Some thoughts

- I want to talk a little about uncertainty (risk?) empirically
- We've seen in the presentation that it is an important aspect in the models
- But how do we think about it empirically in a meaningful way
- Show a proposal that Jim, Flavio and I made a while ago
- Show why it may matter
- Talk about how limited it is for the context of some of the models we talk about
- More importantly, the data limitations are humongous (NLSY79 vs NLSY97)

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A simple model of consumption and earnings

- $Y_{i,t}$: individual $i's$ income at time t
- $A_{i,t}$ denotes the assets he saves for the next period
- $u(C_{i,t})$ denotes individual utility if the agent consumes $C_{i,t}$
- $\tilde{u}(A_{i,T})$ denotes the utility in the terminal period
- ρ: discount rate.
- Let $\mathscr{I}_{i,t}$ be the information available to the agent at t
  - Includes past and current realizations of earnings, the (assumed constant) interest rate r and the asset stock.
  - May contain information about future earnings. Exactly how much?.

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Consumption

- Live for $T+1$ periods. All risks arise from labor market risk and are idiosyncratic.

- At t given information set $\mathscr{I}_{i,t}$:

Bellman

$$V_{i,t}(\mathscr{I}_{i,t}) = max_{A_{i,t}} u(C_{i,t}) + \frac{1}{1+\rho} E(V_{i,t+1}(\mathscr{I}_{i,t+1}) \mid \mathscr{I}_{i,t}) \quad (1)$$

$$s.t.\ C_{i,t} = Y_{i,t} + W_{i,t} + (1+r)A_{i,t-1} - A_{i,t},\ A_{i,0}, \quad (2)$$

$$A_{i,T} \geq 0. \quad (3)$$

- $A_{i,T} \geq 0$ imposes a borrowing constraint at every period.

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$$A_{i,t}^{MIN} = \frac{A_{i,t+1}^{MIN} - Y_{i,t+1}^{MIN}}{1+r}, \tag{4}$$

where $Y_{i,s,t}^{MIN}$ is the minimum certain value that income can take at time t and $W_{i,s,t} = \max\{Y_{i,t+1}^{MIN} - Y_{i,t}, 0\}$.

- One can then, for example, take the last period where $A_{i,T} = 0$ and solve backwards for $Y_{i,t+1}^{MIN}$ using the data:

$$A_{i,T-1}^{MIN} = \frac{-Y_{i,T}^{MIN}}{1+r}$$

under the assumption that $A_{i,T-1}^{MIN}$ corresponds to the minimum value of assets we observe in the data (so relax the hard borrowing limit of 0 associated with traditional forms of this model).

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In Principle

- We can think of changing it so that the constraints arise from limited commitment problems
- A "reduced form" solution is to allow the limits to be individual specific depending on income (for example) by taking the $A_{i,t}^{MIN}$ by quartiles of income
- But of course the models predict that it depends on income on particular ways
- Plus they tend to depend on whole histories not only current income in which case (if we want to be "reduced form") we may run out of data fast

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Solution

- From the agent's perspective: A pair of time indexed functions

Policy and Value

$$C^*_{i,t} = \mathscr{C}_{s,t}(\mathscr{I}_{i,t}) \tag{5}$$

$$V^*_{i,t} = \mathscr{V}_t(\mathscr{I}_{i,t}) \tag{6}$$

- Usual problem how to guarantee consumption fits the data?
- Consumption measured with error:

$$\widehat{C}_{i,t} = C_{i,t} e^{K_{i,t}\delta + \xi_{i,t}} \tag{7}$$

- Notice peculiar measurement error ($K_{i,t}$). We usually introduce it ourselves so why not account for it!

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Uncertainty The General Case I

- Following Cunha, Heckman and Navarro (2005) I cast the problem of determining agent information sets as a testing problem.
- Propose information set, $\tilde{\mathscr{I}}_{i,t}$: it follows that

$$\ln \hat{C}_{i,s,t} = \ln \mathscr{C}_{s,t}\left(\tilde{\mathscr{I}}_{i,t}\right) + K_{i,t}\delta + \xi_{i,t}.$$

- True for a whole class of models:

$$\ln \hat{C}_{i,s,t} = g\left(\tilde{\mathscr{I}}_{i,t}\right) + K_{i,t}\delta + \xi_{i,t}. \tag{8}$$

- Consumption will be a function of the information set, independent of the utility function or of the earnings equations

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The General Case II

- Let $\pi_{\tau,t}$ be an auxiliary parameter. Base the likelihood on

$$\ln \widehat{C}_{i,s,t} = g_{s,t}\left(\widetilde{\mathscr{I}}_{i,t}\right) + K_{i,t}\delta + \xi_{i,t} + \sum_{\tau=t+1}^{T}\left[Y_{s,\tau} - E\left(Y_{s,\tau}|\tilde{\mathscr{I}}_{i,t}\right)\right]\pi_{\tau,t}.$$

- Predicted consumption $g_{s,t}\left(\tilde{\mathscr{I}}_{i,t}\right)$ should not depend on the innovations.
- Test which $\{\pi_{\tau,t}\}_{\tau=t+1}^{T}$ equals zero: test whether the proposed agent's information set at time t is correctly specified. (Sims Causality!)
- How to implement it?

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Factor Models

- Assume

$$U_{i,t} = \theta_i \alpha_t + \varepsilon_{i,t} \quad (9)$$

- θ_i: vector of mean zero factors, $\varepsilon_{i,s,t}$ and ω_i: mean zero uniquenesses.
- Uniquenesses, factors and measurement error for consumption all assumed mutually independent of each other for all time periods t.
- Interpret elements of θ_i as "permanent" shocks that hit and influence earnings at different points in time.

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Assumptions (via example)

$$\ln Y_{i,1} = \mu(X_{i,1}) + \theta_{i,1}\alpha_{i,1,1} + \varepsilon_{i,1}$$

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- $\theta_{i,1}$ known at period 1 since it affects earnings. Is $\theta_{i,2}$ known in period 1?

