) which potentially increases with health (healthy individuals are more productive; i.e., $(\partial \pi_I/\partial H)|_{I_0, H_0} > 0$). Diminishing marginal production benefit of health implies $(\partial \varphi_H/\partial H)|_{H_0} < 0$ and diminishing marginal consumption benefit (utility) of health implies $\partial^2 U/\partial H^2|_{C_0, H_0} < 0$.

$(q_A^0 + \Delta q_{0,H}^A)(\sigma_H - \varphi_H)|_{I_0 + \Delta I_{0,H}, H_0 + \Delta H_0}$; first-order effect). Further, the marginal utility of health is lower for higher health (the curve labeled $\partial U/\partial H|_{C_0 + \Delta C_{0,H}, H_0 + \Delta H_0}$ shifts downward) as a result of the diminishing marginal utility of health (first-order effect). The net result is a lower level of health investment (negative $\Delta I_{0,H}$).
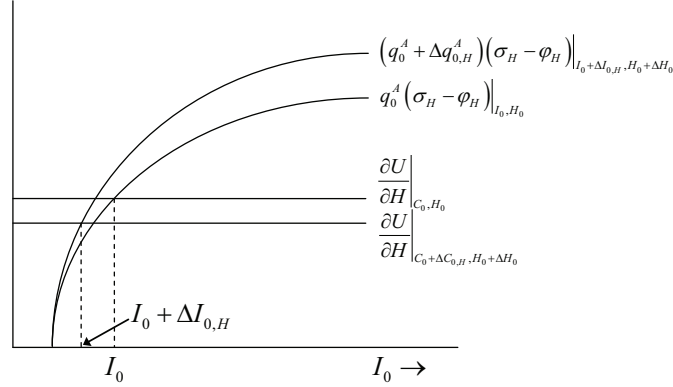

Figure 4: *Differences in initial health: Marginal consumption $\partial U/\partial H$ and marginal production benefit φ_H of health versus the user cost of health capital at the margin σ_H as a function of initial health investment I_0.*

Greater initial health (positive ΔH_0) reduces the initial demand for health investment (negative $\Delta I_{0,H}$). Because one can start the optimization problem at any age by redefining the initial conditions H_0 and A_0 for that age, this result holds for any age. Thus the theory predicts a negative relation between health and health investment in cross-section (see for more details section 3.3). This addresses the criticism by Zweifel and Breyer (1997).

3.3 Structural equations

Empirical tests of health production models have thus far been based on structural and reduced form equations derived under the assumption of a CRTS health production process.[31] Because these structural and reduced form relations suffer from the issue of the indeterminacy of health investment (see section 3.1.2), I derive in this section structural relations for the DRTS health production process presented in this paper.

---

[31]E.g., Grossman (1972a), Wagstaff (1986a), van Doorslaer (1987), Leu and Gerfin (1992), Nocera and Zweifel (1998), Erbsland, Ried and Ulrich (2002).

3.3.1 Simple functional forms

In order to obtain expressions suitable for empirical testing we have to assume functional forms for model functions and parameters that cannot be observed directly, such as the health investment production process I_t and the biological aging rate d_t.

I specify the following constant relative risk aversion (CRRA) utility function:

$$U(C_t, H_t) = \frac{1}{1-\rho} \left(C_t^\zeta H_t^{1-\zeta} \right)^{1-\rho}, \tag{27}$$

where ζ $(0 \leq \zeta \leq 1)$ is the relative "share" of consumption versus health and ρ $(\rho > 0)$ the coefficient of relative risk aversion. This functional form can account for the observation that the marginal utility of consumption declines as health deteriorates (e.g., Finkelstein, Luttmer and Notowidigdo, 2008) which would rule out strongly separable functional forms for the utility function, where the marginal utility of consumption is independent of health.

I make the usual assumption that sick time is a power law in health

$$s_t = \Omega \left(\frac{H_t}{H_{\min}} \right)^{-\gamma}, \tag{28}$$

where $\gamma > 0$ so that sick time decreases with health. This choice of functional form has the properties $\lim_{H_t \to \infty} s_t = 0$ and $\lim_{H_t \downarrow H_{\min}} s_t = \Omega$, where Ω is the total time budget as in (4).

Using equation (8) we have:

$$\begin{aligned} \varphi_{H_t} &= w_t \gamma \Omega H_{\min}^\gamma H_t^{-(1+\gamma)}, \\ &\equiv w_t \Omega^* H_t^{-(1+\gamma)}. \end{aligned} \tag{29}$$

Investment in medical care I_t is assumed to be produced by combining own time and goods/services purchased in the market according to a Cobb-Douglas CRTS production function (Grossman, 1972a, 1972b, 2000)

$$I_t = \mu_{I_t} m_t^{1-k_I} \tau_{I_t}^{k_I}, \tag{30}$$

where μ_{I_t} is an efficiency factor and $1 - k_I$ and k_I are the elasticities of investment in health I_t with respect to goods and services m_t purchased in the market (e.g., medical care) and with respect to own-time τ_{I_t}, respectively.

Analagously, consumption C_t is assumed to be produced by combining own time and goods/services purchased in the market according to a Cobb-Douglas CRTS production function

$$C_t = \mu_{C_t} X_t^{1-k_C} \tau_{C_t}^{k_C}, \tag{31}$$

where μ_{C_t} is an efficiency factor and $1 - k_C$ and k_C are the elasticities of consumption C_t with respect to goods and services X_t purchased in the market and with respect to own-time τ_{C_t}, respectively.

Following Grossman (1972a, 1972b, 2000) I assume that the more educated are more efficient consumers and producers of health investment (based on the interpretation of education as a productivity factor in own time inputs and in identifying and seeking effective care)

$$\mu_{I_t} = \mu_{I_0} e^{\rho_I E}, \tag{32}$$

where E is the level of education (e.g., years of schooling) and ρ_I is a constant.

Further, following Galama and van Kippersluis (2010) I assume a Mincer-type wage equation in which the more educated and more experienced earn higher wages (Mincer 1974)

$$w_t = w_E e^{\rho_w E + \beta_x x_t - \beta_{x^2} x_t^2}, \tag{33}$$

where education E is expressed in years of schooling, x_t is years of working experience, and ρ_w, β_x and β_{x^2} are constants, assumed to be positive.

Lastly, following Wagstaff (1986a) and Cropper (1981) I assume the biological aging rate d_t to be of the form

$$d_t = d_\bullet e^{\beta_t t + \beta_\xi \xi_t}, \tag{34}$$

where $d_\bullet \equiv d_0 e^{-\beta_\xi \xi_0}$ and ξ_t is a vector of environmental variables (e.g., working and living conditions, hazardous environment, etc) that affect the biological aging rate. The vector ξ_t may include other exogenous variables that affect the biological aging rate, such as education (Muurinen, 1982).

3.3.2 Structural relation between health and medical care

A structural relation for the demand for medical goods and services m_t can be obtained from the first-order conditions for health investment (15) and for consumption (18) and the functional relations defined in the previous section 3.3.1 (see section D in the Appendix for details)

$$b_{it}^1 m_{it}^{1-\alpha} - (1-\alpha) m_{it}^{1-\alpha} \widetilde{m_{it}} \;\; = \;\; b_{it}^2 H_{it}^{-1/\chi} + b_{it}^3 H_{it}^{-(1+\gamma)}, \tag{35}$$

where I have defined the following functions

$$b_{it}^1 \;\; \equiv \;\; \left[d_\bullet e^{\beta_t t_i + \beta_\xi \xi_{it}} + \delta - (1-\alpha k_I) \widetilde{p_{m_{it}}} - \alpha k_I \widetilde{w_{it}} \right], \tag{36}$$

$$b_{it}^2 \;\; \equiv \;\; b_*^2 \left(q_{0i}^A \right)^{-1/\rho\chi} e^{\alpha \rho_I E_i} p_{m_{it}}^{-(1-\alpha k_I)} w_{it}^{-[k_C(1/\rho\chi - 1) + \alpha k_I]} p_{X_{it}}^{-(1-k_C)(1/\rho\chi - 1)} \left(\frac{1 + \beta_i}{1 + \delta} \right)^{-t_i/\rho\chi} \tag{37}$$

$$b_{it}^3 \;\; \equiv \;\; b_*^3 e^{\alpha \rho_I E_i} p_{m_{it}}^{-(1-\alpha k_I)} w_{it}^{1-\alpha k_I}, \tag{38}$$

