Preliminary; Comments Welcome

Financial Crises and the Optimality of Debt for Liquidity Provision[+]

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Abstract

Systemic financial crises involve debt (leverage). We provide a theory of the optimality of debt, which nevertheless can lead to a crisis. Trade is best implemented by debt because it provides the smallest incentive for private information production, which creates trade-reducing adverse selection. Debt preserves symmetric ignorance between counterparties. Debt is least *information-sensitive*: the value (in utility terms) to producing private information or learning public information about the payoff is lowest. Even if one party is privately informed, so there is adverse selection in the market, debt is still optimal because it maximizes the amount of trade. Moreover, when there can be no adverse selection, but public signals, debt maximizes the amount that can be traded. For the economy as a whole there is a systemic risk of using debt to provide liquidity: an aggregate shock, if bad enough, can be made worse because the amount traded is reduced further. A public signal can cause debt to become information-sensitive. Then agents try to prevent triggering private information production; they trade an amount below the expected value conditional on the shock. The shock is amplified, leading to a crisis.

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1. Introduction

In 2007-2008 the global economy experienced a systemic financial crisis reminiscent of the financial crises that have occurred repeatedly in U.S. history and in the histories of other countries as well. Systemic crises have the common feature that they involve debt, without which there would be no crises. Yet current theories of crisis assume the existence of debt, and current theories of debt do not explain the origins of crises. In this paper we provide a theory of the existence and optimality of debt, a theory that also shows that debt is vulnerable to a crisis in which trade collapses.

While our theory concerns debt generally, it applies with most force to forms of debt that are used for transactions. Historically, this would include private banks notes, and demand deposits prior to deposit insurance. Currently, sale and repurchase agreements ("repo") are a prime example (see Gorton (2009), and Gorton and Metrick (2009a)). The optimality of debt is linked to the notion of "liquidity." Roughly, "liquidity" refers to the ability to trade a given amount quickly without the transaction moving prices, and without an uninformed party losing money to a privately informed party. Symmetric information facilitates trade. One form of symmetric information is symmetric ignorance. Debt optimally facilitates trade because debt provides the smallest incentive for private information production, which creates adverse selection. Debt is designed to preserve symmetric ignorance, creating liquidity.

We analyze a strategic security issuance, information acquisition, and trading game and show that in equilibrium debt is issued in the primary market and traded in the secondary market. An agent may have a strategic incentive to acquire private information to exploit other agents. The security design problem faced initially is to design a security to minimize this incentive (at initial issuance and in later trading). We define the *value of information* of a security to be the value in terms of expected utility to producing private information about the payoff on the security. This leads to the definition of the *information-sensitivity* of a security: the value (in utility terms) from a trader producing private information, or learning a public signal, about the security's payoff. A "least information-sensitive security" is one which minimizes the incentive to produce private information and hence maximizes trade or liquidity because it is common knowledge that no agent will pay to produce private information about the security. We show that debt is an optimal security for the provision of liquidity because it minimizes the incentive for private information production. Information-sensitivity and liquidity are essentially the same. While reminiscent of the result of Gorton and Pennacchi (1990) that the purpose of banks is to create debt that is immune to adverse selection for trading (also see Holmström (2008)), here debt is shown to be the optimal trading security from first principles.

After the debt is issued in the primary market there comes another trading date. But just prior to this date an interim public information signal is learned. If the interim news is bad, it causes the information-sensitivity of debt to rise. The expected value of the debt, its price, falls (when news is bad). But the amount of trade that occurs in equilibrium may be much lower (than the expected value of the debt conditional on the signal) because, in equilibrium, agents want to avoid triggering the production of private information. One way to do this is to scale down or haircut the debt that is traded, that is, agents trade only a fraction of the original bond to avoid triggering information production. Instead of trading at the new (lower) expected value of the debt, agents trade much less or even not at all. In fact, at the interim date, the best outcome may be to allow adverse selection (in which case we show that debt is still optimal). In all these possible outcomes in response to the bad news, there is a collapse of trade, a systemic crisis.[1]

With both primary and secondary trading, and endogenous information acquisition, we show that the optimal security is always debt. But debt has different implications for information acquisition. (i) If only the buyer can produce information, the optimal debt contract will either avoid information acquisition or induce endogenous adverse selection in equilibrium (if the cost of information production is small). Equity is never optimal. (ii) If only the seller can produce information the optimal debt contract never induces adverse selection. For any cost of information production, information acquisition by the seller is avoided by reducing the face value of debt or increasing the price so as to "bribe" him not to produce information. Combining these results, we formalize the notion that in secondary markets it is more difficult for an uninformed agent to sell than to buy assets. We discuss later how this resale concern has severe implications for trading behavior in a financial crisis where uninformed agents face potential adverse selection because assets endogenously become information-sensitive.

