Credit and Insurance for Investments in Human Capital

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1 Disclaimer

The views expressed here are those of the author and do not necessarily reflect those of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

2 Introduction

Extensive literature on credit constraints and human capital:

Micro:

structural and IV estimates.

Macro:

social mobility, inequality;

cross-country differences.

Most papers use simple, ad-hoc models of credit constraints.

This paper: Simplest Human Capital Investment Model

Credit limitations and lack of insurance

distorted investments and consumption; lower welfare.

Incentive problems

limited commitment (complete and incomplete markets)

moral hazard (in school, in labor markets)

costly state verification

Main message:

Useful to move beyond standard notions of credit constraints and look at endogenous constraints on financing.

cross section variation; response to economy-wide changes.

default highlights the importance of insurance and incentives.

Ongoing/Future work:

Integrate endogenous labor market risk (e.g. unemployment, disability) with optimal investment in human capital.

3 Government Student Loans and Limited Commitment

GSL programs

lending is directly tied to investment.

upper loan limits

extended enforcement vis-a-vis private loans.

$$d_g \leq \min\left\{\tau h, \bar{d}\right\}. \tag{1}$$

Private Lending

punishment for default (credit bureaus, costly avoidance actions)

foreseen by rational lenders

A fraction $0 < \tilde{\kappa} < 1$

$$d_p \leq \tilde{\kappa} R^{-1} a f(h). \tag{2}$$

Overall credit:

Simple two-period model:

$$d = d_g + d_p \leq \min\left\{h, \bar{d}\right\} + \tilde{\kappa} R^{-1} a f\left(h\right). \tag{3}$$

Life-cycle model

$$d_p \leq \kappa_1 \Phi a h^{\alpha} + \kappa_2 d_g, \quad 0 \leq \kappa_1 \leq 1 \ \& \ \kappa_2 > -1. \tag{4}$$

Empirical Implications.

(1) Schooling is strongly positively correlated with ability over time.

(2) The correlation between schooling and family income (conditional on ability and family background) has grown since the early 1980s.

(3) There has been a sharp increase in the fraction of undergraduates borrowing the maximum amount from GSL programs since the 1990

(4) There has been a dramatic rise in student borrowing from private lenders since the mid-1990s.

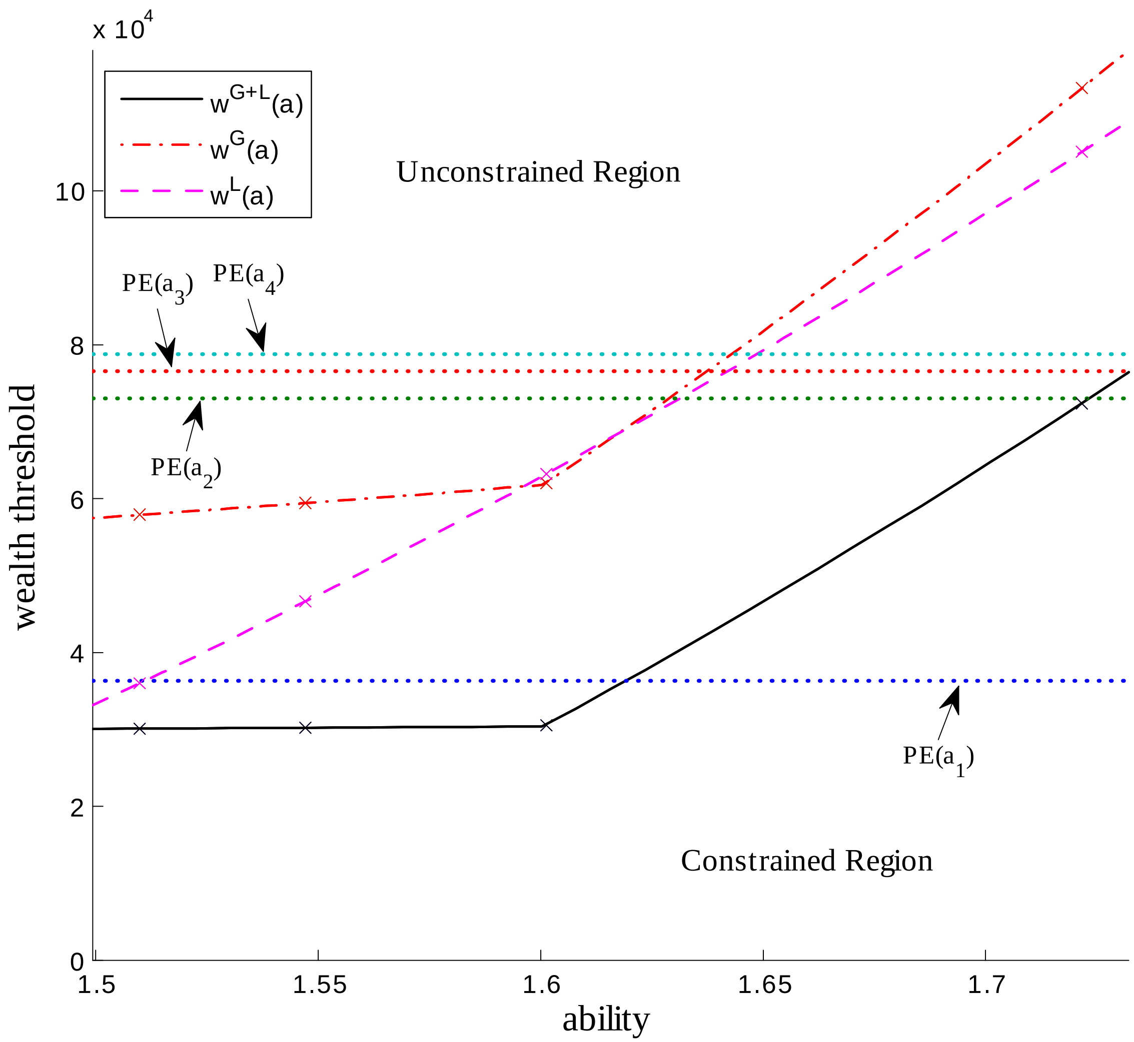
A Rise in the Costs of and Returns to Schooling