and the following constants

$$b_*^2 \equiv [(1-\zeta)\Lambda]^{1/\chi}\, \alpha k_I^{\alpha k_I}(1-k_I)^{1-\alpha k_I}\mu_{I_0}^\alpha \left[k_C^{k_C}(1-k_C)^{1-k_C}\mu_{C_t}\right]^{1/\rho\chi-1}, \tag{39}$$

$$b_*^3 \equiv \alpha k_I^{\alpha k_I}(1-k_I)^{1-\alpha k_I}\mu_{I_0}^\alpha \Omega_*, \tag{40}$$

$$\Lambda \equiv \zeta^{\frac{1-\rho}{\rho}}\left(\frac{\zeta}{1-\zeta}\right)^{1-\chi}, \tag{41}$$

$$\chi \equiv \frac{1+\rho\zeta-\zeta}{\rho}, \tag{42}$$

where the subscript i indexes the ith individual, and where the notation $\widetilde{f}_t$ is used to denote the relative change $\widetilde{f}_t \equiv 1 - \frac{f_{t-1}}{f_t}$ in a function f_t. Further, I have assumed small relative changes (much smaller than one) in the price of medical care $\widetilde{p_{m_{it}}}$, wages $\widetilde{w_{it}}$ and the efficiency of the health investment process $\widetilde{\mu_{I_{it}}}$ and, for simplicity, assumed a constant discount factor $\beta_t = \beta$ and constant rate of return to capital $\delta_t = \delta$.

A similar expression for own-time inputs $\tau_{I_{it}}$ can be obtained using (83). Further, one can substitute the expression (33) for the wage rate w_{it} to obtain an expression in terms of years of schooling E_i and years of experience x_{it}.

3.3.3 Pure investment and pure consumption models

Analytical solutions to the Grossman model are usually based on two sub-models (1) the "pure investment" model in which the restriction $\partial U_t/\partial H_t = 0$ is imposed and (2) the "pure consumption" model in which the restriction $\partial Y_t/\partial H_t = 0$ is imposed. In this section I explore the characteristics of these two sub models for the following reasons. First, the two sub models represent two essential characteristics of health: health as a means to produce (investment) and health as a means to provide utility (consumption) and exploring them separately provides insight into these two distinct properties of health. Second, these restrictions allow one to obtain linearized structural expressions. Last, the two sub-models are widely used in the health production literature and exploring them allows for comparisons with previous research.

In the pure investment model health does not provide utility and hence $\zeta = 1$ (see equation 27) and $b_{it}^2 = 0$, whereas in the pure consumption model health does not provide a production benefit and hence $\varphi_{H_{it}} = 0$ and $b_{it}^3 = 0$. We can obtain a structural linear relation for the demand for health investment goods / services m_{it} in the pure investment and pure consumption models as follows.

Pure investment For small $\widetilde{m}_{it}$ and $b_{it}^2 = 0$ we have (see equation 35)

$$
\begin{aligned}
(1-\alpha)\ln m_{it} \quad \sim \quad & \ln b_{it}^3 - \ln b_{it}^1 - (1+\gamma)\ln H_{it}, \\
= \quad & \ln b_*^3 - (1+\gamma)\ln H_{it} + \alpha\rho_I E_i - (1-\alpha k_I)\ln p_{m_{it}} + (1-\alpha k_I)\ln w_{it} \\
- \quad & \ln d_\bullet - \beta_i t - \boldsymbol{\beta_\xi \xi_{it}} - \ln\left\{1 + \left[\frac{\delta - (1-\alpha k_I)\widetilde{p_{m_{it}}} - \alpha k_I \widetilde{w_{it}}}{d_\bullet e^{\beta_i t_i + \boldsymbol{\beta_\xi \xi_{it}}}}\right]\right\}.
\end{aligned} \tag{43}
$$

Pure consumption For small $\widetilde{m}_{it}$ and $b_{it}^3 = 0$ we have (see equation 35)

$$
\begin{aligned}
(1-\alpha)\ln m_{it} \quad \sim \quad & \ln b_{it}^2 - \ln b_{it}^1 - \frac{1}{\chi}\ln H_{it}, \\
= \quad & \ln b_*^2 - \frac{1}{\chi}\ln H_{it} - \frac{1}{\rho\chi}\ln q_{0i}^A + \alpha\rho_I E_i - (1-\alpha k_I)\ln p_{m_{it}} \\
- \quad & \left[k_C\left(1/\rho\chi - 1\right) + \alpha k_I\right]\ln w_{it} - (1-k_C)\left(1/\rho\chi - 1\right)\ln p_{X_{it}} - \ln d_\bullet - \beta_i t_i - \boldsymbol{\beta_\xi \xi_{it}} \\
- \quad & \frac{1}{\rho\chi}\left[\ln(1+\beta_i) - \ln(1+\delta)\right]t_i - \ln\left\{1 + \left[\frac{\delta - (1-\alpha k_I)\widetilde{p_{m_{it}}} - \alpha k_I \widetilde{w_{it}}}{d_\bullet e^{\beta_i t_i + \boldsymbol{\beta_\xi \xi_{it}}}}\right]\right\}.
\end{aligned} \tag{44}
$$

It is customary to assume that the term $\ln d_\bullet$ in equations (43) and (44) is an error term with zero mean and constant variance $\xi_1(t) \equiv -\ln d_\bullet$ (as in Wagstaff, 1986a, and Grossman, 1972a, 1972b, 2000) and that the term $\ln[1 + \delta_t/d_t - \widetilde{\pi_{I_t}}/d_t]$ (the last term in equations 43 and 44) is small or constant (see, e.g., Grossman, 1972a, 2000),[32] or that it is time dependent $\ln[1 + \delta_t/d_t - \widetilde{\pi_{I_t}}/d_t] \propto t$ (e.g., Wagstaff, 1986a).

3.3.4 Reduced form relations

The solution for the health stock H_t follows from the dynamic equation (2) and using expressions (32) and (82)

$$
H_t \quad = \quad H_0 \prod_{j=0}^{t-1}(1-d_j) + \left(\frac{1-k_I}{k_I}\right)^{-\alpha k_I} \mu_{I_0}^\alpha e^{\alpha\rho_I E}\sum_{j=0}^{t-1} p_{m_j}^{\alpha k_I} w_j^{-\alpha k_I} m_j^\alpha \prod_{k=j+1}^{t-1}(1-d_k), \tag{45}
$$

where I have suppressed the index i for the individual.

The health stock H_t is a function of past levels of consumption of medical goods / services m_j ($j \le t-1$) and past biological aging rates d_j ($j \le t-1$). In principle one can obtain reduced

---

[32]This would require that the rate of return to capital δ_t and changes in the wage rate w_t and the price p_{m_t} and efficiency μ_{I_t} of health investment goods/services in producing health investment are much smaller than the health deterioration rate d_t or that such changes follow the same pattern as changes in d_t (so that the term is approximately constant).

form expressions for the health stock H_t[33] and for the demand for medical goods / services m_t.[34] This excercise, however, results in complex expressions with arguably limited value for empirical analyses. The reduced form solutions for the health stock H_t and the demand for medical goods / services m_t are functions of the initial health stock H_0, wealth q_0^A (endowed assets and life-time earnings) and the *history* of past prices of medical care p_{m_s}, past prices of consumption goods / services p_{X_s}, past wage rates w_s, past biological aging rates d_s and past rates of return to capital δ_s ($s < t$). In addition, the demand for medical goods / services is also a function of the current price of medical care p_{m_t}, the current price of consumption goods / services p_{X_t}, the current wage rate w_t, the current biological aging rate d_t and the current rate of return to capital δ_t (the health stock does not depend on current values).

