Optimal Taxation in a Life-Cycle Economy with Endogenous Human Capital Formation: A Review

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Introduction

- **This presentation:** Overview of selected results on dynamic optimal taxation in an environment where
  - Human capital is endogenous
  - Individual's abilities are unobservable and permanent
- I will cover several cases:
  - Human capital is observable and deterministic
  - Human capital is unobservable and deterministic
  - Human capital is observable and has unobservable stochastic returns
  - Human capital is unobservable and has unobservable stochastic returns
- I will focus on the case when the cost of accumulating human capital is time, not physical resources
- This presentation will focus on qualitative results. However, one goal of this research is to allow for quantitative characterization of the optimal tax policies

0. Static Optimal Taxation

- A standard static optimal taxation model (Mirrlees 1971):
  - Question: How should a government design a tax system that maximizes some utilitarian social welfare function?
  - Individuals have abilities $\in \Theta$
  - Abilities are private information: The government only knows their distribution $Q(\ )$
  - People choose labor supply l and consumption c. Income is $y = \ l$.
  - c and y is observable, l and is not.
  - Taxation principle: Anything that can be achieved by an income tax function can be achieved in a direct revelation mechanism with incentive constraints

0. Static Optimal Taxation

- The planner's problem:
  - Maximize some social welfare function by choosing $\{c(\ );y(\ )\}$
  - subject to an incentive constraint

$$U(c(\ )) - V(\frac{y(\ )}{\ }) \geq U(c(\hat{\ })) - V(\frac{y(\hat{\ })}{\ });$$

  - and a resource constraint

$$\int [c(\ ) - \ l(\ )]dQ(\ ) \leq 0:$$

  - Replace the incentive constraint by an envelope condition:

$$U(c(\ )) - V(l(\ )) = w_0 + \int_{\underline{\theta}}^{\theta} V_l(l(''))l('')\frac{d''}{''};$$

0. Static Optimal Taxation

- Diamond (1998) shows that with $U(c) = c$ the optimal intratemporal wedge $\ (\ ) = 1 - \frac{V'(l)}{\theta}$ satisfies

$$\frac{\ (\ )}{1 - \ (\ )} = (1 + \ ^{-1})X(\ )$$

where $\ $ is the elasticity of labor supply, $X(\ )$ is given by

$$X(\ ) = \frac{1 - Q(\ )}{q(\ )} C(\ )$$

and $C(\ )$ depends on the social welfare function.

- For instance, if the planner is Rawlsian then $C(\ ) = 1$.
- If the shock support is finite, $X(\bar{\ }) = 0$ (no distortion at the top)

1. Deterministic Human Capital

- Agents live for $J > 1$ periods. They consume, work, and learn.
- Preferences:

$$\sum_{j=1}^{J} \quad ^{j-1}\left[U(c_j) - V(l_j; s_j)\right]; \quad 0 < \quad < 1; \tag{1}$$

- Labor Earnings:

$$y_j = \quad h_j l_j \tag{2}$$

- Human capital formation:

$$h_{j+1} = F(h_j; s_j) \tag{3}$$

- c is consumption, l is labor, s is schooling effort, h is beginning of period human capital.
- c and y are always observable. is always unobservable.

1.1. Deterministic and Observable Human Capital

- Allocation: $(c; y; h) = \{c_j(\ ); y_j(\ ); h_{j+1}(\ )\}_{j=1}^{J}$

- Lifetime utility:

$$W_{y,c,h}(\hat{\ }|\ ; h_1) = \sum_{j=1}^{J} \ ^{j-1} \left[U(c_j(\hat{\ })) - V\left(\frac{y_j(\hat{\ })}{h_j(\hat{\ })}; S(h_j(\hat{\ }); h_{j+1}(\hat{\ })) \right) \right]$$

- Incentive compatibility:

$$W_{y,c,h}(\ |\ ; h_1) \geq W_{y,c,h}(\hat{\ }|\ ; h_1) \quad \forall \hat{\ } \in \Theta: \tag{4}$$