Debt is optimal for the economy, which needs a certain amount of leverage to implement efficient trade. But, a systemic event can occur because the debt is not riskless (as it is in Gorton and Pennacchi (1990)). The systemic event corresponds to information-insensitive debt becoming information-sensitive, giving rise to concerns of adverse selection, requiring a response from agents that reduces the amount of trade below what could be implemented if the agents just traded at the lower expected value of the debt. The problem is that information, whether privately produced or a public signal, can reduce efficient trade. A loss of "confidence" corresponds to an increase in information-sensitivity, leading to a fear of adverse selection, which may be realized. In any case, fear of adverse selection reduces trade.

The financial crisis in our economy comes from an entirely different source than crises and amplification mechanisms in the literature. It is a different crisis mechanism than that of, for example, Kiyotaki and Moore (1997) where the collateral value is subject to a feedback effect from the initial shock causing its value to decline further. The cause of the systemic event here is also distinct from coordination failure models of bank runs based on self-fulfilling expectations, as in Diamond and Dybvig (1983). In our theory, the crisis is linked to the underlying rationale for the existence of debt as the optimal trading security.

The two issues that we focus on, liquidity and the optimality of debt, have not been previously linked. On the one hand, Diamond and Dybvig (1983) and Gorton and Pennacchi (1990) study liquidity provision but assume the existence of debt. Diamond and Dybvig (1983) associate "liquidity" with intertemporal consumption smoothing and argue that a banking system with demand deposits provides this type of liquidity. But, as Jacklin (1987) argued, there is no explanation for the optimality of demand deposits in their setting, and demand deposits only arise because other markets and securities are arbitrarily ruled out. Gorton and Pennacchi argue that debt is an optimal trading security because it minimizes trading losses to informed traders when used by uniformed traders. Hence debt provides liquidity in that sense. Gorton and Pennacchi, however, focus on explaining the existence of banks, institutions that attract informed traders to be the bank equity holders, so that the uninformed traders

[1] Arguably, this is the type of collapse which occurred in U.S. financial markets starting in August 2007; information-insensitive debt used as collateral in the sale and repurchase –"repo"—market became information-sensitive when house prices did not rise. See Gorton (2009) and Gorton and Metrick (2009).

can use the banks' demand deposits to trade (minimizing their losses). In Gorton and Pennacchi the debt is riskless, and it is not formally shown that debt is an optimal contract.

On the other hand, a large literature in corporate finance studies the optimality of debt, but does not study liquidity. Following Townsend (1979) and Myers and Majluf (1984) a large literature has developed on debt in firms' capital structures based on agency issues in corporate finance. Examples of other related papers include Hart and Moore (1995, 1998), Aghion and Bolton (1992), Bolton and Scharfstein (1990), Gale and Hellwig (1985), and DeMarzo and Duffie (1999).[2] The settings studied by these authors are not that of trading (in secondary markets) or liquidity, but concern a privately-informed firm issuing a security in the primary market. Our setting is very different. We analyze the design of a security for a sequence of bilateral trades, including the secondary market as well as the primary market. We do not assume ex ante asymmetric information, but incorporate endogenous information production. We ask how security design can prevent information production and asymmetric information from arising in the first place. In our setting there are some potential buyers and sellers who can produce information at some cost so that adverse selection may arise endogenously.[3]

In our setting efficient trade is inhibited by "transparency." There are a few papers that raise the issue of whether more information is better in the context of trading or banking. These include, for example, Andolfatto (2009), Kaplan (2006), and Pagano and Volpin (2009). Andolfatto (2009) considers an economy where agents need to trade, and shows that when there is news about the value of the "money" used to trade, some agents cannot achieve their desired consumption levels. Agents would prefer that the news be suppressed. Kaplan (2006) studies a Diamond and Dybvig-type model and in which the bank acquires information before depositors do. He derives conditions under which the optimal deposit contract is non-contingent. Pagano and Volpin (2009) study the incentives a security issuer has to release information about a security, which may enhance primary market issuance profits, but harm secondary market trading. These authors assume debt contracts.

The paper proceeds as follows. In Section 2 we present the model. There are three dates (t=1,2,3), a primary market, a secondary market, and three agents. In Section 3 we define *information-sensitivity* and derive a least information-sensitive security as well as some further results that are needed for the equilibrium analysis of the full game. The full model is solved by backward induction. In Section 4 we analyze optimal security design for an uninformed agent (investor) who faces a potentially informed buyer in the secondary market at t=2 when he trades the security. In Section 5 we analyze optimal security design for an uninformed agent when he faces a potentially informed seller in the primary market at t=1. In Section 6 we analyze optimal security design when there is public information at t=2 but no agent can produce private information. In Section 7 we use the previous results to characterize the equilibrium of the full game with public information and private information production by some agents. Section 8 contains a brief discussion of the results and presents some extensions. Section 9 is a conclusion.