3.3.5 Discussion

The structural form (35) of the first order condition for health investment describes a direct relationship between the demand for health investment goods / services m_t (e.g., medical care), the relative change in the demand for health investment goods / services $\widetilde{m}_t$ and the health stock H_t. For slow changes in the demand for health investment goods / services with time (small $\widetilde{m}_t$), the demand for health investment goods / services m_t falls with the level of health H_t. This is further reflected in the elasticity of health investment goods / services with respect to health H_t, which, for small $\widetilde{m}_t$, is negative (and a function of the health stock H_t)

$$\sigma_{m_t,H_t} = \frac{\partial m_t}{\partial H_t}\frac{H_t}{m_t} = -\frac{1}{1-\alpha}\left[\frac{\frac{1}{\chi}b_t^2 H_t^{-1/\chi} + (1+\gamma)b_t^3 H_t^{-(1+\gamma)}}{b_t^2 H_t^{-1/\chi} + b_t^3 H_t^{-(1+\gamma)}}\right], \tag{46}$$

where I have suppressed the index i for the individual. Similarly, the elasticity of health investment goods / services m_t with respect to health H_t (see equation 46) for the pure investment model

$$\sigma_{m_t,H_t}^{PI} = -\frac{1+\gamma}{1-\alpha}, \tag{47}$$

and the pure consumption model

$$\sigma_{m_t,H_t}^{PC} = -\frac{1}{\chi(1-\alpha)}, \tag{48}$$

are negative, where the labels PI and PC refer to the pure investment and pure consumption model, respectively. In other words, I find that the less healthy demand more and the healthy demand less

---

[33]Substitute the solutions for past consumption of medical goods / services m_j ($j \le t - 1$) obtained from (35) in (45) and recursively substitute the expression for the health stock (45) for past values of the health stock to obtain an expression for the health stock from which past levels of the health stock H_j ($j \le t - 1$) and past values of consumption of medical goods / services m_j ($j \le t - 1$) are removed (with the exception of initial health H_0).

[34]Use (35) and recursively substitute the expression for the health stock (45) to obtain an expression from which past consumption of medical goods / services m_j ($j \le t - 1$) and past levels of the health stock H_j ($j \le t - 1$) are removed (again with the exception of initial health H_0).

medical goods / services. This prediction from the theoretical model is in line with what has been observed in numerous empirical studies and addresses the criticism by Zweifel and Breyer (1997).

Assuming that both medical goods / services m_t and time input τ_{I_t} increase health investment suggests $0 \leq k_I \leq 1$ (see equation 30), and if education E increases the efficiency of medical care then $\rho_I > 0$ (see equation 32). Similarly we have $0 \leq k_C \leq 1$ (see equation 31).

For these assumptions and small changes $\widetilde{m}_t$, the demand for health investment goods/services m_t (see relations 35 and 36) decreases with the biological aging rate d_t (and hence with environmental factors that are detrimental to health ξ_t), the rate of return to capital δ_t (an opportunity cost – individuals can invest in health or in the stock market) and increases with price increases $\widetilde{p_{m_t}}$ and wage increases $\widetilde{w}_t$ (it is better to invest in health now when prices p_{m_t} and the opportunity cost of time w_t are higher in the future). In addition, due to the *consumption* aspect of health (health provides utility) the demand for health investment goods/services m_t (see relations 35 and 37 or 44) increases with wealth q_0^{A-1} (the shadow price of wealth is a decreasing function of wealth and life-time earnings),[35] education E (through assumed greater efficiency of health investment with the level of education) and decreases with the price of health investment goods/services p_{m_t}. For $\rho\chi < 1$ the demand for health investment goods/services m_t decreases with the price of consumption goods/services p_{X_t} (for $\rho\chi > 1$ it increases) and with the wage rate w_t (opportunity cost of time) (for $\rho\chi > 1$ the effect of the wage rate w_t is ambiguous)). And, due to the *production* aspect of health (health increases earnings) the demand for health investment goods/services m_t (see relations 35 and 38 or 43) increases with education E (through assumed greater efficiency of health investment with the level of education) and the wage rate w_t (a higher wage rate increases the marginal production benefit of health, and this outweighs the opportunity cost of time associated with health investment) and decreases with the price of health investment goods/services p_{m_t}.

The above discussion masks important effects of earnings and education. In our model of perfect certainty an evolutionary wage change (along an individual's wage profile) does not affect the shadow price of wealth q_0^A as the change is fully anticipated by the individual. Thus comparing panel data for a single individual may reveal a higher wage rate w_t to be associated with a lower demand for medical goods / services m_t due to a higher opportunity cost of time. However, comparing across individuals, those who currently have a higher wage rate will in most cases also have higher life-time earnings and thus a lower shadow price of wealth q_0^A. This wealth effect increases the demand for medical goods / services and competes with the opportunity cost of time effect. Similarly, to account for the effect of education it is important not only to consider the possible effect of a higher efficiency of health investment (the parameter ρ_I), as in the structural

---

[35]In principle, an expression for the shadow price of wealth q_0^A can be obtained by using the life-time budget constraint (which follows from integrating the dynamic equation 3), substituting the solutions for consumption goods/services X_t, health H_t, and health investment goods/services m_t and solving for q_0^A (see, for example, Galama et al. 2008). In practice, the shadow price of wealth q_0^A cannot be solved analytically: it is a very complicated function of the shadow price of health q_{H_t}, wealth (assets), education E and earnings, wages w_t, prices p_{m_t}, p_{X_t}, and health deterioration rates (terms $d_\bullet$, β_t and β_ξ) over the life cycle.

relations (35), (43) and (44), but also the effect that education has on earnings (opportunity cost of time effect; see equation 33) and in turn on wealth (wealth effect). Plausibly, the wealth effect dominates the opportunity cost effect. For example, Dustmann and Windmeijer (2000) and Contoyannis et al. (2004) find a positive effect on health from a permanent wage increase and a negative effect from a transitory wage increase. We expect then that the effect of education and earnings is to increase the demand for health investment goods / services through a wealth effect that may dominate the opportunity cost of time effect associated with higher earnings.[36]

Thus, in testing the theory it will be important to account for wealth. This can be done by employing measures of wealth (endowed assets, life-time earnings) as proxies for the shadow price of wealth q_0^A or, following Wagstaff (1986a), by utilizing an approximation for q_0^A (his equations 15 and 16).

3.4 Numerical simulations

In this section I present simulations of the model with a DRTS health production process and a simple step process. I first discuss the step process for fixed length of life (section 3.4.1). I then illustrate the properties of the model with numerical simulations accounting for endogenous length of life (section 3.4.2).

3.4.1 Step process and fixed length of life

We start with the initial condition for health H_0. Initial consumption C_0 then follows from the first-order condition for consumption (18), which, for the assumed functional forms in section 3.3.1, can be written as

$$C_t = \left[\frac{q_0^A}{\zeta} \pi_{C_t} \frac{\prod_{j=1}^{t}(1+\beta_j)}{\prod_{j=1}^{t}(1+\delta_j)} \right]^{-1/\rho\chi} H_t^{\frac{\chi-1}{\chi}}, \tag{49}$$

where π_{C_t} is given by (19). Initial consumption C_0 is a function of initial health H_0, the price of goods and services p_{X_0}, the wage rate w_0 (the opportunity cost of not working) and the shadow price of wealth q_0^A. Next, the initial level of health investment I_0 follows from the initial marginal cost of curative care π_{I_0} (see expression 12) which is a function of the Lagrange multiplier q_0^H and the shadow price of wealth q_0^A ($\pi_{I_0} = q_0^A/q_0^H$; see equations 59 and 60). The initial level of health investment I_0 is (through the initial marginal cost of curative care π_{I_0}) a function of the price of goods and services p_{m_0}, the wage rate w_0, education E, and the multipliers q_0^A and q_0^H. Thus, given

---

[36]Further, one may be tempted to conclude that individuals invest less in health care during middle and old age because of the high opportunity cost of time associated with high earnings at these ages (see equation 33). However, as health deteriorates with age the demand for curative care increases (see sections 3.2 and 3.3). If the latter effect dominates, the model is capable of reproducing the observation that young individuals invest little, the middle-aged invest more and the elderly invest most in curative care.

exogenous education, prices and wage rates, the initial level of health investment I_0 and the initial level of consumption C_0 are functions of the endogenous Lagrange multipliers q_0^A and q_0^H.

Health in the next period H_1 is determined by the dynamic equation (2). Assets in the next period A_1 follow from the initial condition for assets A_0 and the dynamic equation for assets (3). For the assumed functional forms in section 3.3.1 we have

$$A_{t+1} = (1 + \delta_t)A_t + w_t\left[\Omega - \tau_{I_t}^* I_t - \tau_{C_t}^* C_t - s_t\right] - p_{X_t} X_t^* C_t - p_{m_t} m_t^* I_t, \tag{50}$$

where $s_t, m_t^*, \tau_{I_t}^*, X_t^*, \tau_{C_t}^*$ are defined in (28), (82), (83), (85) and (86).