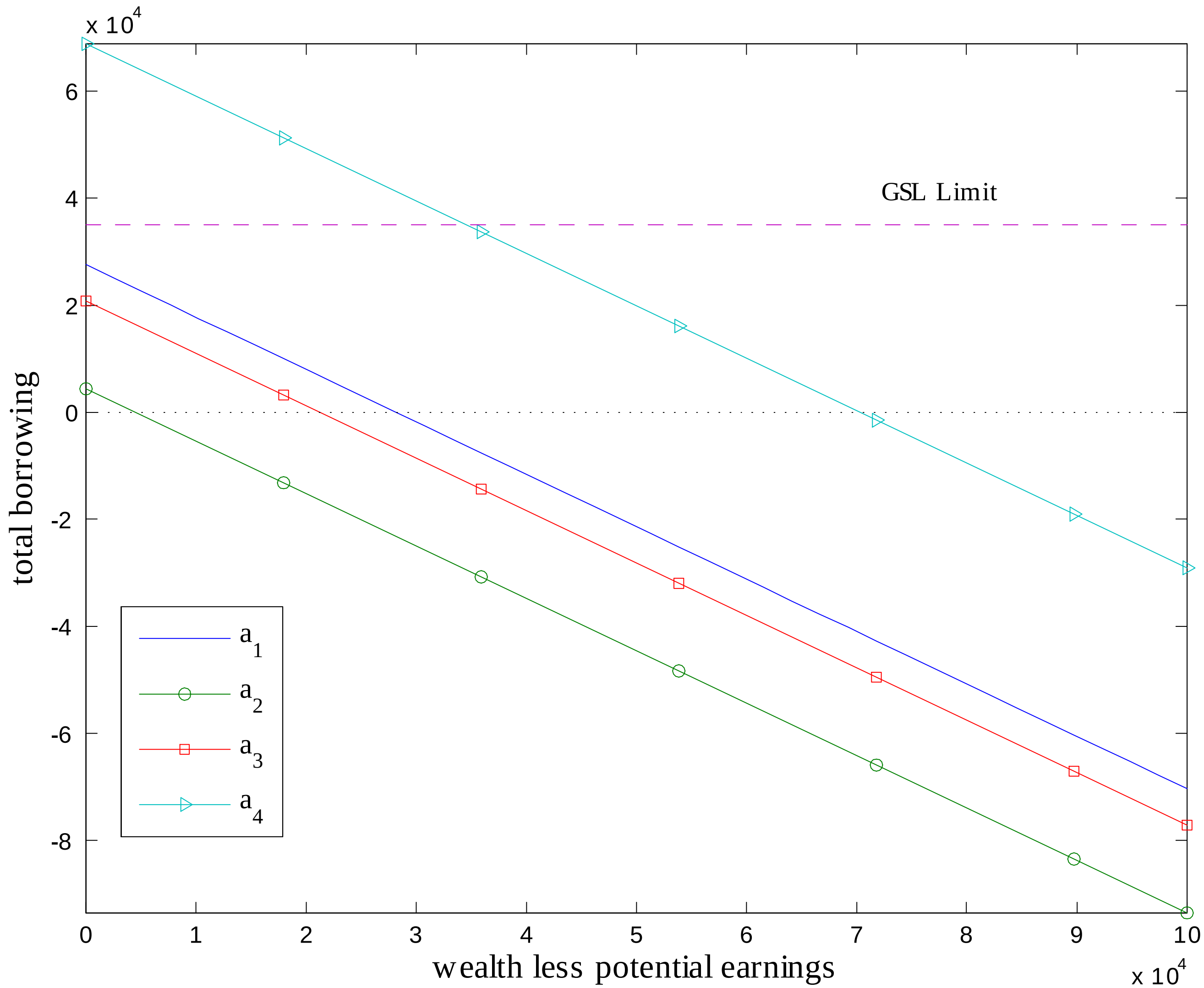
Aim: compare the U.S. economy in 1980s vs. 2000s.