[2] Banerjee and Maskin (1996) study repeated trade and focus on what goods will be used as the medium of exchange. There, agents are exogenously privately informed about their own goods.

[3] In contrast to stock trading in a centralized exchange, bonds and all securitized assets are traded in decentralized markets where buyers and sellers negotiate the terms of trade. In such a setting, the seller does not necessarily have better information than a potential buyer.

2. The Model

We consider an exchange economy with three dates (t=1, 2, 3) and three agents {A, B, C} whose utility functions are given as follows:

$$U_A=C_{A1}+C_{A2}+C_{A3}$$
$$U_B=C_{B1}+\alpha C_{B2}+C_{B3}$$
$$U_C=\tfrac{1}{\alpha}C_{C1}+C_{C2}+C_{C3},$$

where $\alpha>1$ is a constant and where C_{ht} denotes consumption of agent h at date t. The endowment of agent h is described by the vector $\varpi_h=(\varpi_{h1},\varpi_{h2},\varpi_{h3})$, where the second subscript refers to the time at which the endowment arrives. We assume that $\varpi_A=(0,0,X)$, $\varpi_B=(w,0,0)$, $\varpi_C=(0,w,0)$ where w is a constant and X is a random variable. So, agent A has no endowment of goods at date 1 and 2 but receives x units of goods at date 3, where x is a realization of the random variable X (a project or "Lucas tree"). Agent B possesses w units of goods at date 1 and nothing at the other dates. Agent C only has w units of goods at date 2. Goods are nonstorable. The agents start with identical information about the random variable X. We assume that X is a continuous random variable with positive support on $[x_L, x_H]$ and density $f_m(x)$. The information about the endowments and the project (tree) is common knowledge.

Since agents have different marginal valuations of consumption at different dates, gains from trade can be realized by a reallocation of goods. Given the assumed form of the utility functions, it is socially efficient for agent A to consume at date 1, for agent B to consume at date 2, and for agent C to consume at date 3. In order to implement the efficient allocation and for agent B to consume at t=2, at t=1 agent B trades some of his t=1 goods to agent A and in exchange agent A promises agent B some of his t=3 (uncertain) endowment. Technically speaking, a promise is a contract s(x) that maps the outcome of X to a repayment s(x).[4] At date 2, agent B can use s(x) to trade for agent C's t=2 goods.

<u>The set of contracts:</u> Let S denote the set of all possible securities (contracts), i.e., functions, s(x), which satisfy the resource feasibility (or limited liability) constraint, $s(x)\leq x$. Any mapping s: $X\rightarrow R$ with $s(x)\leq x$ is an element of S. Some examples are:

(i) Equity: $s(x)=\beta x$ where $\beta\in(0,1]$ is the share on the x;
(ii) Debt: s(x)=min[x, D] where D is the face value of the debt;
(iii) Step function contract: y_1 if $x\in[x_L,x_1]$, y_i if $x\in[x_{i-1},x_i]$ where $y_i\leq x_i$.
(iv) State contingent securities: $s(x_i)=y_i$ where $y_i\leq x_i$;
(v) Stochastic contracts: $s(x_i)=y_i$ where $y_i : x_i \rightarrow [x_L, x_i]$ with distribution F_i.

In principle, agent A could promise whatever he wants, e.g. $s(x)>x_H$, but agent B would simply not believe it. Therefore, at date 1, the set of (feasible) contracts agent A can issue to agent B is $s\in S=\{s: s(x)\leq x\}$. At date 2 the set of contracts agent B can trade with agent C is given by $\hat{S}=\{\hat{s}:\hat{s}(y)\leq y\}$ where y=s(x) denotes the payoff of the security that agent B has bought from agent A. In other words, agent B need not simply trade the original security that he received from agent A to agent C. He can

[4] We assume that the realization x at date 3 is verifiable. For example, cash flows in asset backed securities deals are by design verifiable since third parties, the trustee and servicer, monitor and collect the underlying loans and distribute the cash flow to investors.

redesign the security by issuing a new security, $\hat{s}(y)$, using the original security as collateral, i.e., agent B can securitize the bond that he got from agent A.

Two examples of feasible contracts at t=2, which will play roles later, are:

(i) A "vertical strip," i.e., $\hat{s}(x) = \kappa s(x)$ where $\kappa \in [0,1]$ is a pro rata share of the original contract (i.e. agent B sells the fraction κ of his security to agent C.)