Consumption C_1 follows from the first-order condition for consumption (49), health investment I_1 follows from the first-order condition for health investment (11), (13), (14) or (15), which for the assumed functional forms in section 3.3.1 can be expressed as

$$\pi_{I_t} = \frac{1}{1 - d_t}\left[\pi_{I_{t-1}}(1 + \delta_t) - \frac{1 - \zeta}{q_0^A}C_t^{1 - \rho\chi}H_t^{\rho(\chi-1)-1} - w_t\Omega_* H_t^{-(1+\gamma)}\right]. \tag{51}$$

Health H_2 and assets A_2 in the next period are determined by the dynamic equations (2) and (50) and so on. The solutions for consumption C_t, health H_t and health investment I_t for every period t are functions of the two Lagrange multipliers q_0^A and q_0^H. In the final period, the two end conditions for the final level of health $H_T = H_{\min}$ and the final level of assets A_T determine the Lagrange multipliers q_0^A and q_0^H.[37]

Some have argued that length of life is determined in an iterative process by the condition that health at the end of life H_T equal the minimum health stock $H_{\min}$ (e.g., Grossman, 1998; Reid, 1998). These results are however based on a CRTS health production process and are the result of the indeterminacy of health investment. The results do not hold for a DRTS health production process as advocated here. This can be seen as follows. As the preceding discussion shows, the end conditions $H_T = H_{\min}$ and A_T are met for fixed length of life T because the solutions for assets, consumption, health and health investment (and having used the initial conditions H_0 and A_0) are functions of the Lagrange multipliers q_0^A and q_0^H. Applying the end conditions A_T and H_T determines the Lagrange multipliers q_0^A and q_0^H for fixed T. Thus, in the health production literature, as pointed out by Ehrlich and Chuma (1990), length of life T is exogenous (fixed) in the absence of the required terminal (transversality) condition.

3.4.2 Simulations with endogenous mortality

In this section I simulate the model for a particular set of parameter values. The purpose of this exercise is to illustrate some properties of the model. Other parameter choices are possible and a full exploration of the model's properties would require exploring a wide range of parameter values. Ultimately one would like to estimate the model with panel data to test its ability to describe human behavior. This is beyond the scope of this paper.

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[37]Alternatively one could start with the final period $t = T - 1$ and use recursive back substitution. Reaching the initial period, the two initial conditions for health H_0 and assets A_0 determine the Lagrange multipliers q_0^A and q_{T-1}^H.
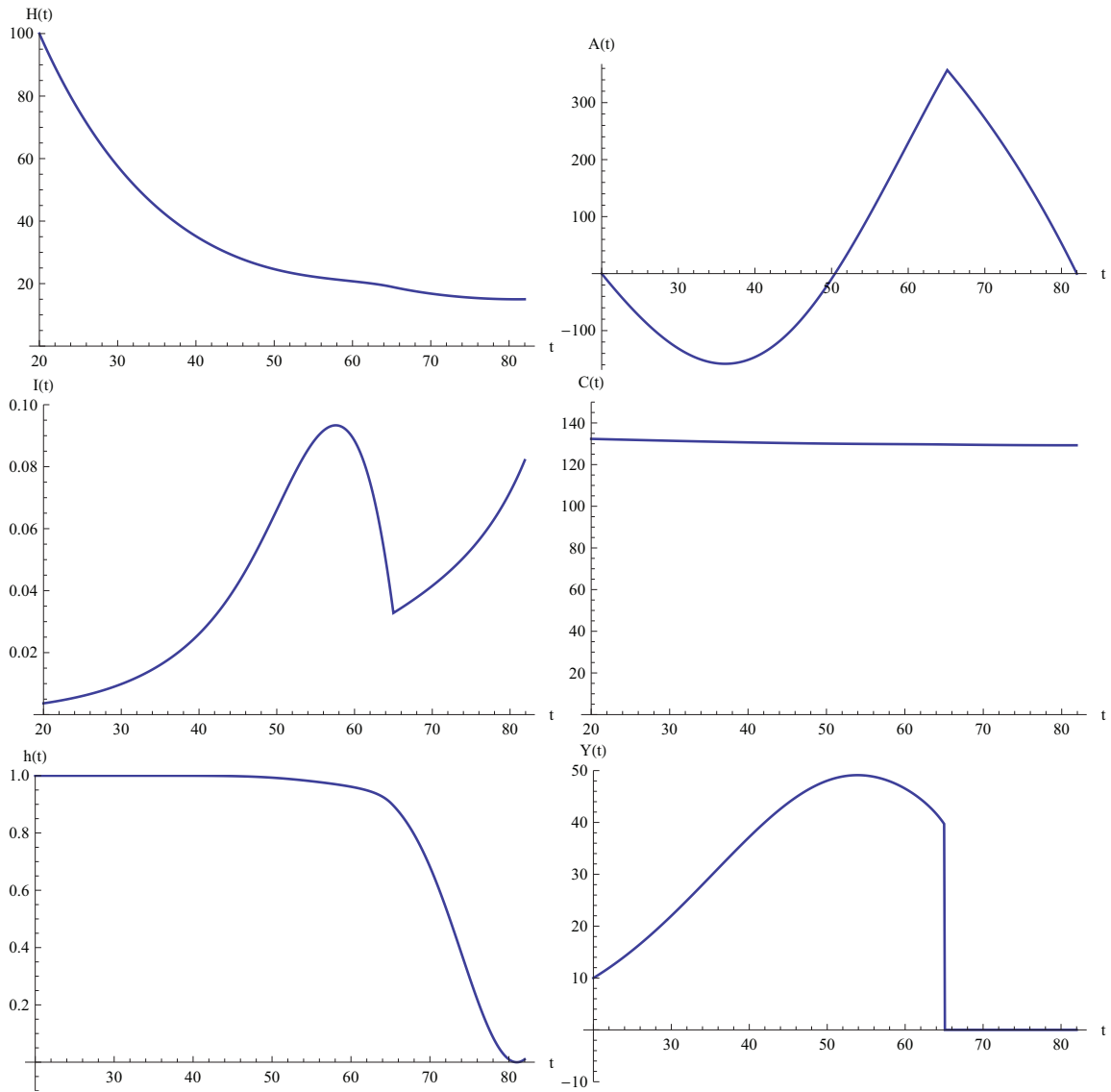
Figure 5: Health ($H(t)$; top-left panel), assets ($A(t)$; \$ thousands; top-right panel), health investment ($I(t)$; center-left panel), consumption ($C(t)$; center-right panel), healthy time ($h(t)$; fraction of total time Ω; bottom-left panel), annual earnings ($Y(t)$; \$ thousands per year; bottom-right panel)

Figure 5 shows the results of model simulations using the step process and equations presented in the above section 3.4.1. In the simulations I have used a period step size of one tenth of a year

and assumed annual wages of the form

$$w_t = 10e^{\{1.31383\times10^{-3}[70(t-20)-(t-20)^2]\}} \ \$ \text{ (thousands)}, \tag{52}$$

starting at age $t = 20$ when the individual begins work life until she retires at a fixed retirement age $R = 65$. This Mincer-type wage equation starts with annual wages of \$ 10,000 per year at age $t = 20$ and peaks at \$ 50,000 per year at age $t = 55$ after which it gradualy declines till the age of retirement $R = 65$ after which wages w_t are zero. In addition I use the following parameters: $\alpha = 0.5$, $\gamma = 10$ (sick time increases significantly only upon approaching end of life, i.e., as H_t approaches $H_{\min}$),[38] $H_0 = 100$, $H_T \equiv H_{\min} = 15$, $A_0 = A_T = 0$ \$ (thousands) (no bequests), $\Omega = 0.1$ year (the total time available in a period equals the time step size), $k_I = k_C = 0$ (health investment I_t and consumption C_t consist of purchases in the market, no own time inputs),[39] $p_{m_t} = 0.2$ \$ per medical good/service unit, $p_{X_t} = 0.2$ \$ per consumption good/service unit, $\mu_{I_t} = 0.01$, $\mu_{C_t} = 1$, $\rho = 0.8$, $\zeta = 0.95$ (high relative "weight" of consumption versus health in providing utility), a constant aging rate $d_t = d_0 = 0.06$ (per year), a constant return to capital $\delta_t = \delta_0 = 0.03$ (per year) and a constant subjective discount factor $\beta_t = \beta_0 = 0.03$ (per year).