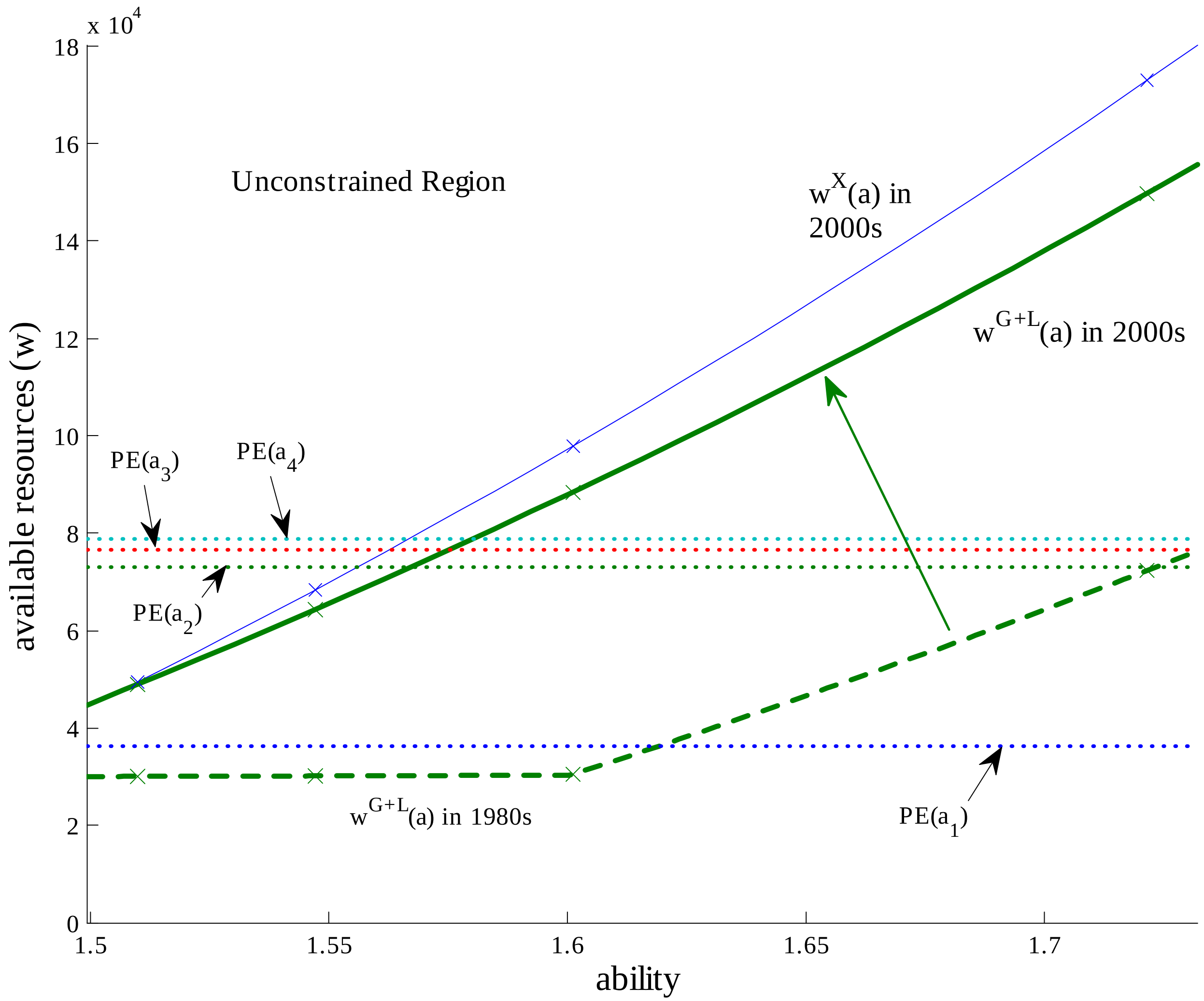
increase the cost of investment and the returns to human capital