(ii) A tranche or "horizontal slice," is as follows. Suppose s(x)=min[x,D] was issued originally at t=1. Then, at t=2, agent B could create a new security using s(x) as the collateral. In particular, agent B could design a new bond $\hat{s}(x) = \min[s(x), \hat{D}]$, with $\hat{D} \leq D$. This is a debt contract that writes-down the original face value D of the original debt contract to the new face value $\hat{D}$. The original bond is used as collateral for the new contract. In particular, the new debt contract is a senior tranche of the collateral, and agent B will hold the equity residual, with payoff $\max[s(x) - \hat{D}, 0]$. The new bond is a "horizontal slice" ("tranche") of the collateral based on seniority.

Public Information: We assume that at date 1 the agents' prior on X is given by the (mixture) distribution F_m, where the density of F_m is given by $f_m(x) = \sum_{k=1}^{K} \lambda_k f_k(x)$, where $\lambda_k \geq 0$, $\sum_{k=1}^{K} \lambda_k = 1$, and there are K distributions, each indicated by F_k. We assume that $\{F_k\}$ is ordered by First Order Stochastic where F_N first-order stochastically dominates all other distributions, while F_1 is first-order stochastically dominated by all other distributions. At date 2, the agents receive a public signal about the "true" distribution of X (i.e. public news about which distribution x will be drawn from).

Private Information Production: We assume that agents A and C can produce information about the final payoff at the cost γ (in terms of utility) at date 1 and 2.[5] If an agent produces information, he privately learns the *true* realization x. Information acquisition is not observable by other agents.[6]

Sequence of Moves and Events: The timing of events at t=1 is as follows:

t=1.0: The realization of X (namely, x) occurs but is not publicly known.
t=1.1: Agent B makes a take-it-or-leave-it contract offer of s(x) to agent A.
t=1.2: Agents A chooses whether to produce private information about the *true* x at the cost γ or not.
t=1.3: Agent A accepts the contract or not.

If there is no trade between agents A and B, the game ends. If agent B trades with agent A, then at t=2 agent B has the claim s(x) available to use to trade with agent C. The timing of events at t=2 is as follows:

t=2.0: The public signal F_k is observed.
t=2.1: Agent B makes a take-it-or-leave-it contract offer $\hat{s}(y)$ to agent C.
t=2.2: Agents C chooses whether to produce private information about the *true* x at the cost γ or not (if the information was not produced earlier).
t=2.3: Agent C accepts the offer or not.

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[4] This assumption is made for tractability and is discussed later. At this point, it suffices to note that agent B (the proposer at both dates) cannot acquire information, so no signaling issue arises. For an analysis of a bargaining game with two sided information acquisition see Dang (2008).

[6] In Appendix B we discuss alternative information structures where information production results in less than perfect information. This does not change any results.

t=3: Agent B redeems s(x) with agent A, and agent C redeems $\hat{s}(y)$ with agent B.

To fix ideas the reader may want to think of agent B as an agent who wants to buy a security so as to save his current endowment for future consumption. For that purpose agent B wants to buy the securitized project from agent A at t=1 and later sell it to agent C at t=2 so that he can consume at his desired consumption date. We assume that agent B is fully rational but less sophisticated in the sense that he cannot produce private information about the security he uses to trade. Thus agent B wants to buy a security that is "liquid", i.e. least prone to adverse selection.[7]

We are interested in the following question: What is the "optimal" security, s(x), for such a sequence of transactions when agents A and C are sophisticated in that they can produce private information, while agent B cannot produce information? The problem is to design a security such that given some information production cost, it does not pay for an agent to learn about the final payoff of the security. If no agent has an incentive to learn, then no agent is concerned about facing a privately informed counterparty and adverse selection. If only a subset of agents can produce information, then preventing information production is desirable since symmetric information facilitates trade and efficient intertemporal reallocations of consumption goods.

In the next section we define *information-sensitivity* and derive a least information-sensitive security as well as some further results that are needed for the equilibrium analysis of the full game. We proceed by backward induction. In Section 4 we analyze optimal security design by agent B when he faces a potentially informed buyer (agent C) at t=2. In that section we assume that agent A cannot produce information and there is no public information. In Section 5 we analyze optimal security design by agent B when he faces a potentially informed seller (agent A) at t=1. In that section we assume that agent C cannot produce information and there is no public information. In Section 6 we analyze optimal security design by agent B when there is public information at t=2 but agent A and C cannot produce private information. In Section 7 we characterize the equilibrium of the full game with public information and private information production by both agents A and C and discuss welfare implications.

3. The Design of a Least Information-sensitive Security

In subsection A we define *information-sensitivity*, which is a key parameter for our theory of debt and financial crises. In subsection B we derive some results that are needed for the equilibrium analysis of the whole game. And subsection C contains shows the maximal debt amount that can be issued. Subsection D analyzes securitization, a particular form of debt. In this section we assume that w<E[X]. Otherwise, the efficient allocation requires agent A to sell his whole project (tree) to agent B and it may seem that there is no reason to discuss optimal security design subsequently. However, we show below that this assumption is not crucial for any results.