I start with the initial values for health H_0 and assets A_0 and employ the Nelder-Mead method (also called the downhill simplex or amoeba method; Nelder and Mead, 1965) to iteratively determine the shadow price of wealth q_0^A and of health q_0^H that satisfy the end conditions A_T and H_T. I use the usual values $\alpha_{NM} = 1$, $\gamma_{NM} = 2$, $\rho_{NM} = 0.5$ and $\sigma_{NM} = 0.5$ for the Nelder-Mead reflection, expansion, contraction and shrink coefficients, respectively.

Optimal length of life is determined by maximizing the "indirect utility function" V_T (10) with respect to length of life T. I find $T = 82.0$ years.

Health H_t (top-left panel of Figure 5) gradually declines with age t and life ends at age $T = 82.0$ years. Health deteriorates somewhat slower during the ages 50 to 65, coinciding with increased levels of health investment I_t (center-left panel of Figure 5). The demand for health investment consists of two components. The first component is driven by the production benefit of health and follows a hump shaped pattern similar to the earnings profile Y_t (bottom-right panel of Figure 5). Health investment serves to maintain health in order to reduce sick time and hence increase earnings Y_t. Because of the parameter choice $k_I = 0$ there is no opportunity cost of time as the individual does not spend own-time on health investment. As a result, the production benefit of health is roughly proportional to the wage profile (equation 52). The second component is driven by the desire of individuals to be healthy (consumption benefit) and to live long lives (increases

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[38]Note that this choice of γ allows for a "realistic" relation between health and sick time but does not give rise to large medical expenditures near the end of life. The parameter γ affects sick time, not health, and individuals decide on the level of health investment based on the utility that health provides (note that after retirement there is no production benefit of health).

[39]This simplification helps avoid corner solutions in which the time budget constraint is not satisfied. This is because for this choice healthy time $h_t = \Omega - s_t$ is always positive after retirement, even as s_t approaches Ω as H_t approaches $H_{\min}$. After retirement income Y_t and time spend working τ_{w_t} are zero. Further, for $k_I = k_C = 0$ no time is devoted to health investment $\tau_{I_t} = 0$ or to consumption $\tau_{C_t} = 0$.

life-time utility). This component gradually increases with age. Thus the simulation suggests that solutions are feasible in which health investment increases near the end of life.

One possible explanation for the gradual rise in health investment near the end of life is that the simulations suggest that optimal length of life, at least for this set of parameters, coincides with the condition that the change in the health stock with age equal zero at the end of the last period. If the rate of change were positive, health would be below H_{min} some time before it eventually returned to H_{min} at the end of the last period $T - 1$, a condition that is not allowed since length of life is defined by the first time an individual's health reaches H_{min}.[40] If the rate of change were negative, adding a period extends life and provides additional utility (again, for this set of parameters). Thus as individuals approach end of life they slow their effective rate of change in health ($H_{t+1} - H_t$ approaches zero) through more and more health investment.

At a price $p_{m_t} = 0.2$ \$ per medical good/services unit her expenditures on health investment goods/services $p_{m_t} m_t$ peak at about \$ 1,800 per year at around age 55. The fact that such humped-shape profiles are generally not observed in medical expenditure data sets, at least not as sizeable as the simulation shows, suggests that the production benefit of health (compared to the consumption benefit of health) may be smaller in real life than is simulated. Again, the simulations are to illustrate the model's characteristics and attempts to estimate the model are left to future research.

The individual's assets A_t (top-right panel in Figure 5) initially deplete till about age 50 as she borrows to fund her consumption C_t (center-right panel of Figure 5) and health investment I_t needs. She builds up savings between ages 50 and the age of retirement (65) and depletes these savings by end of life. Consumption is relatively constant with age. At a price $p_{X_t} = 0.2$ \$ per consumption good/service unit her expenditures on consumption goods/services $p_{X_t} X_t$ are about \$ 28,000 per year.

Healthy time h_t (bottom-left panel of Figure 5) starts to decline rapidly around the age of retirement. While some of this can be explained by a drop in health investment I_t following retirement, this is mostly due to the steep functional relation assumed between health H_t and sick time s_t (equation 28 for $\gamma = 10$).

The simulations further show that solutions are feasible for which the biological aging rate is constant, despite the common perception that the biological aging rate needs to increase with age in order to ensure that health falls with age and life is finite (e.g., Grossman, 1972a, 2000; Ehrlich and Chuma, 1990; Case and Deaton, 2005).

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[40]To the best of my knowledge the health production literature has failed to observe that the optimization problem allows for solutions where the health stock falls below H_{min} before the end of life. I explicitly discard such solutions in the numerical simulations.

4   Discussion and conclusions

I have presented a theory of the demand for health, health investment and longevity, building on the human capital framework for health, in particular the work by Grossman (1972a, 1972b) and Ehrlich and Chuma (1990) and related literatures.

My contribution to the health production literature is as follows. First, I argue for a different interpretation of the health stock equilibrium condition, one of the most central relations in the health production literature: this relation determines the optimal level of health investment (not the health stock), conditional on the level of the health stock. Consistently employing the alternative interpretation allows me to simplify the theory and develop the health production literature further than was previously possible. This is because the equilibrium condition for the health stock (15) is of a much simpler form than the condition (11) which is typically utilized to determine the optimal level of health investment. There are several implications of this interpretation that I discuss in more detail below.

Second, the alternative interpretation of the first-order condition necesitates DRTS in the health production process or no solution for health investment exists. I therefore revisit the debate on the indeterminacy of health investment under the widely used assumption in the health production literature of a CRTS health production process and show that under this assumption the first-order condition for health investment (11 or 15) is not a function of health investment, and thus health investment is not determined. This widely used assumption represents a degenerate case with problematic properties. While this is no new result (Ehrlich and Chuma, 1990) I provide intuitive, less technical and additional arguments in its support. Revisiting this debate is important because the implications of the indeterminacy are significant and because the debate does not appear to have settled in favor of a DRTS health production process as illustrated by its lack of use in the health production literature. Besides technical reasons that suggest a CRTS health production process is restrictive, the different experiences of developing and developed countries suggest that the economic principle of eventually diminishing returns applies to health production. Quite modest increases in expenditures on health input (food, sanitation) have relatively large impacts on health in the developing world whereas large increases in resources in the developed world have a relatively modest impact on health (e.g., Wagstaff, 1986b).

Third, I explore in detail the implications of the alternative interpretation of the first-order condition and of the properties of a DRTS health production process. In particular the simpler form (15) allows me to utilize a stylized representation of the first-order condition for health investment to obtain an intuitive understanding of its properties. I find that for a DRTS health production process and the usual assumptions of diminishing marginal utility and diminishing marginal benefits a unique optimal solution for health investment exists (i.e., the indeterminacy is removed). The optimal level of health investment decreases with the user cost of health capital (i.e., with the price of medical goods / services, the wage rate [the opportunity cost of not working], the biological aging rate and the return to capital [the opportunity cost of investing in, e.g., the stock market rather than in health]) and increases with wealth (endowed assets and life-time

earnings) and with the marginal consumption and marginal production benefit of health (because both are decreasing in health, the demand for health investment decreases with health).

Further, I find that for every level of the health stock a unique optimal level of health investment exists. Thus I find no support for the concept of an "optimal" level of the health stock as utilized in the health production literature, in particular the notion that individuals may seek to adjust their health to this "optimal" level in case their health deviates from it. I find that individuals do not aspire to a certain level of health. Instead, given any level of their health stock, individuals decide about the optimal level of health investment. Thus one does not need to assume that any discrepancy between the actual and the "desired" health stock is dissipated instantaneously and on a continual basis.[41] Theoretically there is no justification for the assumption of a continual adjustment process and empirical work by Wagstaff (1993) suggests non-instantaneous adjustment better describes the health production process.

The simpler form of the first-order condition for health investment (15) allows me to investigate the effect of changes in initial conditions such as initial assets, initial health and education on the level of health investment and consumption by studying the effect of variations on the optimal solutions (see section 3.2). I find that the wealthy and more educated invest more in health and that their health deteriorates at a slower pace. As a result, given similar initial health endowments, they remain healthier as they age and live longer. Not only does this confirm the directional predictions made earlier by Ehrlich and Chuma's (1990) analysis, but the relations I derive in section 3.2 also allow for an understanding of the underlying mechanisms that lead to the predicted outcome. Calibrated simulations by Ehrlich and Yin (2005) of a related model (Ehrlich, 2000) which treats length of life as uncertain, and life expectancy as partly the product of individuals' efforts to self-protect against mortality and morbidity risks also finds that greater endowed wealth and higher wages over the life cycle increase life expectancy.