- Resource constraint

1.1. Deterministic and Observable Human Capital

- Replace the incentive constraint by an envelope condition:

$$W_{y,c,h}(\ |\ ; h_1) = \int_{\underline{\theta}}^{\theta} \sum_{j=0}^{J} {}^{t-1} V_{l,j}('') l_j('') \frac{d''}{''} + W_{y,c,h}(\_|\_; h_1); \qquad (5)$$

  where $V_{l,j}(\ ) = V_l(l_j(\ ); s_j(\ ))$
- Characterize the optimum by the following:
  - The *intratemporal wedge*

$$_j \equiv 1 - \frac{V_l(l_j; s_j)}{h_j U_c(c_j)}$$

  - The *human capital wedge*

$$\Delta_j \equiv \frac{V_{s,j}}{F_{s,j}} - \left(V_{l,j+1} \frac{l_{j+1}}{h_{j+1}} + V_{s,j+1} \frac{F_{h,j+1}}{F_{s,j+1}} \right) \qquad (6)$$

1.1. Deterministic and Observable Human Capital

Example 1

Assumption (Example 1)

$V(l;s) = \frac{l^{1+\nu^{-1}}}{1+\nu^{-1}} + \frac{s^{1+\epsilon^{-1}}}{1+\epsilon^{-1}}$, $U(c) = c$ *and* $F(h;s) = s$.

- The intratemporal wedge

$$\frac{\ _j(\ )}{1-\ _j(\ )} = (1+\ ^{-1})X(\ ) \tag{7}$$

- The intertemporal wedge

$$\Delta_j(\ ) = \ (1+\ ^{-1})X(\ )\frac{l_{j+1}(\ )^{1+\nu^{-1}}}{s_j(\ )} \geq 0:$$

- Main implications:
  - Intratemporal wedge is the same as in the static model
  - Schooling subsidies provided to encourage investment in human capital

1.1. Deterministic and Observable Human Capital

Example 1

- Separable utility drives the constant intratemporal wedge
- Human capital wedge corrects for the fact that private benefits from investment in human capital are smaller than social benefits from investment in human capital
- DaCosta and Maestri (2007) show that the result is modified if human capital can help to separate people of different skills

1.1. Deterministic and Observable Human Capital

Example 2

Assumption (Example 2)

$V(l;s) = \frac{(l+s)^{1+\nu^{-1}}}{1+\nu^{-1}}$, $U(c) = c$ and $F(h;s) = s$.

- The intratemporal wedge

$$\frac{\ _j(\ )}{1-\ _j(\ )} = (1+[\ (1+\frac{s_j(\ )}{l_j(\ )}]^{-1})X(\ ) \quad (8)$$

- The elasticity term is now endogenous
- Higher schooling to labor ratio decreases the intratemporal wedge
  - Makes the tax system initially more regressive relative to a static economy
  - Intratemporal wedge should be increasing over the life-cycle

1.2. Deterministic and Unobservable Human Capital

- The incentive compatibility constraint:

$$h = \arg\max_{\hat{h}} W_{c,y,\hat{h}}(\ \ |\ \ ; h_1) \tag{9}$$

$$W_{c,y,h}(\ \ |\ \ ; h_1) \geq \max_{\hat{h}} W_{c,y,\hat{h}}(\ \hat{}\ |\ \ ; h_1) \qquad \forall\ \hat{}\ \in \Theta: \tag{10}$$

- Necessary conditions for incentive compatibility:
  - Envelope condition (the same as before)
  - Euler equation in HC investment:

$$\frac{V_{s,j}}{F_{s,j}} = \left(V_{l,j+1}\frac{l_{j+1}}{h_{j+1}} + V_{s,j+1}\frac{F_{h,j+1}}{F_{s,j+1}} \right) \tag{11}$$

- Incentives to accumulate human capital must now be provided differently

1.2. Deterministic and Unobservable Human Capital

Example 1 cont'd

Assumption (Example 1)

$$V(l;s) = \frac{l^{1+\nu^{-1}}}{1+\nu^{-1}} + \frac{s^{1+\epsilon^{-1}}}{1+\epsilon^{-1}},\ U(c) = c \text{ and } F(h;s) = s.$$

- The intratemporal wedge is given by

$$\frac{{}_1(\ \ )}{1 - {}_1(\ \ )} = (1 + \ \ ^{-1})X(\ \ )$$

$$\frac{{}_j(\ \ )}{1 - {}_j(\ \ )} = (1 + \hat{\ }^{-1})X(\ \ ); \quad j = 2 ::: J;$$

where $\hat{\ } = \frac{2+\nu^{-1}+\epsilon^{-1}}{\nu^{-1}\epsilon^{-1}-1} > \ \ .$

- Lower wedge for $j \geq 2$: labor supply is complementary with previous human capital investment