A. Information-Sensitivity

[7] Asset-backed securities are designed to reflect the demand of rational but less sophisticated investors (agent B) who use these assets to "store" their wealth and are concerned about facing agents with the ability to produce information and thus potential adverse selection in the secondary market when they have to sell the security. Examples include insurance companies and pension funds. Or we can interpret agent B as a bank that has excess cash at t=1 that he wants store by using s(x). At t=2, he has a shortage of cash and thus wants to sell s(x) to agent C. Since agent B cannot produce information, agent B wants to buy a security that is least prone to potential adverse selection. Sale and repurchase agreements (repo), using securitized assets as collateral, is a kind of private money endogenously created by the banking system.

In this subsection we assume that there is no interim public information. Thus we omit the subscript m and just write F in this section. We start with analyzing a (pure) decision problem of the following type: Suppose the agent (with utility function $U=C_1+C_2+C_3$) has wealth w and can buy a security s(x) with p=E[s(x)] where p denotes the price and s(x) specifies the payoff as a function of x. If the agent is informed, then he knows the true realization of x. We ask which security s(x), with p=E[s(x)], gives rise to the lowest value of information or, in other words, which security is least information-sensitive.

Definition (The value of information): Suppose the decision of the agent is whether to buy a particular security or not. With respect to that security, the *value of information of a buy (B) transaction* is defined as $\pi_B = EU(PI) - EU(IG)$, where EU(PI) is the expected utility based on the optimal transaction decision in each state under perfect information about x (PI), and EU(IG) denotes the expected utility of a buy transaction based on the initial information, ignorance of the true state (IG). An analogous definition applies to value of information of a sell transaction, π_S, where the agent must decide to either sell or not sell the asset. Formally, the value of information is given by:

$$\pi_B = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x)dx \qquad \text{and}$$

$$\pi_S = \int_{x_L}^{x_H} \max[s(x) - p, 0] \cdot f(x)dx .$$

To understand these expressions first consider the point of view of the buyer. Under ignorance a (risk neutral) agent is willing to buy the security since E[s(x)]=p. The value of information to a potential buyer concerns the region where s(x)<p. That is the area where the buyer is overpaying. If the buyer knew that s(x)<p, then he would not trade and instead he would consume the unspent amount p. Define $Q_< = \{x | s(x) < p\}$ to be the set of such states and define $Q_> = \{x | s(x) \geq p\}$. Thus, $Q_< + Q_> = Q = [x_L, x_H]$. So, the value of information for a potential buyer is $\pi_B = EU_B(PI) - EU_B(IG)$:

$$\Leftrightarrow \qquad \pi_B = \left(\int_{Q<} p \cdot f(x)dx + \int_{Q>} s(x) \cdot f(x)dx \right) - \int_{Q} s(x) \cdot f(x)dx$$

$$\Leftrightarrow \qquad \pi_B = \left(\int_{Q<} p \cdot f(x)dx + \int_{Q>} s(x) \cdot f(x)dx \right) - \left(\int_{Q<} s(x) \cdot f(x)dx + \int_{Q>} s(x) \cdot f(x)dx \right)$$

$$\Leftrightarrow \qquad \pi_B = \int_{Q<} (p - s(x)) \cdot f(x)dx = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x)dx$$

This is the value of avoiding overpayment.

For a seller (S) information is valuable when s(x)>p, that is, when the security is worth a lot, the seller would be undervaluing it were he to sell for the price p. Again, define $Q_> = \{x : s(x) \geq p\}$ and $Q_< = \{x : s(x) < p\}$. Proceeding similarly, the value of information for the seller is $\pi_S = EU_S(PI) - EU_S(IG)$

$$\Leftrightarrow \qquad \pi_S = \left(\int_{Q<} p \cdot f(x)dx + \int_{Q>} s(x) \cdot f(x)dx \right) - p$$

$$\Leftrightarrow \qquad \pi_S = \int_{Q>} (s(x) - p) \cdot f(x)dx = \int_{x_L}^{x_H} \max[s(x) - p, 0] \cdot f(x)dx.$$

This is the value of avoiding selling the security for too little.

Example: The agent's utility function is $U=c_1+c_2$ and wealth is 1.5. Suppose there are two outcomes, s(x=L)=1 or s(x=H)=2, both states are equally likely, and the price of buying this asset is 1.5. Suppose the decision of the agent is whether to buy the asset or not. If the agent buys the asset without information acquisition, or simply consumes his endowment, then EU(IG)=1.5. If the agent acquires information, then in state 1 he does not buy the asset and consumes his endowment at t=1; in state 2 he buys the asset and consumes s(x=H)=2 at t=2. Thus, EU(PI)=0.5·1.5+0.5·2=1.75 and π_B=0.25.