Further, I find a negative relation (in cross-section) between health and the level of health investment: the healthy demand fewer medical goods / services than the less healthy. This is an important new result that addresses a significant critique of health production models by Zweifel and Breyer (1997; see for more details below).

The simpler form of the first-order condition for health investment (15) also allows me to derive structural relations between health and health investment (e.g., medical care) that are

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[41]For the CRTS health production process the model is characterised by a so-called "bang-bang" solution as one has to assume that in the first period individuals adjust their health to its "optimal" level by investing a large positive or negative (depending on the direction of the adjustment) amount of medical care (or other forms of health investment; e.g., Wolfe, 1985; Ehrlich and Chuma, 1990; Grossman, 2000). But even if the health stock is at the "desired" level, health investment (and with it the health stock; see equation 2) is still undetermined as any level of investment is allowed (see discussion in section 3.1.2). Further, even if at some age t an individual's health stock is at the "desired" level H_t^*, it is not guaranteed that the health stock will subsequently evolve along this particular health path H_s^* (ages $s > t$) because for a given level of health both the marginal benefit of health $\partial U_t / \partial H_t$ and the cost of maintaining the health stock $q_0^A(\sigma_{H_t} - \varphi_{H_t})$ are determined by exogenous parameters and there is no mechanism to ensure that the two are equal. Thus in a formulation with a CRTS health production process one has to assume that any discrepancy between the actual and the "desired" health stock is dissipated not just once but on a continual basis.

suitable for empirical testing. These structural relations contain the CRTS health production process as a special case, thereby allowing empirical tests to verify or disproof this common assumption in the health production literature.

Finally, I show that for a DRTS health production process length of life is not endogenously determined and that an additional condition for optimal length of life is needed (see also, Ehrlich and Chuma, 1990; Seierstad and Sydsaeter, 1987; Kirk, 1970). I numerically solve the model to properly include the role of endogenous length of life. These simulations show that for plausible parameters health investment increases near the end of life and that length of life is finite as a result of limited life-time resources (the budget constraint) and if medical technology cannot fully offset biological aging.

Thus I find that a DRTS health production process addresses five consistent criticisms of the characteristics and predictions of health production models that have been made in the literature. First, as Ehrlich and Chuma (1990) have also shown, by introducing DRTS in the health production process the indeterminacy problem of health investment is addressed.

Second, I have shown that a DRTS health production process is capable of reproducing the observed negative relation between health and the demand for medical care (sections 3.2 and 3.3), addressing the criticism by e.g. Wagstaff (1986a) and Zweifel and Breyer (1997). This result follows directly from the first-order condition for health investment (as one would expect for such a fundamental feature of the demand for medical care). A CRTS health production process, on the other hand, predicts that the relation between health and investment in health is positive. In other words, the healthy are those that invest more in health (e.g., equation 13 in Wagstaff, 1986a; see also Galama and Kapteyn, 2009). Empirical studies strongly reject this prediction: the negative relationship between health and medical care is found to be the most statistically significant of any relationship between medical care and any of the independent variables in several empirical studies (see, e.g., Grossman, 1972a; Wagstaff, 1986a, 1993; Leu and Doppman, 1986; Leu and Gerfin, 1992; van Doorslaer, 1987; Van de Ven and van der Gaag, 1982; Erbsland, Ried and Ulrich, 2002).[42]

Third, Case and Deaton (2005) argue that while health production models can explain differences in the *level* of health between socioeconomic status (SES) groups they cannot explain differences in the *rate* of health deterioration between SES groups. In other words, health production models cannot account for the observed widening of disparities in health by SES with age. In section 3.2 I show that for a DRTS health production process the wealthy and more

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[42]Grossman (2000; pp. 369-370) shows that the model does not always produce the incorrect sign for the relationship between health and investment in medical care. For the pure investment model and assuming that the biological aging rate d_t increases with age (a necessary assumption for the health stock to decline with age in a CRTS formulation), he finds that investment in medical care increases with age while the health stock falls with age if the elasticity of the marginal production benefit of health with respect to health is less than one (Grossman refers to this as the MEC schedule). The requirement that the biological aging rate increase with age is another artifact of the indeterminacy of health investment. I do not have to rely on characteristics of exogenous functions such as the biological aging rate (apart from assuming that aging is detrimental to health, i.e. $d_t > 0$) to obtain a negative relation between health and health investment or to ensure that life is finite (see the criticism by Case and Deaton, 2005).

educated invest more in health and consume more and that their health deteriorates at a slower pace.[43] As a result, given similar initial health endowments, they remain healthier as they age and live longer. It is plausible that as the disparity in health widens the deteriorating health of low-SES individuals induces them to begin to invest more in health than their high-SES peers (e.g., due to the negative relation between health and investment in health). Thus the model could reproduce both the observed initial widening and the subsequent narrowing of the SES health gradient.

Fourth, Usher (1975) has pointed to the lack of "memory" in health production model solutions (e.g., Usher 1975, p. 220).[44] Casual observation and introspection suggests that our health depends on initial and past conditions: healthy individuals are those that began life healthy and that have invested in health over time. Indeed, in the alternative interpretation of the first-order condition presented here, health is not determined by the condition for "optimal" health (15) but by the dynamic equation (2), which can be written (using 5) in the form (21). Thus, the solution for the health stock H_t is a function of the initial health stock H_0 and the history of past health investments I_s and past biological aging rates d_s ($s < t$). As a result I find the health stock to be a complex function of the initial health stock H_0, initial assets A_0, education E and the entire history of prices, wages and environmental conditions (see the discussion in section 3.3.4).

Fifth, Case and Deaton (2005) note that in the health production literature " ... *if the rate of biological aging is constant, which is perhaps implausible but is hardly impossible, (and if the interest rate is as least as large as the rate of time preference), people will "choose" an infinite life ... "*[45] Thus, to ensure that life is finite and health falls with age it is necessary to assume that the biological aging rate increases with age ($\partial d_t / \partial t > 0$). In the interpretation of the theory presented here, however, it is not required that the biological aging rate d_t increase with age in order for health to decrease with age and in order for life to be finite. This follows intuitively from the dynamic relation (2; or in alternative form: equation 21) for health. If medical technology cannot fully repair the health of individuals for certain diseases (e.g., low efficiency of medical care will result in small I_t) then the health stock will decrease with age. Solutions are possible not only for a biological aging rate that increases with age, but also for constant or decreasing biological

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[43]For a CRTS health production process, however, the effective health deterioration rate $\partial H_t / \partial t$ depends on the rate of the biological aging rate $\partial d_t / \partial t$ (another artifact of the indeterminacy of health investment). Thus if low SES individuals have more rapidly increasing aging rates $\partial d_t / \partial t$ a model with a CRTS health production process could reproduce the observed widening of health disparities by SES. It seems plausible that the aging process d_t is more rapid for low SES individuals through, e.g., environmental factors such as detrimental living and working conditions, but it is not a priori clear that low SES individuals also have faster rates of the biological aging rate $\partial d_t / \partial t$ (e.g., Case and Deaton, 2005).

[44]The structural- and reduced-form solutions for health H_t and the reduced-form solution for health investment I_t and the demand for medical goods / services m_t derived and utilized in the health production literature are functions only of current parameter values (e.g., equations 4-2, 4-6 and 4-7 in Grossman, 1972a; equations 42, 45 and 46 in Grossman, 2000; equations 11, 12 and 14 in Wagstaff, 1986a; see also Galama and Kapteyn, 2009). Initial and past conditions appear to have been "forgotten".

[45]For a constant biological aging rate health decreases with age only if the time preference rate β_t exceeds the return to capital δ_t (and increases if the reverse is true). This follows from the first-order condition (15) if interpreted as a condition for the "optimal" level of the health stock. See also equation 13 in Grossman, 1972b, equation 11 in Grossman, 2000, or equation 6 in Case and Deaton (2005).

aging rates with age. The numerical simulations in section 3.4 provide an illustration based on a constant biological aging rate with age. This addresses the criticism by Case and Deaton (2005) that health production models are characterized by complete health repair.