1.2. Deterministic and Unobservable Human Capital

Example 2 cont'd

Assumption (Example 2)

$$V(l;s) = \frac{(l+s)^{1+\nu^{-1}}}{1+\nu^{-1}},\ U(c) = c \text{ and } F(h;s) = s.$$

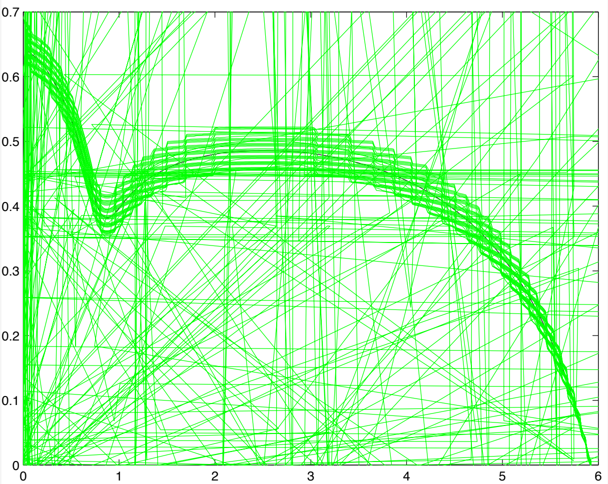
- For $J = 3$ one can show that the intratemporal wedge satisfies

$$_1(\quad) > \ _2(\quad) > \ _3(\quad) > 0:$$

- Complementarity of current schooling with future labor and substitutability of current schooling with current labor imply decreasing wedge
- Changes in labor elasticity imply increasing wedge
- The first effect dominates

Bohacek/Kapicka (2007): A calibrated in nite horizon economy

$$h' = (1 - \quad)h + (hs)^{\alpha}$$



2. Stochastic Human Capital

- Human capital accumulation

$$h_{j+1} = e^{z_{j+1}}(h_j + (h_j s_j)^{\alpha}); \quad z_j \sim i.i.d.$$

- z and s are both private information
- Private information from unobservability of individual abilities and labor
- Moral hazard problem from unobservable schooling effort and rates of return
- Can be mapped into Huggett, Ventura, Yaron (AER, 2011)

2.1. Stochastic and Observable Human Capital

- Social planner chooses $(c; y; s) = \{c_j(\;\; ; h^j); y_j(\;\; ; h^j); s_j(\;\; ; h^j)\}$
- Lifetime utility

$$W_{c,y,s}(\hat{\;}| \;\; ; h_1)$$
$$= \sum_{j=1}^{J} {}^{j-1} \int_{H^{j-1}} \left[U(c_j(\hat{\;}; h^j)) - V\left(\frac{y_j(\hat{\;}; h^j)}{a h_j}; s_j(h^j) \right) \right] P^j\left(h^j | h_1; s^{j-1}(h^{j-1})\right) dh^j$$

- Incentive compatibility

$$s = \arg\max_{\hat{s}} W_{c,y,\hat{s}}(\;\; | \;\; ; h_1) \qquad (12)$$

$$W_{c,y,s}(\;\; | \;\; ; h_1) \geq \max_{\hat{s}} W_{c,y,\hat{s}}(\hat{\;}| \;\; ; h_1) \quad \forall \;\; ; \hat{\;} \in \Theta \forall h_1 \in H \qquad (13)$$

2.1. Stochastic and Observable Human Capital

Relaxed Planning Problem in a 2 period version

Assume 2 periods, expected utility maximization, and let $W(\ ) = W_{c,y,s}(\ |\ ; h_1)$

$$\max \int W(\ )q(\ )d$$

subject to

$$\int_{\theta} \left[c_1(\ ) - y_1(\ ) + R^{-1} \int_{H} [c_2(\ ; h_2) - y_2(\ ; h_2)]\, P(h_2|s_1)\, dh_2 \right] q(\ )\, d \quad \leq 0;$$

$$V_s\left(\frac{y_1(\ )}{h_1}; s_1\right) = \int_{\underline{h}}^{h} \left[U(c_2(\ ; h_2)) - V\left(\frac{y_2(\ ; h_2)}{h_2}; 0\right)\right] P_s(h_2|s_1)\, dh_2$$

$$W(\ ) = W(\_) + \int_{\underline{\theta}}^{\theta} \left[V_\ell\left(\frac{y_1('')}{''h_1}; s_1('')\right) \frac{y_1('')}{''h_1} + \right.$$

$$\left. \int_{\underline{h}}^{h} V_\ell\left(\frac{y_2(''; h_2)}{''h_2}; 0\right) \frac{y_2(''; h_2)}{''h_2} P(h_2|s_1(''))\, dh_2 \right] \frac{d''}{''}$$