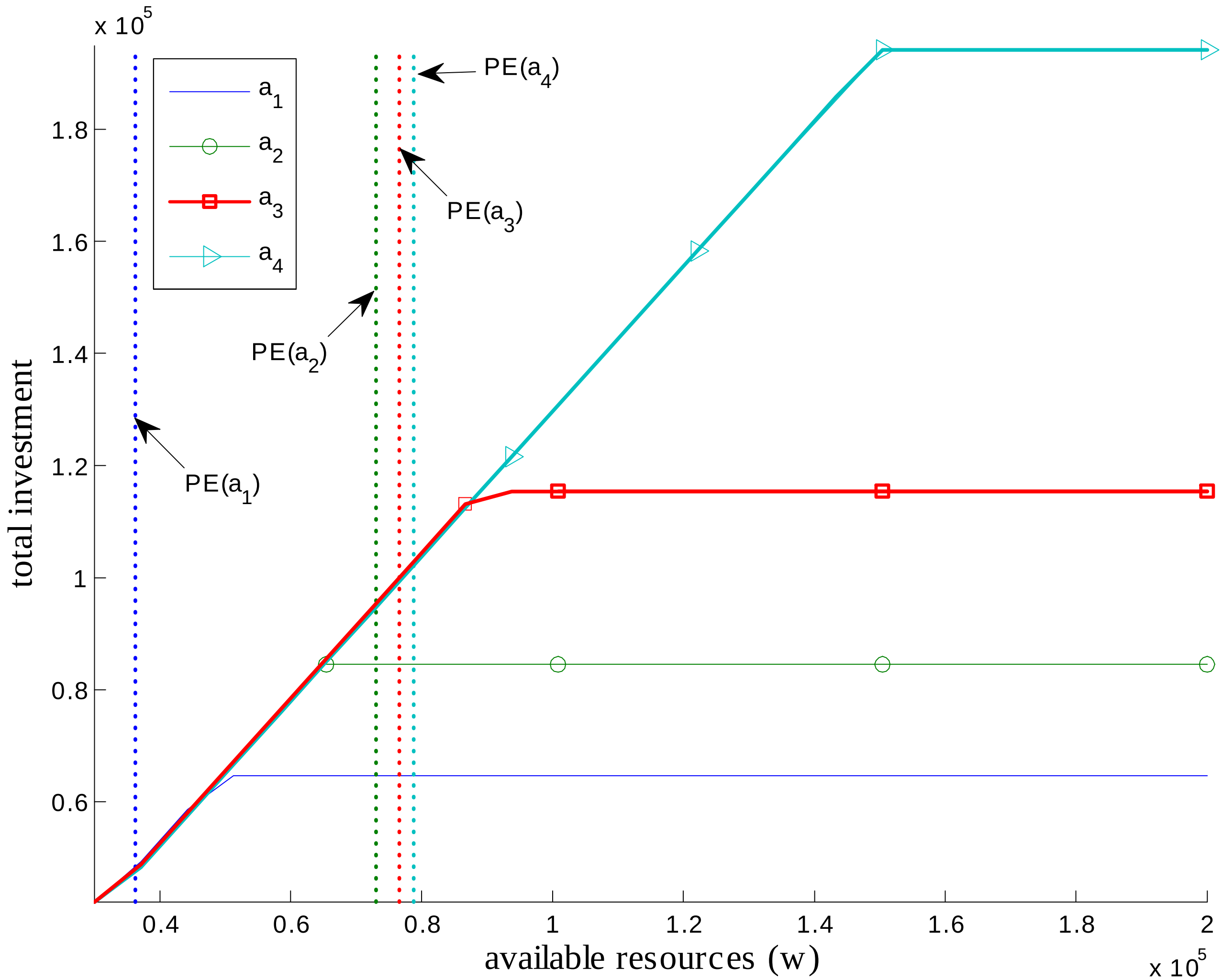
keep constant GSL borrowing limits

keep constant private enforcement


x 10^4
wealth threshold
ability
w^{G+L}(a)
w^G(a)
w^L(a)
Unconstrained Region
Constrained Region
PE(a_3)
PE(a_4)
PE(a_2)
PE(a_1)
0
2
4
6
8
10
1.5
1.55
1.6
1.65
1.7


x 10^4
6
4
2
0
-2
-4
-6
-8
total borrowing
GSL Limit
a1
a2
a3
a4
0
1
2
3
4
5
6
7
8
9
10
wealth less potential earnings
x 10^4


x 10^4
18
16
14
12
10
8
6
4
2
0
available resources (w)
Unconstrained Region
w^X(a) in 2000s
w^{G+L}(a) in 2000s
PE(a_3)