In sum, I find health investment to be a decreasing function of health, that the health of wealthy individuals declines more slowly and that they live longer, that current health status is a function of the initial level of health and the histories of prior health investments made, that health investment rapidly increases near the end of life and that length of life is finite as a result of limited life-time resources (the budget constraint) and if medical technology cannot fully offset biological aging. I find no support for the common notion that individuals aspire to a certain "optimal" level of the health stock. Rather, given any level of their health stock, individuals decide about the optimal level of health investment.

Empirical estimation of the model is needed to test the assumptions and the theoretical predictions presented in this work and to contrast these with the predictions of alternative health production models. To this end I have provided structural form relations in section 3.3.2.

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A First-order conditions

Associated with the Hamiltonian (equation 9) we have the following conditions:

$$q^A_{t-1} = \frac{\partial \Im_t}{\partial A_t} \Rightarrow$$

$$q^A_{t-1} = (1 + \delta_t) q^A_t \Leftrightarrow$$

$$q^A_t = \frac{q^A_0}{\prod_{k=1}^t (1 + \delta_k)}, \tag{53}$$

$$q^H_{t-1} = \frac{\partial \Im_t}{\partial H_t} \Rightarrow$$

$$q^H_{t-1} = q^H_t (1 - d_t) + \frac{\partial U(C_t, H_t)/\partial H_t}{\prod_{k=1}^t (1 + \beta_k)} + q^A_0 \frac{\partial Y(H_t)/\partial H_t}{\prod_{k=1}^t (1 + \delta_k)} \Leftrightarrow \tag{54}$$

$$q^H_t = -\sum_{i=1}^t \left[\frac{\partial U(C_i, H_i)/\partial H_i}{\prod_{j=1}^i (1 + \beta_j)} + q^A_0 \frac{\partial Y(H_i)/\partial H_i}{\prod_{j=1}^i (1 + \delta_j)} \right] \frac{1}{\prod_{k=i}^t (1 - d_k)}$$

$$+ \frac{q^H_0}{\prod_{k=1}^t (1 - d_k)} \tag{55}$$

$$= \sum_{i=t+1}^T \left[\frac{\partial U(C_i, H_i)/\partial H_i}{\prod_{j=1}^i (1 + \beta_j)} + q^A_0 \frac{\partial Y(H_i)/\partial H_i}{\prod_{j=1}^i (1 + \delta_j)} \right] \prod_{k=t+1}^{i-1} (1 - d_k)$$

$$+ q^H_T \prod_{k=t+1}^T (1 - d_k) \tag{56}$$

$$\frac{\partial \Im_t}{\partial X_t} = 0 \Rightarrow$$

$$\frac{\partial U(C_t, H_t)}{\partial C_t} = q^A_0 \frac{p_{X_t}}{\partial C_t/\partial X_t} \frac{\prod_{j=1}^t (1 + \beta_j)}{\prod_{j=1}^t (1 + \delta_j)}$$

$$\equiv q^A_0 \pi_{C_t} \frac{\prod_{j=1}^t (1 + \beta_j)}{\prod_{j=1}^t (1 + \delta_j)}, \tag{57}$$

$$\frac{\partial \Im_t}{\partial \tau_{C_t}} = 0 \Rightarrow$$

$$\frac{\partial U(C_t, H_t)}{\partial C_t} = q^A_0 \frac{w_t}{\partial C_t/\partial \tau_{C_t}} \frac{\prod_{j=1}^t (1 + \beta_j)}{\prod_{j=1}^t (1 + \delta_j)}$$

$$\equiv q^A_0 \pi_{C_t} \frac{\prod_{j=1}^t (1 + \beta_j)}{\prod_{j=1}^t (1 + \delta_j)}, \tag{58}$$

$$\frac{\partial \mathfrak{I}_t}{\partial m_t} = 0 \Rightarrow$$

$$q_t^H = q_0^A \left\{ \frac{p_{m_t} I_t^{1-\alpha}}{\alpha[\partial I_t / \partial m_t]} \right\} \frac{1}{\prod_{j=1}^t (1+\delta_j)}$$

$$\equiv q_0^A \pi_{I_t} \frac{1}{\prod_{j=1}^t (1+\delta_j)}, \tag{59}$$

$$\frac{\partial \mathfrak{I}_t}{\partial \tau_{I_t}} = 0 \Rightarrow$$

$$q_t^H = q_0^A \left\{ \frac{w_t I_t^{1-\alpha}}{\alpha[\partial I_t / \partial \tau_{I_t}]} \right\} \frac{1}{\prod_{j=1}^t (1+\delta_j)}$$

$$\equiv q_0^A \pi_{I_t} \frac{1}{\prod_{j=1}^t (1+\delta_j)}, \tag{60}$$

where I have used the following definitions

$$\sum_{k}^{k-1} (\bullet) \equiv 0,$$

$$\prod_{k}^{k-1} (\bullet) \equiv 1.$$

Combining (59) or (60) with (55) we obtain the first-order condition for health investment (see equations 11 and 13). The first-order condition for consumption C_t is provided by equation (57) or (58) (see equation 18).

B Mathematical equivalency of first-order conditions

Taking the difference between period t and $t-1$ of either expression (11) or (13) one arrives at (14) and (15). In other words

$$(11) \Rightarrow (15), \tag{61}$$

$$(13) \Rightarrow (15). \tag{62}$$

Using recursive backward or forward substitution of relation (14) (which is equivalent to expression 15) one arrives at (11) or (13). Thus we have

$$(11) \Leftarrow (15), \tag{63}$$

$$(13) \Leftarrow (15). \tag{64}$$

Naturally, this result is also true in a continuous time formulation. In this case the first-order condition for health investment I_t can be written as (notation follows the discussion in section 2)

$$
\begin{aligned}
\pi_I(t)e^{-\int_0^t \delta(u)du} &= \pi_I(0)e^{\int_0^t d(u)du}\\
&\quad - \int_0^t \left[\frac{1}{q_A(0)}\frac{\partial U(s)}{\partial H(s)}e^{-\int_0^s \beta(u)du} + \varphi_H(s)e^{-\int_0^s \delta(u)du}\right]e^{\int_s^t d(u)du}ds,
\end{aligned}
\tag{65}
$$

or, in terms of the terminal point $t = T$

$$
\begin{aligned}
\pi_I(t)e^{-\int_0^t \delta(u)du} &= \pi_I(T)e^{-\int_0^T \delta(u)du}\, e^{-\int_t^T d(u)du}\\
&\quad + \int_t^T \left[\frac{1}{q_A(0)}\frac{\partial U(s)}{\partial H(s)}e^{-\int_0^s \beta(u)du} + \varphi_H(s)e^{-\int_0^s \delta(u)du}\right]e^{-\int_t^s d(u)du}ds.
\end{aligned}
\tag{66}
$$

Differentiating (65) or (66) with respect to t one obtains

$$
\frac{\partial U(t)}{\partial H(t)} = q_A(0)\left[\sigma_H(t) - \varphi_H(t)\right]e^{\int_0^t [\beta(s)-\delta(s)]ds}.
\tag{67}
$$

Using the Leibniz integral rule to differentiate (analogous to taking the difference between two time periods in discrete time) the first-order condition for health investment (65) or the alternative expression (66) with respect to t one obtains the alternative expression (67). In other words

$$(65) \Rightarrow (67), \tag{68}$$

$$(66) \Rightarrow (67). \tag{69}$$

From (67) we obtain a first-order differential equation in $\pi_I(t)$

$$
\frac{\partial \pi_I(t)}{\partial t} = \pi_I(t)[d(t) + \delta(t)] - \varphi_H(t) - \frac{\partial U(t)}{\partial H(t)}\frac{1}{q_A(0)}e^{-\int_0^t [\beta(u)-\delta(u)]du},
\tag{70}
$$

which can be solved (analogous to backward or forward substitution in discrete time)

$$
\begin{aligned}
\pi_I(t) &= \pi_I(t')e^{\int_{t'}^t [d(u)+\delta(u)]du}\\
&\quad - \int_{t'}^t \left[\frac{1}{q_A(0)}\frac{\partial U(s)}{\partial H(s)}e^{-\int_0^s [\beta(u)-\delta(u)]du} + \varphi_H(s)\right]e^{\int_s^t [d(u)+\delta(u)]du}ds.
\end{aligned}
\tag{71}
$$