(I-1) The information revelation process of the agent is such that he either knows element l of $\theta_i - \theta_{i,l}$ $(l = 1, \ldots L)$ – or he does not.
(I-2) At period t, the agent observes his outcomes for the period and so he knows $\{\varepsilon_{i,\tau}\}_{\tau=1}^{t}$ and the elements of $\theta_i(t)$
(I-3) Agents have rational expectations

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(I-2) At period t, the agent observes his outcomes for the period and so he knows $\{\varepsilon_{i,\tau}\}_{\tau=1}^{t}$ and the elements of $\theta_i(t)$
(I-3) Agents have rational expectations

Implementing the Test

- Only source of earnings innovations "knowable" to the agent is given by $\bar{\theta}_i(t)$ (i.e. the factors that affect earnings only in periods after t).

- Assuming agents do not know the different elements of θ_i until they hit earnings.

- Estimate using a candidate information set $\widetilde{\mathscr{I}}_{i,t}$ that contains no elements of $\overline{\theta}_i(t)$ before time t. Basing the likelihood on

$$\ln \widehat{C}_{i,t} = g_t\left(\widetilde{\mathscr{I}}_{i,t}\right) + K_{i,t}\delta + \xi_{i,t} + \overline{\theta}_i(t)\pi_t.$$

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Data Requirements

- A couple of things to notice: we need lifecycle data. NLSY79 doesn't have a full lifecycle for the earnings process (merge with PSID)
- More importantly, we need the choices in order to figure out how much of the future they know
- NLSY97 probably have no hope since
- Made it about consumption, but any choice will do.
- Can make loadings depend on variables
- How to test for asymmetries in information?

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Finally, add schooling

- $Y_{i,s,t}$: individual $i's$ income in schooling level s at time t

$$V_{i,s,t}(\mathscr{I}_{i,t}) = max_{A_{i,t}} u(C_{i,t}) + \frac{1}{1+\rho} E(V_{i,s,t+1}(\mathscr{I}_{i,t+1}) \mid \mathscr{I}_{i,t}) \tag{10}$$

$$s.t. \; C_{i,t} = Y_{i,s,t} + W_{i,s,t} + (1+r) A_{i,t-1} - A_{i,t}, \; A_{i,0}, \tag{11}$$

$$A_{i,T} \geq 0. \tag{12}$$

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Schooling decision

- At $t = 0$ the agent attends college if

Schooling

$$E\left(\mathscr{V}_{c,1}\left(\mathscr{I}_{i,1}\right) - \mathscr{V}_{h,1}\left(\mathscr{I}_{i,1}\right) - Cost_i \mid \mathscr{I}_{i,0}\right) > 0. \qquad (13)$$

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- Earnings, $X_{i,s,t}$ observable and $U_{i,s,t}$ unobservable:

$$lnY_{i,s,t} = \mu_{s,t}(X_{i,s,t}) + U_{i,s,t} \quad (14)$$

- Agent knows X. $U_{i,s,t}$ is revealed to him at period t.
- May also know all or part of each $(U_{i,s,\tau}, \tau = t+1, ..., T)$ at time t. Uncertainty is thus associated with $\{U_{i,s,\tau}\}_{\tau=t+1}^{T}$.
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$$u(C) = \frac{C^{1-\psi}}{1-\psi}, \tag{16}$$

$$\tilde{u}(A_{i,s,t}) = b\frac{(\varepsilon A_{i,s,T})^{1-\chi}}{1-\chi} \tag{17}$$

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Data and Parametrization I

- Sample of white males who either graduated high school or are college graduates individuals from both NLSY79 and PSID: 2,986 white males born between 1923 and 1975

- $\theta_{i,1}$: Ability since I use five components from the ASVAB battery of tests modeled as a function of only the first factor

$$M_{i,j} = \beta_{0,j} + X_i^M \beta^M + \theta_{i,1} \alpha_j^M + \varepsilon_{i,j}^M.$$

- Individual earnings life cycles are then simplified to six 8 year long periods:

$$lnY_{i,s,t} = X_i \beta_{s,t} + \theta_{i,1} \alpha_{s,t,1} + \theta_{i,2} \alpha_{s,t,2} + \theta_{i,3} \alpha_{s,t,3} + \varepsilon_{i,s,t}.$$

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Data and Parametrization II

- Cost function allowed to be a function of all three factors:

$$Cost_i = Z_i\gamma + \theta_{i,1}\lambda_1 + \theta_{i,2}\lambda_2 + \theta_{i,3}\lambda_3 + \omega_i.$$