PE(a_4)
PE(a_2)
PE(a_1)
w^{G+L}(a) in 1980s
1.5
1.55
1.6
1.65
1.7
ability



x 10^5
a_1
a_2
a_3
a_4
PE(a_1)
PE(a_2)
PE(a_3)
PE(a_4)
total investment
available resources (w)
x 10^5
0.6
0.8
1
1.2
1.4
1.6
1.8
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Comparison with Exogenous Constraints Model

calibration as in the 1980s

comparable credit limits in both economies

4 Uncertainty, Default and other Incentives

A Simple Environment

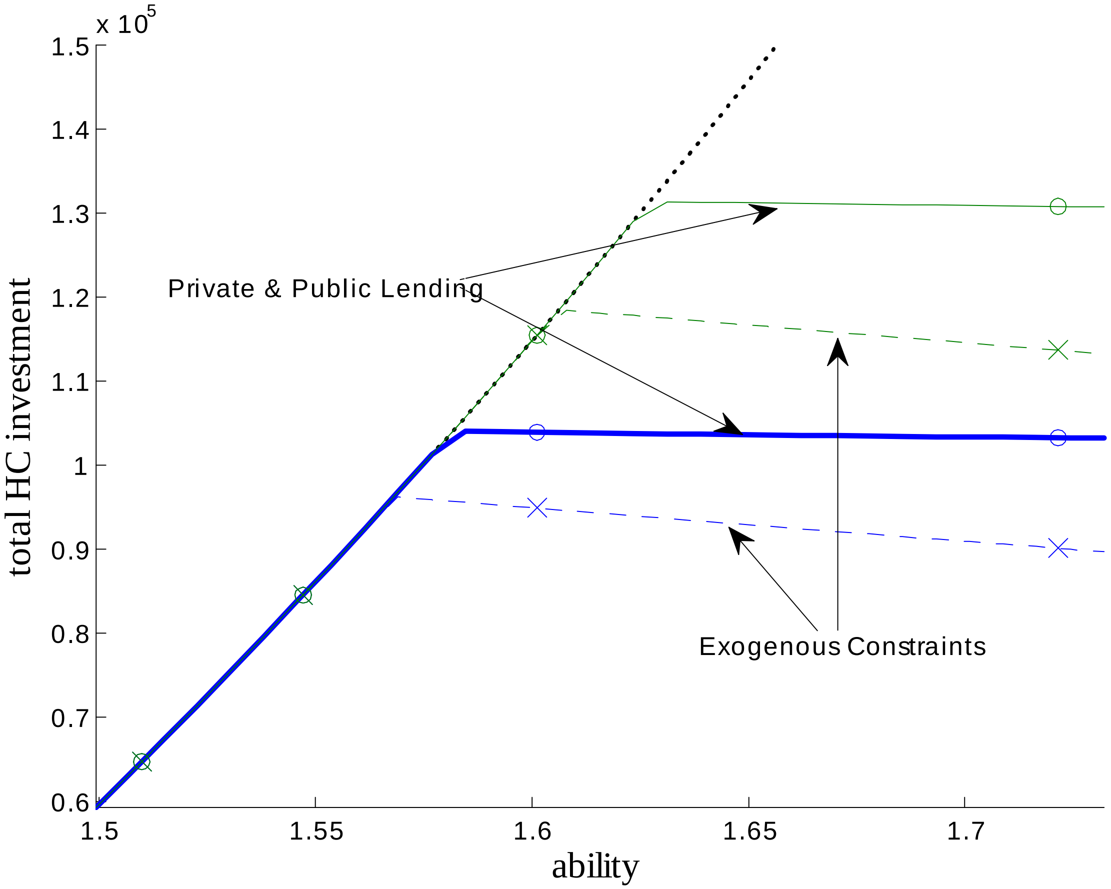
stochastic second period price of human capital