For $t' = 0$ we obtain (65) and for $t' = T$ we obtain (66). Thus we have

$$(65) \Leftarrow (67), \tag{72}$$

$$(66) \Leftarrow (67). \tag{73}$$

C Variation in initial assets

Consider a variation in initial assets as in (23). The first order condition for health investment (15)

$$
\left.\frac{\partial U}{\partial H}\right|_{C_t+\Delta C_{t,A},\, H_t+\Delta H_{t,A}} = \left\{ \left.\pi_{I_t}\right|_{I_t+\Delta I_{t,A},\, H_t+\Delta H_{t,A}} (d_t+\delta_t) - \left.\Delta\pi_{I_t}\right|_{I_t+\Delta I_{t,A},\, H_t+\Delta H_{t,A}} (1+\delta_t) - \left.\varphi_{H_t}\right|_{H_t+\Delta_{t,A}} \right\}
$$

$$
\times \ \left(q_0^A + \Delta q_{0,A}^A \right) \frac{\prod_{j=1}^{t}(1+\beta_j)}{\prod_{j=1}^{t}(1+\delta_j)}, \tag{74}
$$

leads, using a first-order Taylor expansion, to the following expression

$$
\frac{\left.\frac{\partial^2 U}{\partial C \partial H}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial H}\right|_{C_t,H_t}} \Delta C_{t,A} = \frac{\Delta q_{0,A}^A}{q_0^A} + \left[\frac{\left.\frac{\partial \pi_I}{\partial H}\right|_{I_t,H_t}(d_t+\delta_t) - \left.\frac{\partial \varphi_H}{\partial H}\right|_{H_t}}{\left.\sigma_H\right|_{I_t,H_t} - \left.\varphi_H\right|_{H_t}} - \frac{\left.\frac{\partial^2 U}{\partial H^2}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial H}\right|_{C_t,H_t}} \right] \Delta H_{t,A}
$$

$$
+ \ \frac{\left.\frac{\partial \pi_I}{\partial I}\right|_{I_t,H_t}(d_t+\delta_t)}{\left.\sigma_H\right|_{I_t,H_t} - \left.\varphi_H\right|_{H_t}} \Delta I_{t,A}, \tag{75}
$$

where I have omitted second-order terms.[46]

Similarly, from the first order condition for consumption (18)

$$
\left.\frac{\partial U}{\partial C}\right|_{C_t+\Delta C_{t,A},\, H_t+\Delta H_{t,A}} = \left(q_0^A + \Delta q_{0,A}^A \right) \left.\pi_C\right|_{C_t+\Delta C_{t,A},\, H_t+\Delta H_{t,A}} \frac{\prod_{j=1}^{t}(1+\beta_j)}{\prod_{j=1}^{t}(1+\delta_j)}, \tag{76}
$$

we have

$$
\left(\frac{\left.\frac{\partial \pi_C}{\partial C}\right|_{C_t,H_t}}{\left.\pi_C\right|_{C_t,H_t}} - \frac{\left.\frac{\partial^2 U}{\partial C^2}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial C}\right|_{C_t,H_t}} \right) \Delta C_{t,A} = - \left(\frac{\left.\frac{\partial \pi_C}{\partial H}\right|_{C_t,H_t}}{\left.\pi_C\right|_{C_t,H_t}} - \frac{\left.\frac{\partial^2 U}{\partial C \partial H}\right|_{C_t,H_t}}{\left.\frac{\partial U}{\partial C}\right|_{C_t,H_t}} \right) \Delta H_{t,A} - \frac{\Delta q_{0,A}^A}{q_0^A}. \tag{77}
$$

Last, combining (75) with (77) to eliminate $\Delta C_{t,A}$ we obtain (24).

D Structural relations for empirical testing

From the utility function (27) and the first-order conditions (15) and (18) it follows that

$$
C_t = \frac{\zeta}{1-\zeta} \frac{\sigma_{H_t} - \varphi_{H_t}}{\pi_{C_t}} H_t, \tag{78}
$$

---

[46]Such as, e.g., terms in $(\partial \Delta\pi_I/\partial I)|_{I_t,H_t} \Delta I_{t,A}$ and $\Delta q_{0,A}^A (\partial \pi_I/\partial H)|_{I_t,H_t} \Delta H_{t,A}$.

and

$$H_t = (1-\zeta)\Lambda(q_0^A)^{-1/\rho}\left(\sigma_{H_t}-\varphi_{H_t}\right)^{-\chi}\pi_{C_t}^{\chi-1/\rho}\frac{\prod_{j=1}^t(1+\beta_j)^{-1/\rho}}{\prod_{j=1}^t(1+\delta_j)^{-1/\rho}}, \tag{79}$$

$$C_t = \zeta\Lambda(q_0^A)^{-1/\rho}\left(\sigma_{H_t}-\varphi_{H_t}\right)^{1-\chi}\pi_{C_t}^{\chi-1/\rho-1}\frac{\prod_{j=1}^t(1+\beta_j)^{-1/\rho}}{\prod_{j=1}^t(1+\delta_j)^{-1/\rho}}, \tag{80}$$

where Λ and χ are defined in (41) and (42).

Using equations (12) and (30) we have

$$\pi_{I_t} = \frac{p_{m_t}^{1-k_I}w_t^{k_I}}{\alpha k_I^{k_I}(1-k_I)^{1-k_I}\mu_{I_t}}I_t^{1-\alpha}\equiv\pi_{I_t}^*I_t^{1-\alpha}, \tag{81}$$

$$m_t = \left(\frac{1-k_I}{k_I}\right)^{k_I}\mu_{I_t}^{-1}p_{m_t}^{-k_I}w_t^{k_I}I_t\equiv m_{I_t}^*I_t, \tag{82}$$

$$\tau_{I_t} = \left(\frac{1-k_I}{k_I}\right)^{-(1-k_I)}\mu_{I_t}^{-1}p_{m_t}^{1-k_I}w_t^{-(1-k_I)}I_t\equiv\tau_{I_t}^*I_t. \tag{83}$$

Using equations (19) and (31) we have

$$\pi_{C_t} = \frac{p_{X_t}^{1-k_C}w_t^{k_C}}{k_C^{k_C}(1-k_C)^{1-k_C}\mu_{C_t}}, \tag{84}$$

$$X_t = \left(\frac{1-k_C}{k_C}\right)^{k_C}\mu_{C_t}^{-1}p_{X_t}^{-k_C}w_t^{k_C}C_t\equiv X_{C_t}^*C_t, \tag{85}$$

$$\tau_{C_t} = \left(\frac{1-k_C}{k_C}\right)^{-(1-k_C)}\mu_{C_t}^{-1}p_{X_t}^{1-k_C}w_t^{-(1-k_C)}C_t\equiv\tau_{C_t}^*C_t. \tag{86}$$

From (16), (29), (79), (81) and (84) follows

$$a_t^1 I_t^{1-\alpha}-(1-\alpha)I_t^{1-\alpha}\widetilde{I_t} = a_t^2 H_t^{-1/\chi}+a_t^3 H_t^{-(1+\gamma)}, \tag{87}$$

where

$$a_t^1 \equiv \left[d_t+\delta_t-(1-k_I)\widetilde{p_{m_t}}-k_I\widetilde{w}_t+\widetilde{\mu_{I_t}}\right], \tag{88}$$

$$a_t^2 \equiv \left[(1-\zeta)\Lambda(q_0^A)^{-1/\rho}\right]^{1/\chi}(\pi_{I_t}^*)^{-1}(\pi_{C_t})^{1-1/\rho\chi}\frac{\prod_{j=1}^t(1+\beta_j)^{-1/\rho\chi}}{\prod_{j=1}^t(1+\delta_j)^{-1/\rho\chi}}, \tag{89}$$

$$a_t^3 \equiv w_t(\pi_{I_t}^*)^{-1}\Omega_*. \tag{90}$$

where the notation $\widetilde{f_t}$ is used to denote the relative change $\widetilde{f_t}\equiv 1-\frac{f_{t-1}}{f_t}$ in a function f_t and we have assumed small relative changes in the price of medical care $\widetilde{p_{m_t}}$, wages $\widetilde{w}_t$ and the efficiency of the health investment process $\widetilde{\mu_{I_t}}$.

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Using (82) and (87) and the functional relations defined in section 3.3.1 we obtain a structural relation (35) between health investment goods and services m_t purchased in the market and the stock of health H_t.