- $\theta_{i,l} \sim \sum_{j=1}^{J_l} \pi_{l,j} f\left(\theta_{i,l}; \mu_{l,j}, \sigma_{l,j}^2\right)$. $\varepsilon_{i,s,t}$ and $\xi_{i,s,t}$ are also allowed to be distributed as mixtures of normals.
- In any t, $A_{i,s,t}^{MIN}$ is set equal to the lowest level of assets observed in t. Automatically defines the $Y_{i,s,t}^{MIN}$.
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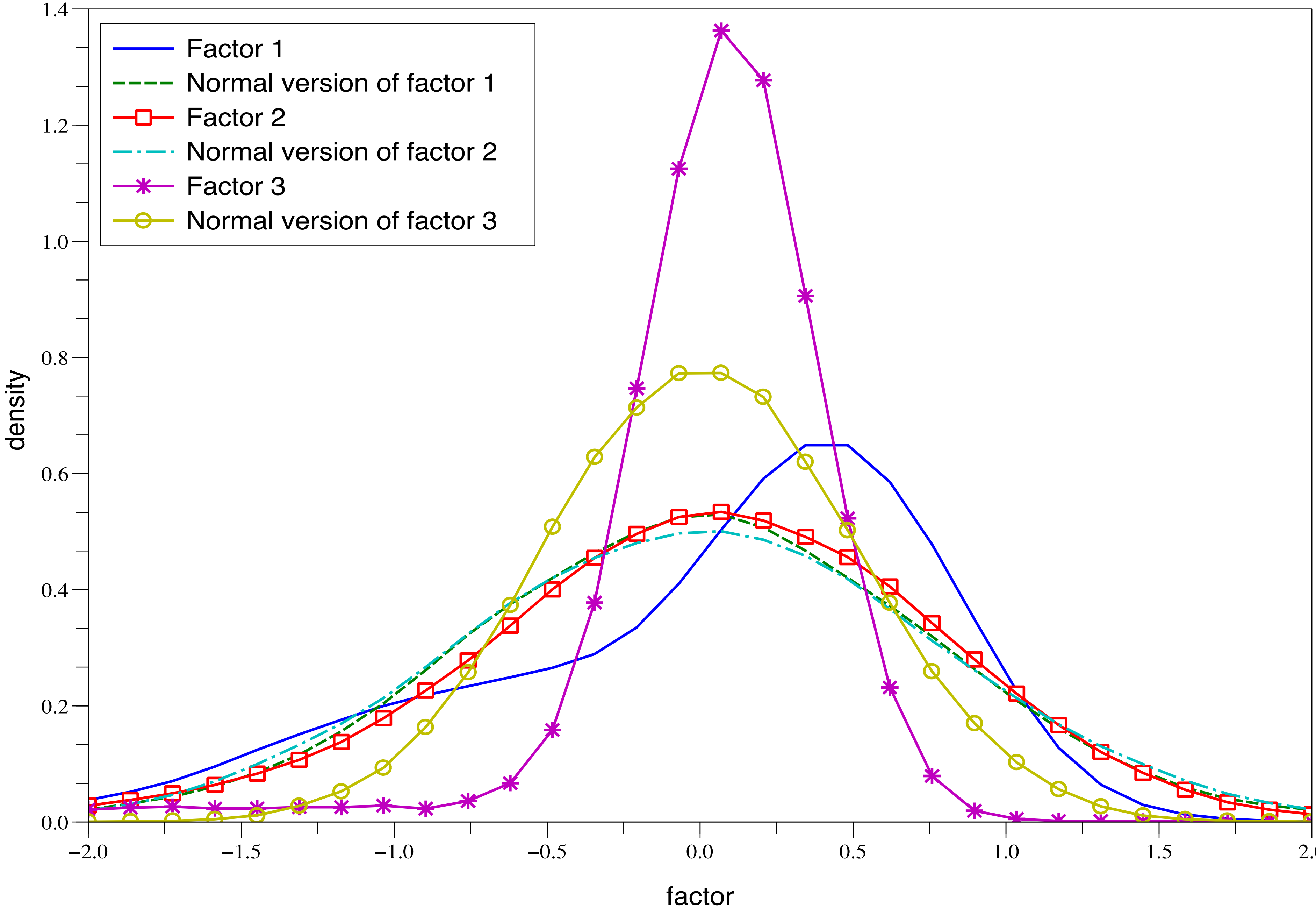
Likelihood

- The contribution of individual i who chooses schooling $S_i = s$

$$\int_{\Theta}\left[\begin{array}{c}\prod_{t=1}^{6} f_{\varepsilon_{i,s,t}}\left(\ln Y_{i,s,t} - \mu_{s,t}\left(X_{i,s,t}\right) - \theta_i \alpha_{s,t} | \theta_i, X_{i,s,t}\right) \\ f_{\varepsilon_{i,j}^{M}}\left(M_{i,j} - \mu_j^M\left(X_{i,j}^M\right) - \theta_i \alpha_j^M | \theta_i, X_{i,j}^M\right) \\ \prod_{t=1}^{6} f_{\xi_{i,t}}\left(\ln \widehat{C}_{i,t} - \ln \mathscr{C}_{s,t}\left(\mathscr{I}_{i,t}\right) - K_{i,t}\delta | \theta_i, X_{i,s,t}, Z_i, K_{i,t}\right) \\ \Pr\left(S_i = s | X_{i,s,t}, Z_i, \theta_i\right)\end{array}\right] dF\left(\theta\right).$$