$i = 1, ..., N$, realizations, p_i =prob. $w_{1,i}$; $w_{1,1} < w_{1,2} < ... < w_{1,N}$.

preferences

$$U = u(c_0) + \beta \sum_{i=1}^{N} p_i u\left(c_{1,i}\right),$$

where $c_{1,i}$ is the second period consumption in realization i.



x 10^5
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
total HC investment
1.5
1.55
1.6
1.65
1.7
ability
Private & Public Lending
Exogenous Constraints

D_i (possibly negative) debt in second period

$$c_0 = W - h + \sum_{i=1}^{N} q_i D_i,$$
$$c_{1,i} = af(h)\, w_{1,i} - D_i, \quad i = 1, ..., N.$$

Arrow prices: $q_i = \beta p_i$.

Unrestricted optima. Let $\bar{w}_1 \equiv \sum_{i=1}^{N} p_i w_{1,i}$ is the expected period 1 price of skill.

$$\bar{w}_1 a f' \left[h^U(a)\right] = \beta^{-1},$$

neither preferences nor initial wealth W are factors.

$$u'(c_0) = u'\left(c_{1,i}\right), \text{ for all } i = 1, ..., N.$$

Limited Commitment with Complete Markets.

Participation constraints: $u\left[w_{1,i}af(h)-D_i\right]\geq V^D(w_{1i},a,h)$. This limits the set of assets/debts individuals can hold as well as their ability to insure against some future states.

Letting $\lambda_i\geq 0$ denote the LM. (discounted) multipliers

$$u'(c_0)=(1+\lambda_i)\,u'\left(c_{1,i}\right).$$

Assume: $V^D(w_{1i},a,h)=u\left[(1-\tilde{\kappa})\,w_{1i}af(h)\right]$.

Then: solvency constraints $D_i\leq\tilde{\kappa}w_{1,i}af(h)$ for all $i=1,...,N$.

Optimal human capital investment $h^{LC}(a, W)$ satisfies

$$\bar{w}_1 a f' \left[h^{LC}(a, W) \right] \left[\frac{\sum_{i=1}^{N} p_i w_{1,i} \left(\frac{1+\lambda_i \tilde{\kappa}}{1+\lambda_i} \right)}{\bar{w}_1} \right] = \beta^{-1}.$$

...underinvestment.

........but not really a model for default.

Limited commitment with incomplete markets.

Same temptation for default.

Non-contingent debt.

Threshold: $\tilde{w}_1(D, a, h) \equiv \frac{D}{\tilde{\kappa} a f(h)}$.

Probability of default, $\Pr\left[w_{1,i} < \tilde{w}_1(D, a, h)\right]$.

Credit:

$$Q(D,a,h)=\beta\left\{D-\sum_{w_{1,i}<\tilde{w}_1}p_i\left[D-\tilde{\kappa}w_{\mathbf{1},i}af\left(h\right)\right]\right\}.$$

A 'hard' borrowing constraint is given by $\sup_D\left\{Q\left(D,a,h\right)\right\}<\infty$.

Consumption: first order condition for D:

$$u'(c_0) = E[u'\left(c_{1,i}\right)|w_{1,i} \geq \tilde{w}_1].$$

Human capital: optimal h is

$$\bar{w}_1 a f'(h) \left[\frac{\sum_{i=1}^{N} p_i u'\left(c_{1,i}\right) w_{1,i} - \tilde{\kappa} \sum_{w_{1,i}<\tilde{w}_1} p_i u'\left(c_{1,i}\right) w_{1,i}}{\bar{w}_1 u'(c_0)(1-Q_h)} \right] = \beta^{-1},$$

where $0 < Q_h < 1$ is the partial derivative (subgradient) of Q wrt h.