Definition (Information-sensitivity of a security): Suppose an agent can buy either of two securities that have the same expected value and the same price. Security i is said to be less *information-sensitive* than security j for agent h if the value of information for buying security i is lower than the value of information for buying security j, i.e. $\pi_h^i < \pi_h^j$. The analogous definition applies to a sell transaction.[8]

B. Debt as a Least Information-sensitive Security

A *standard debt contract* is given by:

$$s^D(x) = D \qquad \text{if } x > D$$
$$s^D(x) = x \qquad \text{if } x \le D.$$

That is, a standard debt contract is a security that pays $s^D(x)=x$ up to a specified amount, the face value of the debt, D. In the range $x \le D$ the payoff function has slope 1 due to the resource constraint or limited liability. If x>D, then the investor receives D. In order to implement trade we will be interested in the following (standard) debt contract, which has price w and face value D, where D solves the following equation:

$$w = \int_{x_L}^{D} xf(x)dx + \int_{D}^{x_H} Df(x)dx.$$

The price of debt equals its expected value. By design this debt contract can potentially implement the transaction needed to achieve the efficient allocation. That is, the amount *w* can be traded. Given f(x) and *w*, D is determined.

Now we derive a contract with the minimal information-sensitivity subject to the constraint that any contract should have the same expected payoff and that the prices of all contracts are p=E[s(x)]. (Note, if traded, the contract should implement efficient consumption, i.e. p=E[s(x)]=w.) In other words, we are minimizing π_B and π_S over a set {s} of functions where {s: s(x)≤x and E[s(x)]=p}, and we require the solution to hold for any distribution F for x. In each of these cases, this is a non-trivial mathematical problem in functional space, but the solution and the proof turn out to be surprisingly

[8] Although similar in spirit, Demarzo and Duffie (1999) define information-sensitivity differently.

simple. The two minimization problems have the same solution for *all* distributions F(x), as will be seen.

Proposition 1: Assume for all s that $E[s^D(x)] = E[s(x)] = p$ and that $s(x) \leq x$. Debt is a least information-sensitive security for the buyer and the seller.

Proof: The proof follows from two lemmas.

Lemma 1: Debt is a least information-sensitive security for the buyer of the security.

Proof: Consider two securities, $s^D(x)$ and $s(x)$, where $s^D(x)$ is debt, i.e., $s^D(x) = D$, for $x \geq D$ and $s^D(x) = x$, for $x<D$. Recall that we have assumed that for all s that $E[s^D(x)] = E[s(x)] = p$, and that there is the resource constraint $s(x) \leq x$. The lemma says that debt is a contract that minimizes:

$$\pi_B = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x)dx \ .$$

Equivalently, for all s,

$$\int_{Q_<^D} (p - s^D(x)) \cdot f(x)dx \leq \int_{Q_<^S} (p - s(x)) \cdot f(x)dx \qquad (*)$$

where $Q_<^D = \{x : s^D(x) \leq p\}$ and $Q_<^S = \{x : s(x) \leq p\}$. Note, $s^D(x) = x \geq s(x)$ for all $x \leq p$ implies (i) $Q_<^D = [x_L, p] \subseteq Q_<^S$, (ii) $p - s^D(x) \leq p - s(x)$ for all $x \in Q_<^D$, and (iii) prob($Q_<^D$)≤prob($Q_<^S$) for all s. //

Lemma 2: The value of information to the seller of a security is equal to the value of information to the buyer of a security.

Proof: E[s(x)]=p can equivalently be written as:

$$E[s(x) - p] = 0$$

$$\Leftrightarrow \quad \int_{Q<} (s(x) - p) \cdot f(x)dx + \int_{Q>} (s(x) - p) \cdot f(x)dx = 0$$

$$\Leftrightarrow \quad \int_{Q>} (s(x) - p) \cdot f(x)dx = -\int_{Q<} (s(x) - p) \cdot f(x)dx$$

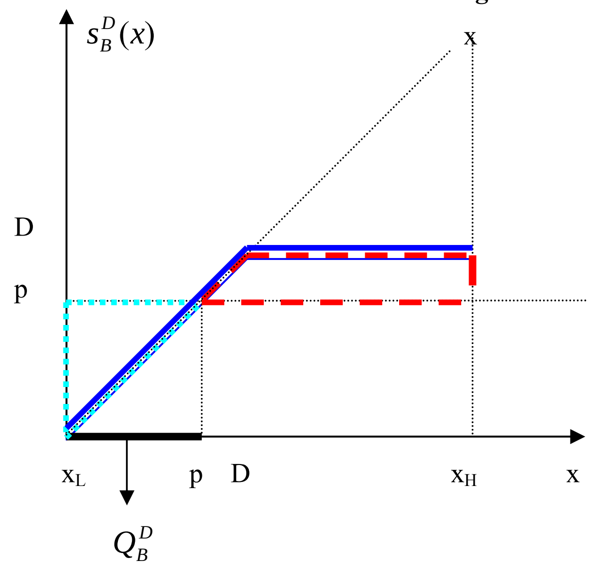
$$\Leftrightarrow \quad \int_{Q>} (s(x) - p) \cdot f(x)dx = \int_{Q<} (p - s(x)) \cdot f(x)dx \ //$$

End of the proof of Proposition 1.