2.1. Stochastic and Observable Human Capital

Inverse Euler Equation

Proposition

$$\frac{1}{U'(c_1(\ ))} = \int_H \frac{1}{U'(c_2(\ ;h_2))} P(h_2|s_1(\ ))\, dh_2$$

by Jensen's Inequality,

$$U'(c_1(\ )) < \int_H U'(c_2(\ ;h_2)) P(h_2|s_1(\ ))\, dh_2$$

2.1. Stochastic Observable Human Capital

Example 1 cont'd

Proposition

Suppose that $V(l; s) = \frac{l^{1+\nu^{-1}}}{1+\nu^{-1}} + \frac{s^{1+\epsilon^{-1}}}{1+\epsilon^{-1}}$. *Then*

$$\frac{1}{{}_1(\ )} = \int_H \frac{1}{{}_2(\ ; h_2)} P(h_2 | s_1(\ ))\, dh_2 :$$

by Jensen's Inequality,

$$ {}_1(\ ) < \int_H {}_2(\ ; h_2) P(h_2 | s_1(\ ))\, dh_2$$

Intratemporal wedge increases over time!

Intratemporal Wedges

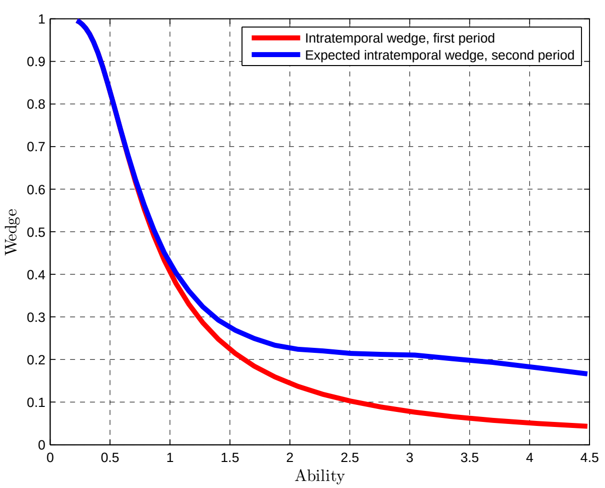


Figure: Intratemporal Wedges

Intratemporal Wedge Second Period

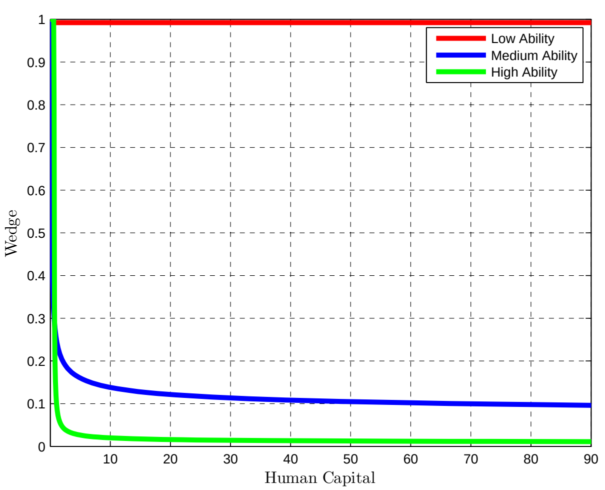


Figure: Intratemporal Wedge in the Second Period

2.2. Stochastic Unbservable Human Capital

- The most complicated: large number of deviations
- Grochulski and Piskorski (2007): solution for a special case:
  - investment in human capital only in the initial period
  - no labor supply in the initial period
  - depreciation shocks take only two values $e^z \in \{0; 1\}$
  - the low shock is absorbing

2.2. Stochastic Unbservable Human Capital

- Two novel results:
  - No distortion at the top does not apply: negative intratemporal wedge at the top
  - Stronger front-loading of consumption
- Both features relax the incentive constraints
- Not obvious if the first result survives in an economy with e.g. continuum of shocks
- The second result is similar to the higher tax in the initial period

3. Conclusions

- Main results:
- Corrective schooling subsidies whenever schooling effort is observable
- If not possible, use intertemporal variations in taxes to provide incentives to accumulate human capital
- The exact pattern depends on
    - substitutability/complementarity of human capital and schooling
    - changes in labor supply elasticity
    - riskiness of human capital investments
- More generally, one of the important questions is if human capital investments are useful in separating people of different skills