Investment: (relative to full insurance)

investment expand credit, which encourages investment.

some benefits of investment lost when default, discourages investment.

a precautionary motive, which may or may not encourage investment.

Default:

can occur in equilibrium.

if it happens, it is for low realizations of $w_{1,i}$

serves an insurance.

may enhance investments.

Interest rates: $R(D, a, h) \equiv D/Q(D, a, h)$.

premium for default.

reduced by ability and investment, increased by debt. ($Q_a > 0$, $Q_h > 0$, $Q_{ah} > 0$; $Q_D < 1$).

Pros:

model of default and interest rates

provides a more interesting policy trade-off

Cons:

Conceptual: Exogenous incompleteness.

Applicability: Abstracts from potential interesting incentive problems.

Question:

Quantitative: Can the model explain observed data?

Moral Hazard (while investing)

Assume z is a continuous rv, support $Z = [0, \infty)$.

Earnings $w_1(z) = w_1 z$

Effort (while in school) is:

costly: if $e_0 < e_1$, then disutility $v(e_1) > v(e_0)$.

productive: $e_1 > e_0$, then $\Phi(z|e_1) \leq \Phi(z|e_0)$ (first order dominance).

For now: $e \in \{e_0, e_1\}$.

Optimal Contract

$$\max_{h,e,d,\{R(z)\}} u\left[W-h+d\right]-v(e)+\beta\int_Z u[w\left(z\right)af(h)-R\left(z\right)]\phi(z|e)dz$$

BEC of the lender (with the LM λ):

$$[\lambda]:-d+\beta\int_Z [R\left(z\right)\phi(z|e_H)dz\geq 0,$$

ICC (assuming e_H is optimal;LM $\mu\geq 0$)

$$\begin{aligned}[\mu] \quad : \quad & -v(e_H)+\beta\int_Z u[w\left(z\right)af(h)-R\left(z\right)]\phi(z|e_H)dz \\ \geq \quad & -v(e_L)+\beta\int_Z u[w\left(z\right)af(h)-R\left(z\right)]\phi(z|e_L)dz.\end{aligned}$$

FOCs:

$$
\begin{aligned}
u'\left[c_0\right] &= \left[1+\mu\left(1-\frac{\phi(z|e_L)}{\phi(z|e_H)}\right)\right]u'\left[c_1\left(z\right)\right], \\
u'\left[c_0\right] &= \beta a f'\left(h\right)w_1\{\int_0^\infty zu'\left[c_1\left(z\right)\right]\phi(z|e_H)dz \\
&\quad +\mu\left[\int_0^\infty zu'\left[c_1\left(z\right)\right]\left[\phi(z|e_H)-\phi(z|e_L)\right]dz\right]\}
\end{aligned}
$$

Key implication: As long as $e = e_H$ optimal investment is

$$\beta^{-1} = af'(h)\, w_1 \left[\int_0^{\infty} z\phi(z|e_H)dz\right].$$

i.e. as long as the first best effort is implemented, then the first best level of investment is also implemented.

Consumption is distorted.

Does it mean that investments are always first best? No. High effort may not be implementable for low W individuals.

For $e = e_L$ and $\mu = 0$ and

$$\begin{aligned} \text{full insurance} &: \quad u'[c_0] = u'[c_1(z)], \\ \text{underinvestment} &: \quad \beta^{-1} = af'(h)\, w_1 \left\{ \int_0^{\infty} z\phi(z|e_L)dz \right\}. \end{aligned}$$

Costly State Verification

Assume no incentive problems in inducing e_H.

However, cost ϑ to verify a borrowers outcome.

Verification: threshold $\bar{z}$

Verification ($z < \bar{z}$): Full insurance $c_1(z) = c_0$

$$R(z) = zw_1 af(h) - [W + d - h].$$

No verification ($z > \bar{z}$):, borrower repays $\bar{R}$.