The intuition for Proposition 1 is as follows. The buyer must decide to buy the bond with price p or not. If he knows the true value of the payoff, x, then he does not buy the bond in states where x<p because his payoff is s(x)=x<p. Since the debt contract has slope one in this region of states, i.e. the buyer receives the maximum amount of repayment that is possible, there exists no other contract that has a smaller set of states that are information-sensitive. Figure 1 depicts the debt contract (the dark

blue line). The set of states where information has value to the buyer is denoted by Q_B^D. It is easy to see that $Q_B^D \subseteq Q_B^S$ for all s∈S.

Figure 1



The value of information to the buyer is that he avoids overpaying p when the realization of x is less than that, x<p. The seller of the security, benefits from being informed to the extent that he avoids paying back too much when the realization of x is larger than p. In the states where x<p, the buyer must be compensated for his low payoff (the blue dotted point triangle), evaluated with the density f(x)), with larger payoffs than w in states x>p (the red dotted line area). The expected payoffs in these high states are exactly the states where information has value to the seller.

Proposition 1 states that for any given distribution F(x), if the trading of debt triggers information acquisition, then so does the trade of any other security with the same expected value (or price). Note, debt is also less information-sensitive than any stochastic contract. Even if s(x) is stochastic, the stochastic repayment s(x) must be backed by the outcome of the underlying X. Thus if the agent knows that x<p, he knows s(x)<p for any stochastic realization s(x). So information about X has value to an agent even if s(x) is stochastic and determined at t=2. The following results are self-evident.

Proposition 2 (Characterization): The set {s} of securities with p=E[s(x)]=w that has the minimal information-sensitivity is given by {s: s(x)=x for x≤w and s(x)≥w for x>w}.

Proposition 2 identifies a class of debt contracts that are least information-sensitive. This class includes standard debt, but also arbitrary senior debt-like securities, of the type shown in panel (c) of Figure 2, below. This proposition shows that the key feature of a least information-sensitive security is not the flat part of a standard debt contract, but seniority of repayment when there is default, i.e. for x<p, the holder of the security is repaid first and receives everything the underlying asset delivers.[9] Later we show that if there is public interim information about the distribution of x, at t=2 the standard

[9] The limited liability (slope 1) assumption is not crucial. If there is insurance and the security issuer can repay m·x for m>1, then a least information-sensitive security has the feature that s(x)=mx for x<p/m. But with asset insurance, agents may also have an incentive to learn about the ability m of the insurer to step in the case of non-full delivery of the issuer. Thus information acquisition may also be about the credibility of the insurer. Another interpretation of security insurance is that insurance changes the distribution of s(x), having less probability mass on the left tail. With this interpretation, it is obvious that Proposition 1 holds without any modification.

debt contract has an expected interim information-sensitivity that is less than the expected information-sensitivity of the other contracts given in Proposition 2.

Corollary 1 (Equity): Assume $E[s^D(x)]=E[s^E(x)]<E[X]$. Equity, $s^E(x)$, is strictly more information-sensitive than debt.

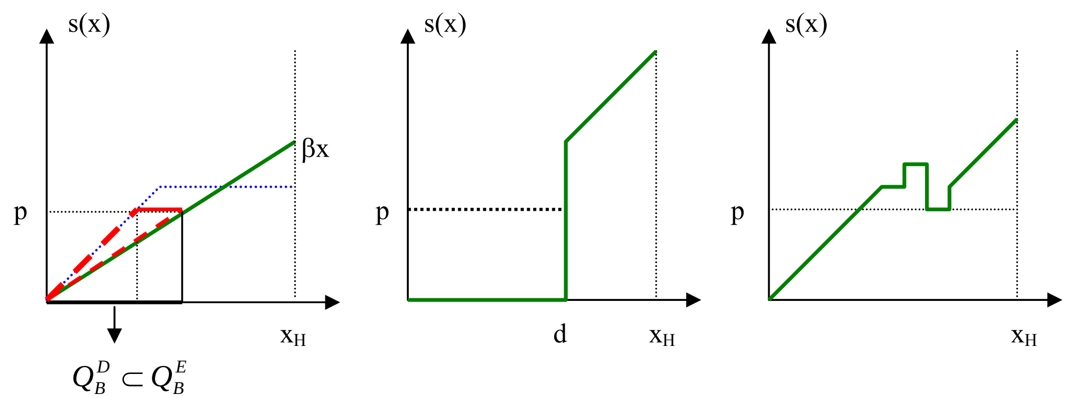
Corollary 2 (Levered Equity): The security with the maximal information-sensitivity is given by: $\bar{s}(x)=0$ for $x \in [x_L, d]$ and $\bar{s}(x)=x$ for $x \in [d, x_H]$ where d solves