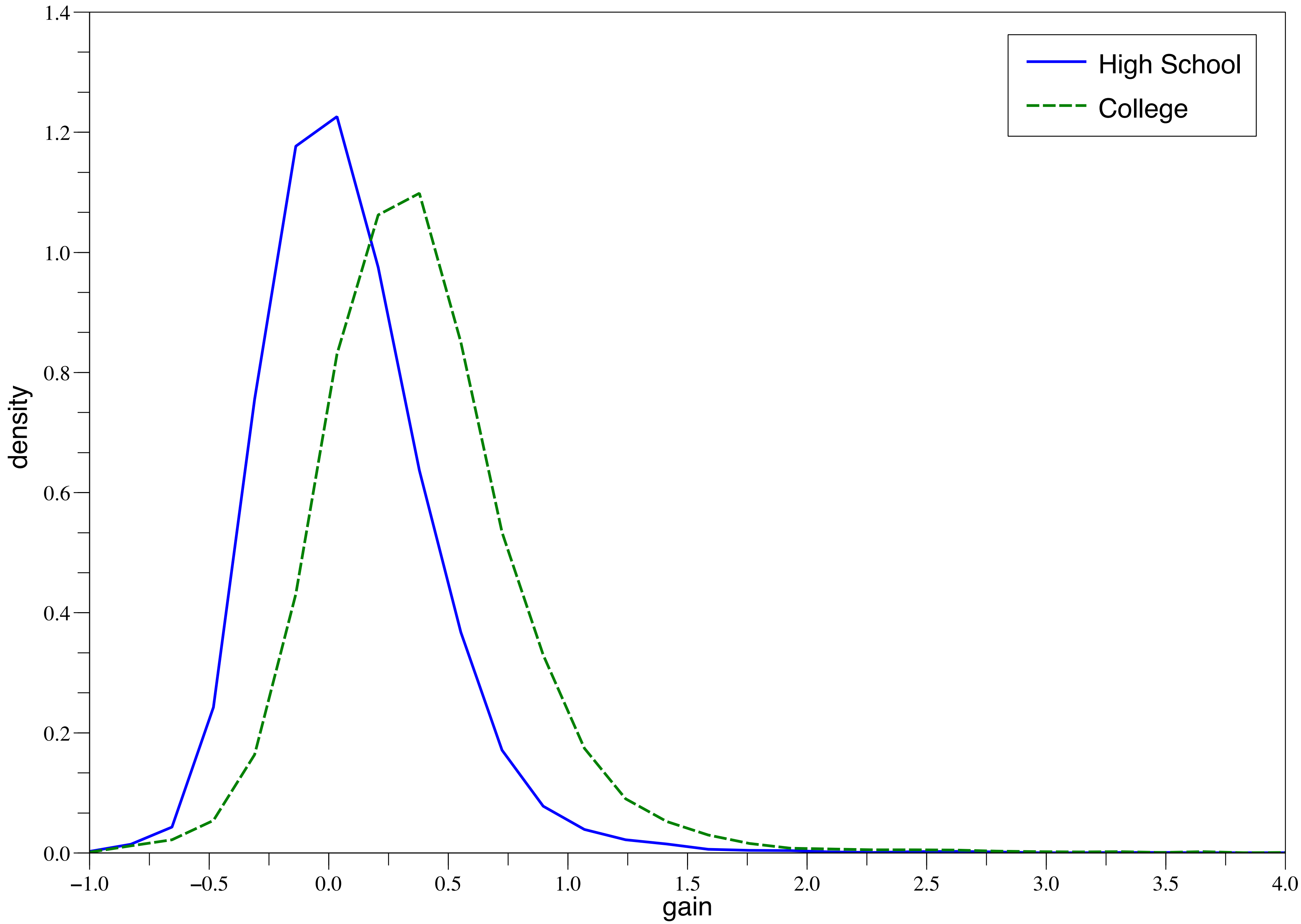
Figure 2

Density of factors and their normal equivalents



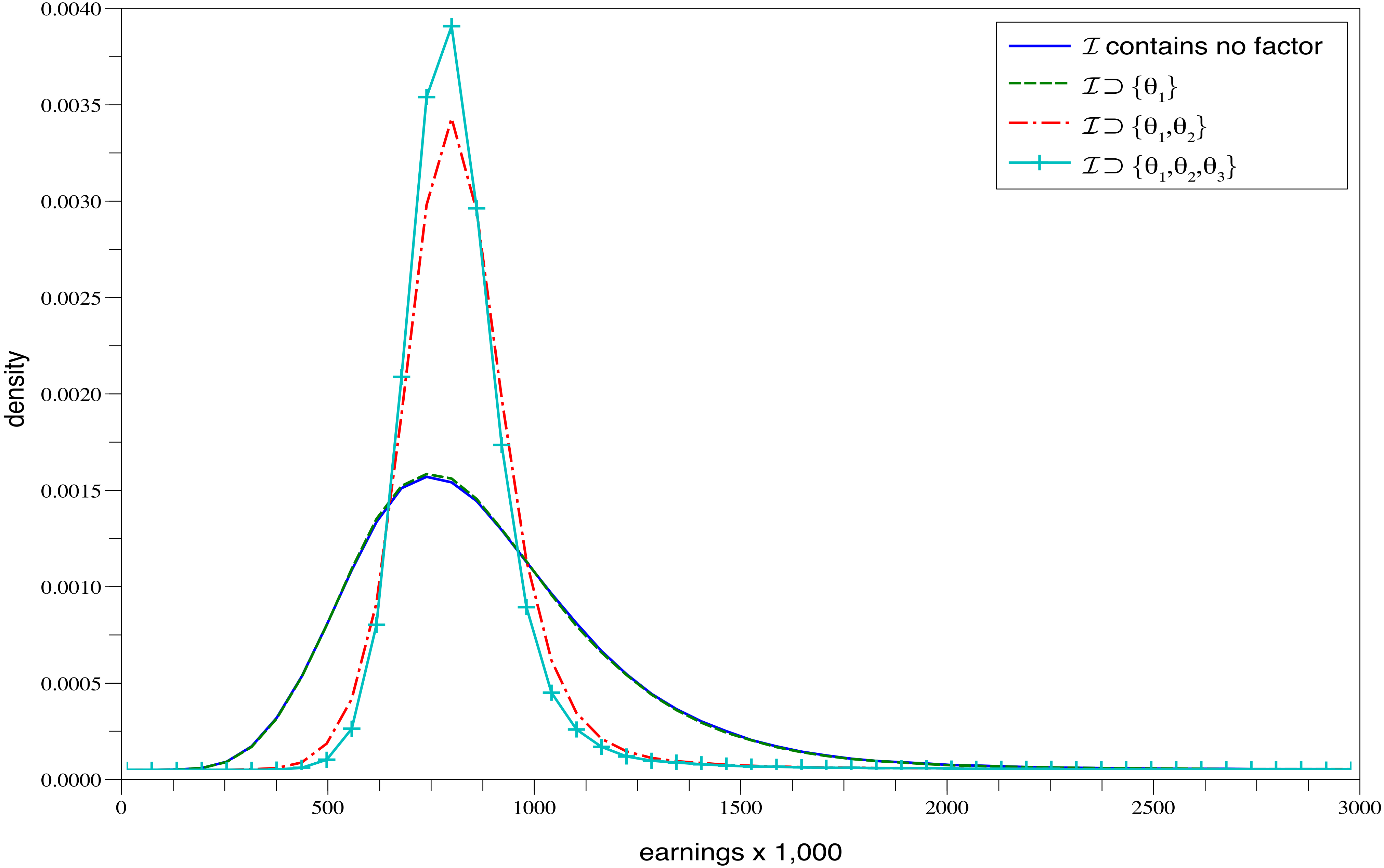
Let $f(\theta_1)$ denote the density of factor θ_1. $f(\theta_1)$ is allowed to be a mixture of normals. Let $m_1=E(\theta_1)$ and $v_1=Var(\theta_1)$. The solid line is the actual density of factor 1 while the dashed line is the density of a normal random variable with mean m_1 and variance v_1. We proceed similarly for the other factors. The plots are smoothed using a Gaussian kernel.

Figure 3.3
Density of realized monetary gains to college conditional on schooling choice