Then,

$$\bar{R} = \bar{z} w_1 a f(h) - [W + d - h] \quad \text{or} \quad \bar{z} = \frac{\bar{R} + W + d - h}{w_1 a f(h)}.$$

The BEC for the lender:

$$\beta \left\{ \begin{array}{c} w_1 a f(h) \int_0^{\bar{z}} z \phi(z|e_H) dz - (W + d + \vartheta - h) \Phi(\bar{z}|e_H) \\ + \bar{R} \left[1 - \Phi(\bar{z}|e_H) \right] \end{array} \right\} \geq d.$$

The Lagrangean boils down to

$$
\begin{aligned}
L \;=\; & \max_{\{h,d,\bar{z}\}} \min_{\lambda} u\left[W-h+d\right]-v(e_H)+\beta u\left[W-h+d\right]F(\bar{z}|e) \\
& +\beta\int_{\bar{z}}^{\infty} u\left[\left(z-\bar{z}\right)w_1 af\left(h\right)+W+d-h\right]f(z|e)dz \\
& +\lambda\left[-d\left(1+\beta\right)+\beta\left\{\begin{array}{c} w_1 af\left(h\right)\left[\int_0^{\bar{z}} z\phi(z|e_H)dz+\bar{z}\left[1-\Phi(\bar{z}|e_H)\right]\right] \\ -\left[W-h\right]-\vartheta\Phi(\bar{z}|e_H) \end{array}\right\}\right]
\end{aligned}
$$

The first order conditions:

$$[h] \quad : \quad u'[c_0] + \beta \left\{ u'[c_0] F(\bar{z}|e) + \int_{\bar{z}}^{\infty} u'[c_1(z)] \phi(z|e_H) dz \right\} = \lambda w_1 a f'(h);$$

$$[d] \quad : \quad u'[c_0] + \beta \left\{ u'[c_0] F(\bar{z}|e) + \int_{\bar{z}}^{\infty} u'[c_1(z)] \phi(z|e_H) dz \right\} = \lambda (1+\beta)$$

$$[\bar{z}] \quad : \quad \int_{\bar{z}}^{\infty} u'[(z-\bar{z}) w_1 a f(h) + W + d - h] f(z|e) dz = \lambda \beta \{ w_1 a f(h) [1 - \Phi(\bar{z}|$$

Consumption:

with verification:

$$c_1(z) = c_0.$$

no verification:

$$c_1(z) = w_1 z a f(h) - \bar{R}.$$

Pros:

endogenous incompleteness.

model of default and interest rates

provides interesting framework for policies on ϑ

Cons:

full insurance in case of default!

Moral Hazard with Costly State Verification

Optimal Contract

$$\max_{\bar{z},h,e,d,\{\bar{R},R(z)\}} u\left[W - h + d\right] - v(e) + \beta \int_Z u[w\left(z\right) af(h) - R\left(z\right)]\phi(z|e)dz$$

BEC of the lender (with the LM λ): $[\lambda]\ :\ -d + \beta \int_Z [R\left(z\right) - \vartheta\chi\left(z\right)]\phi(z|e_H)dz \geq 0$,

the optimal 'auditing' or verification incentive compatibility:

$$R\left(z\right) \leq \bar{R} \text{ for } z < \bar{z}$$

ICC (assuming e_H is optimal;LM $\mu \geq 0$)

$$\begin{aligned}[\mu] \quad &: \quad -v(e_H) + \beta \int_Z u[w(z)\, af(h) - R(z)]\phi(z|e_H)dz \\ &\geq \quad -v(e_L) + \beta \int_Z u[w(z)\, af(h) - R(z)]\phi(z|e_L)dz.\end{aligned}$$

Consumption:

with verification:

$$u'[c_0] = \left[1 + \mu\left(1 - \frac{\phi(z|e_L)}{\phi(z|e_H)}\right)\right] u'[c_1(z)]$$

no verification:

$$c_1(z) = w_1 z a f(h) - \bar{R}$$

Pros:

endogenous incompleteness.

model of default and interest rates

provides interesting framework for policies on ϑ

no full insurance in case of default!

Cons:

abstracts from other incentive problems

Question:

Quantitative implications.

Moral Hazard: effort in labor markets

second period utility: $E\left[u\left(c_1\right)\right]-v\left(s\right)$.

earnings distribution $\phi\left[y|a,h,s\right]$

equilibrium exertion of effort: given $D\left(y,a,h\right)$

$$s^* \in \arg\max_s \left\{ \int u\left[y-D\left(y,a,h\right)\right]\phi\left[y|a,h,s\right]dy - v\left(s\right)\right\}$$

debt-overhang may reduce labor market performances.

5 Conclusions

Simple models of incentive problems applied to human capital investments

Lots of interesting economics to explore