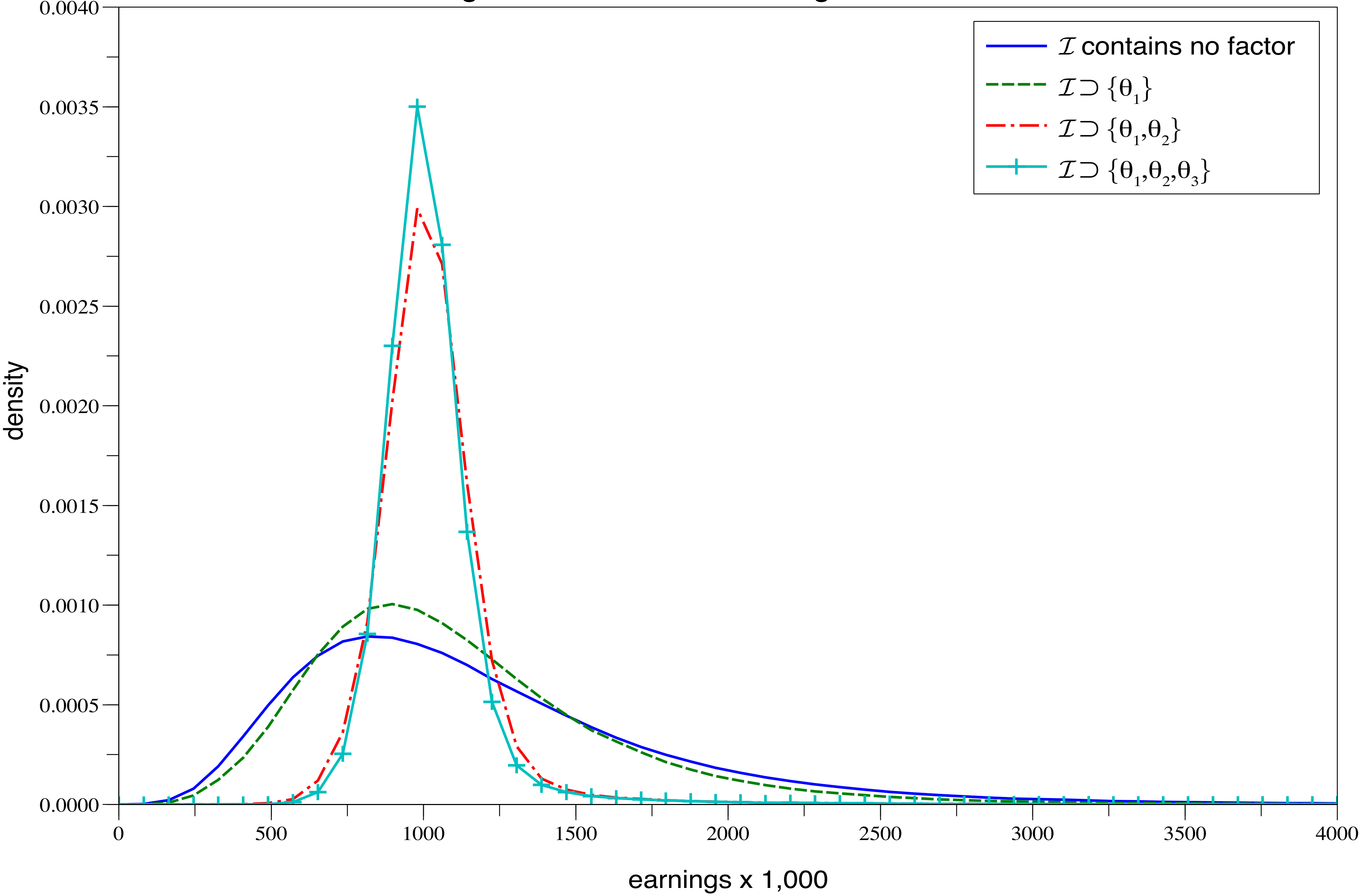
Let (Y_0,Y_1) denote the potential present value of earnings from age 18 to 65 (discounted using an interest rate of 3%) in the high school and college sectors, respectively. Define the realized monetary gain to college as $G=(Y_1-Y_0)/Y_0$. The solid line is the density of gains for people who choose high school, that is, f(g|choice=high school) and the dashed line f(g|choice=college). The plots are smoothed using a Gaussian kernel.

Figure 4.1
Densities of present value of high school earnings under different information sets for the agent at the time schooling decisions are made



Let Y_0 denote the present value of high school earnings from age 18 to 65 discounted at a 3% interest rate
Let $\mathcal{I}$ denote the agent's information set and $f(y_0|\mathcal{I})$ denote the density of the present value of earnings conditional on the information set $\mathcal{I}$. The graph shows $f(y_0|\mathcal{I})$ with $\mathcal{I}$ containing no factors, factor 1, factors 1 and 2, and all three factors. The plots are smoothed using a Gaussian kernel.

Figure 4.2
Densities of present value of college earnings under different information sets for the agent at the time schooling decisions are made



Let Y_1 denote the present value of high school earnings from age 18 to 65 discounted at a 3% interest rate
Let $\mathcal{I}$ denote the agent s information set and $f(y_1|\mathcal{I})$ denote the density of the present value of earnings conditional on the information set $\mathcal{I}$. The graph shows $f(y_1|\mathcal{I})$ with $\mathcal{I}$ containing no factors, factor 1, factors 1 and 2, and all three factors. The plots are smoothed using a Gaussian kernel.

Table 5.1

Agent's forecast* of the variance of the present value of earnings

Under different information sets at schooling choice date

		$Var(Y_H)$	$Var(Y_C)$	$Var(Y_C\text{-}Y_H)$
	Variance with $\mathcal{I}=\varnothing$	349205	402710	459648
$\mathcal{I}_1=\theta_1$	Variance	346736	285190	372504
	Fraction of the variance** with $\mathcal{I}=\varnothing$ explained by $\mathcal{I}_1$	0.71%	29.18%	18.96%
$\mathcal{I}_2=\{\theta_1,\theta_2\}$	Variance	193100	76074	260178
	Fraction of the variance with $\mathcal{I}=\varnothing$ explained by $\mathcal{I}_2$	44.70%	81.11%	43.40%
$\mathcal{I}_3=\{\theta_1,\theta_2,\theta_3\}$	Variance	187993	71090	258965
	Fraction of the variance with $\mathcal{I}=\varnothing$ explained by $\mathcal{I}_3$	46.17%	82.35%	43.66%

*Variance of the unpredictable component of earnings from age 18-65 as predicted at age 18.

**The variance of the unpredictable component of high school earnings with $\mathcal{I}_1=\theta_1$ is (1-0.007)*349205=325916

Table 5.2

Proportion of people who, after observing their realized outcomes (keeping credit constraints in place), regret their choice

Choice under Uncertainty	Choice under Certainty	
	High School Choose HS: 52.3%	College Choose Col: 47.7%
High School Choose HS: 51.1%	87.02%	12.98%
College Choose Col: 48.9%	16.03%	83.97%

Table 5.3

Average Annual Ex-post Gains and Equivalent Variations with and without Uncertainty (keeping credit constraints in place)

	Choice under Uncertainty		**Choice under Certainty**	
	High School	College	High School	College
Ex-post Gain[1]	2.15%	9.70%	0.48%	11.85%
Equivalent Variation[2]	-8.60%	15.20%	-9.95%	18.57%

[1]Let Y_1 be the present value of earnings in college and Y_0 in high school. The lifetime ex-post gain is defined as $G=(Y_1-Y_0)/Y_0$. The annual ex-post gain is simply G/4.

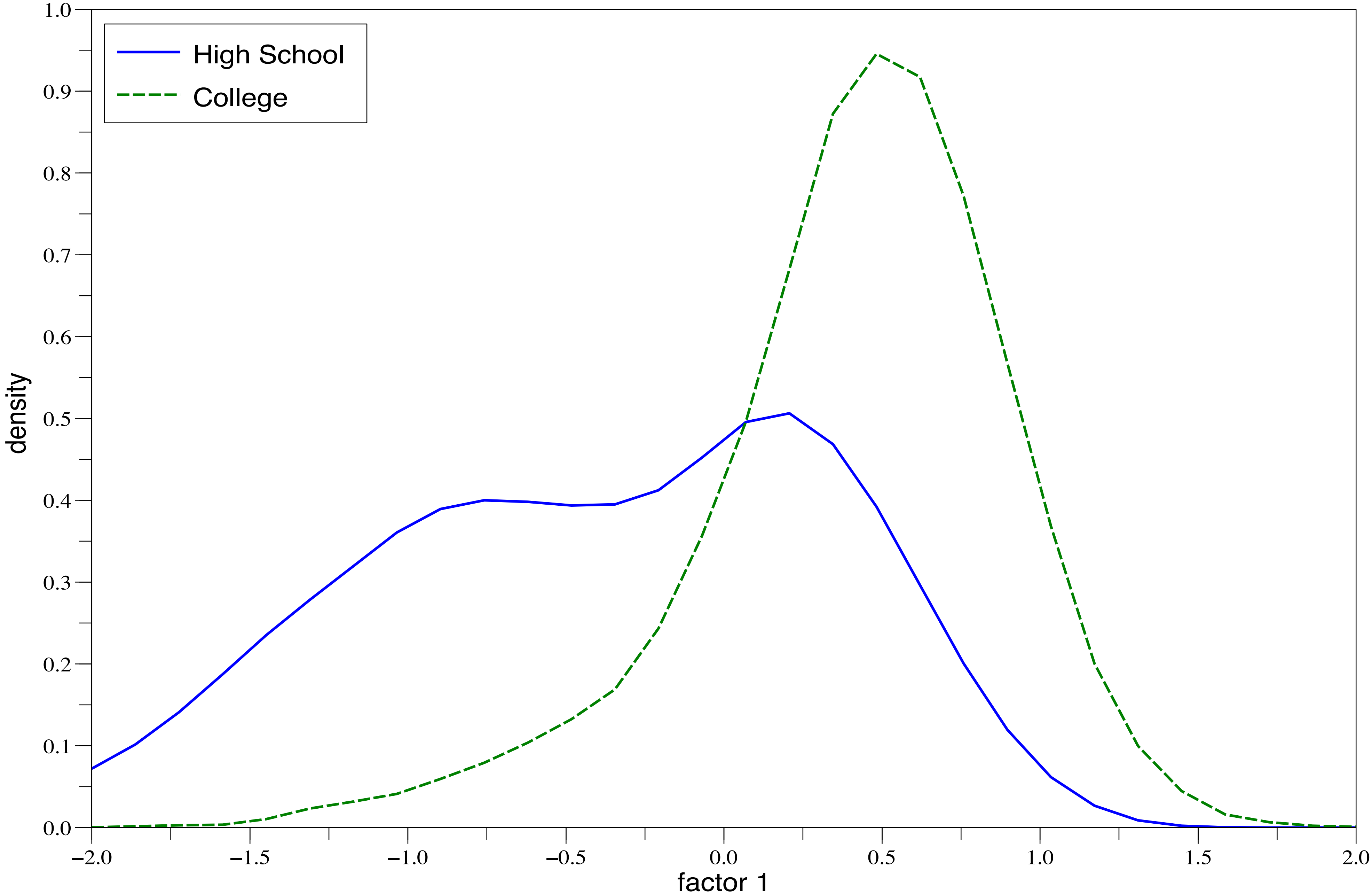
[2]The lifetime equivalent variation is defined as the proportion by which consumption (in each period) in high school needs to be changed so that the individual is indifferent between choosing high school and college. The annual equivalent variation is the lifetime equivalent variation divided by 4.

Table 7

Percentage of people who choose college under different scenarios

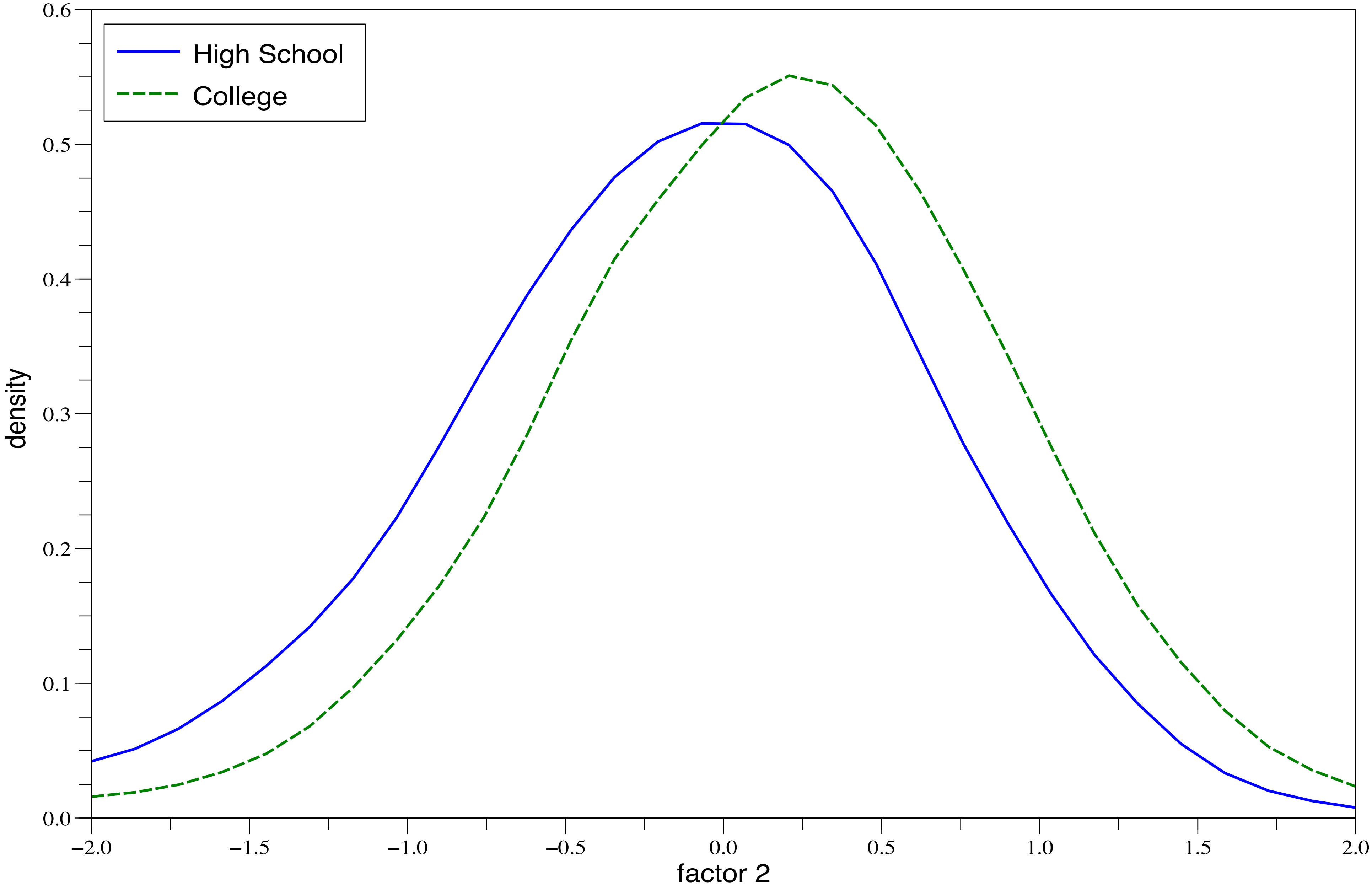
Scenario	*Overall*
Original Economy	48.93%
Zero Tuition Economy	50.48%
Certainty with Credit Constraints	47.75%
Certainty without Credit Constraints	57.40%

Figure 5.1
Density of ability (factor 1) conditional on schooling choice



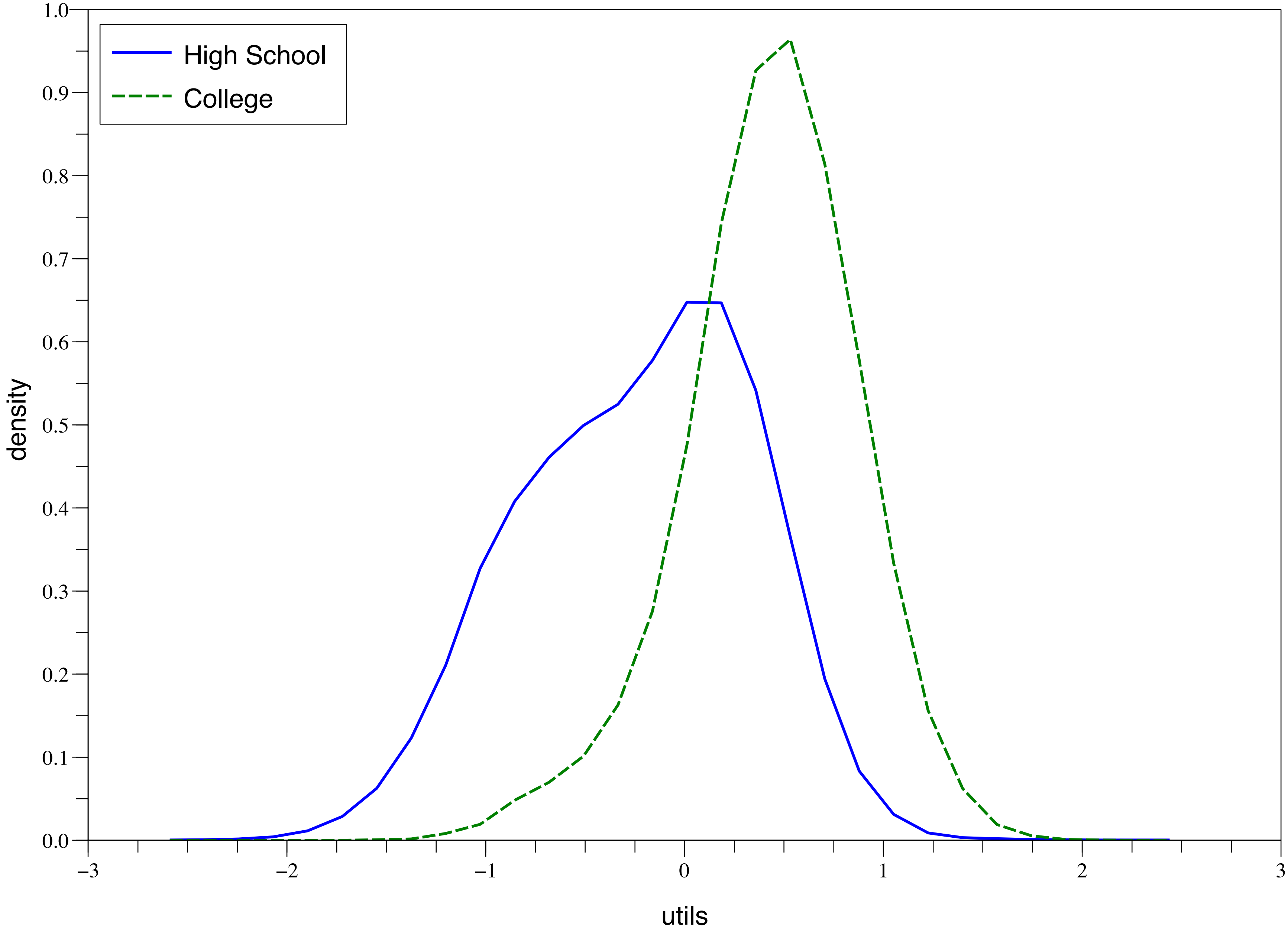
Let $f(\theta_1)$ denote the density function of factor θ_1. $f(\theta_1)$ is allowed to be a mixture of normals. The solid line plots the density of the factor conditional on choosing high school, that is, $f(\theta_1|\text{choice=high school})$. The dashed line plots the density of the factor conditional on choosing college, that is, $f(\theta_1|\text{choice=college})$. The plots are smoothed using a Gaussian kernel.

Figure 5.2
Density of factor 2 conditional on schooling choice



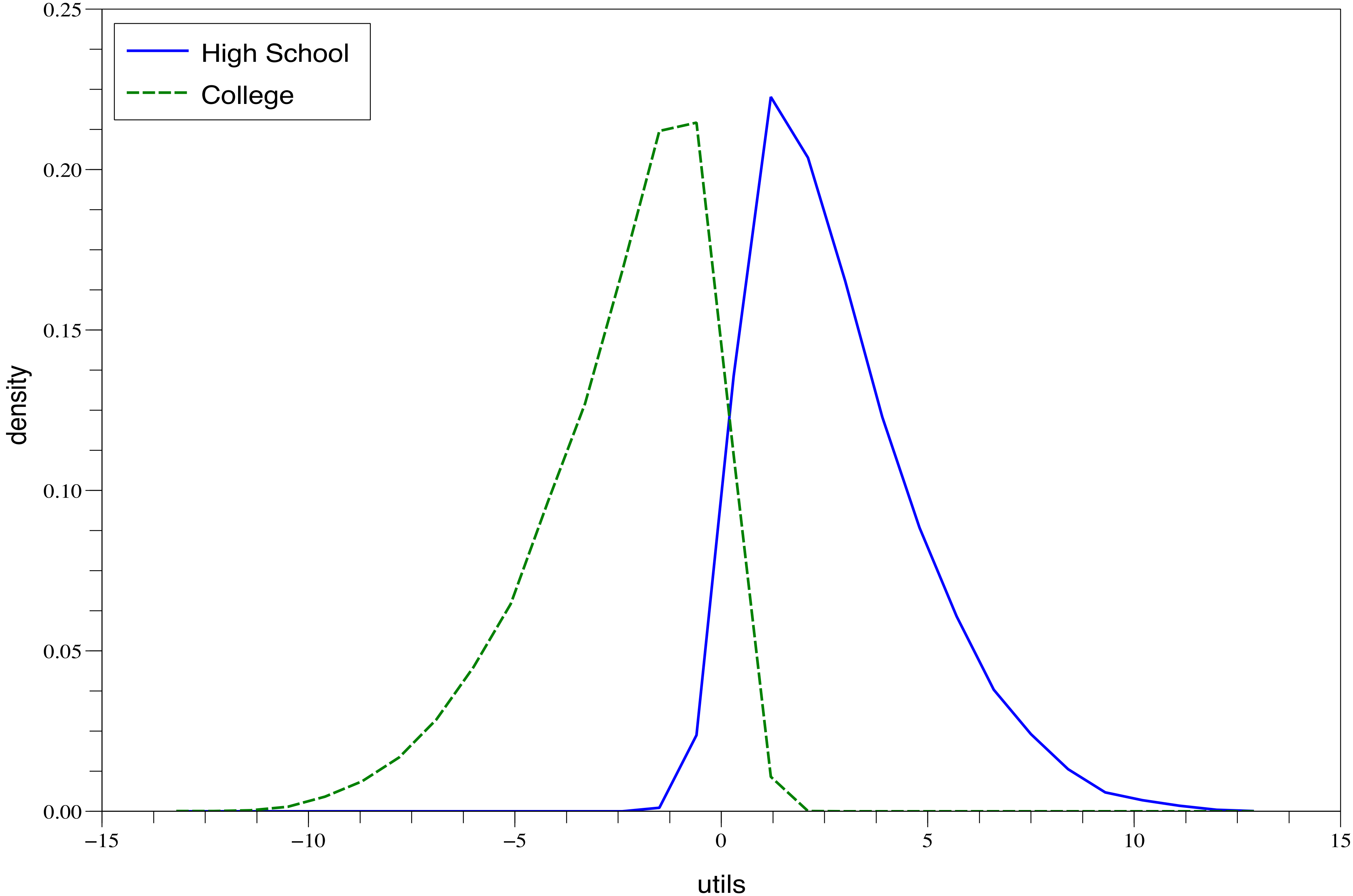
Let $f(\theta_2)$ denote the density function of factor θ_2. $f(\theta_2)$ is allowed to be a mixture of normals. The solid line plots the density of the factor conditional on choosing high school, that is, $f(\theta_2|\text{choice=high school})$. The dashed line plots the density of the factor conditional on choosing college, that is, $f(\theta_2|\text{choice=college})$. The plots are smoothed using a Gaussian kernel.

Figure 6.1
Density of expected gross utility differences conditional on choice



Let $V_{h,1}$ and $V_{c,1}$ denote the value functions for high school and college at period 1. Define the ex−ante gross utility difference, $D=E(V_{c,1}-V_{h,1}|\mathcal{I}_0)$ where the expectation is taken with respect to the information available at period 0. The solid line shows the density of D for agents who choose high school (i.e. f(D|choice=high school)) and the dashed line shows the density of D for agents who choose college (f(D|choice=college). The plots are smoothed using a Gaussian kernel.

Figure 6.2
Density of psychic costs conditional on schooling choice



Let C denote the individual psychic costs associated with attending college. The solid line plots the density of the costs conditional on choosing high school, that is, f(C|choice=high school). The dashed line plots the density of costs conditional on choosing college, that is, f(C|choice=college). The plots are smoothed using a Gaussian kernel.

Open Questions

- How to effectively impose limited commitment?
- How do we impose information asymmetries? How do we test for them?
- How to do this in a sensible matter when we don't have the "right" data?
- Related, how to look at this in recent cohorts?
- Other restrictions possible (i.e. only aggregate restrictions?)
- How to figure out what the right model is? One can do pure statistical model selection but is there a more "economic" way of doing it?
- For policy purposes, can one do model averaging or robustness effectively for example?

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