$$\int_d^{x_H} x f(x)dx = w.$$

Figure 2 compares the payoff on debt to three other securities. Figure 2 shows the payoff to an equity contract, in panel (a), the payoff on levered equity in panel (b); and another least information-sensitive, debt-like, contract is shown in panel (c). In panel (a) the red dotted triangle is the area where equity is more information-sensitive than debt. In panel (b) the information-sensitive area is the rectangle spanned by p and d and evaluated with the density is the value of information. In panel (c) the payoff is non-monotonic, compared to the flat payoff on standard debt.

Figure 2

(a) Equity (b) Levered Equity (c) Another least information-sensitive security



In the remainder of the paper we employ the standard assumption (motivated by moral hazard concerns) that s(x) is non-decreasing.

C. Maximum Debt Issuance

Now we assume that the cost of information production about the true value of X is γ in terms of utility. So what is the maximal amount of debt that can be issued without triggering information production? Proposition 1 shows that for any contract with $p=E[s(x)]$, we have $\pi_B=\pi_S$. Thus information production is not worthwhile for a buyer or a seller if $\pi_B \le \gamma$.

Corollary 3 (Maximum Debt Issuance)**:** The maximum amount of debt (with $p=E[s^D(x)]$) that agent B can buy at t=1 and sell at t=2 without triggering information acquisition by agent A and C is given by min[w, E[X], p] where p solves the following equation:

$$\int_{x_L}^{p} (p-x) f(x)dx = \gamma .$$

Corollary 3 will play an important role in the proof of the main results.

D. Securitization[10]

We can think of securitization in this context. The maximum possible debt that can be issued can be increased if it is backed by a portfolio of projects. Suppose a claim is written on a portfolio of N projects, $X_1,\ldots, X_N$ with distributions $F_1,\ldots, F_N$. Define $Y=X_1+\ldots+X_N$ as the random variable with distribution F_y. Securitization refers to the structure where the random variable Y is a pool of mortgages or automobile loans that are sold to a Special Purpose Vehicle (SPV), which finances the purchase of these loans by issuing asset-backed securities in the capital markets.

In general it is very complicated to calculate the distributions of a sum of random variables. As an example, suppose N=2, and X_1 and X_2 are independently and uniformly distributed on [0,1]. Then Y is not uniformly distributed; it has density:

$$f_Y(y)=\begin{cases} y & \text{for } 0\le y\le 1 \\ 2-y & \text{for } 1<y<2 \\ 0 & \text{otherwise} \end{cases} .$$

The information-sensitivity of a single debt contract with price $p=E[s^D(x)]<E[x]$ is $\int_0^p (p-x) f(x)dx$. If all the X's are independently and identically distributed, trading N individual bonds gives rise to the total value of information, $\Pi_N^{\Sigma} \equiv N\pi$. We compare Π_N^{Σ} to the information-sensitivity of a debt portfolio (DP) (i.e., a single bond backed by N projects) and to the information-sensitivity of a portfolio of debt contracts (PD). Denote $\Pi_N^{DP} = \int_0^{p_{DP}} (p_{DP}-y)\cdot f_{Y_N}(y)dy$ as the information-sensitivity of a single bond backed by a portfolio of N projects where $y=\sum_{n=1}^{N} X_n$. Analogously, denote $\Pi_N^{PD} = \int_0^{p_{PD}} (p_{PD}-s(y))\cdot f_{Y_N}(y)dy$ as the information-sensitivity of a portfolio of bonds where $s(y)=\sum_{n=1}^{N}\min[x_i, D]$. To facilitate comparison, we set $p_{DP}=p_{PD}=Np$. Also, define $\overline{\Pi}_N^{DP} = \frac{1}{N}\int_0^{N\cdot p} (Np-y)\cdot f_{Y_N}(y)dy$, i.e. the information-sensitivity per unit of project debt.

***<u>Lemma 3 (Portfolio Information-Sensitivity)</u>*: Suppose $Y=X_1+\ldots+X_N$, where X_i is independently and uniformly distributed on [0,1] for i=1,.., N. Then:

(i) $\Pi_N^{DP} < \Pi_N^{PD} < \Pi_N^{\Sigma}$ for N≥2.

[10] See Gorton and Souleles (2006) for details of securitization.

(ii) $\overline{\Pi}_{N+1}^{DP} < \overline{\Pi}_{N}^{DP}$ for all N.

Proof: See Appendix A. //

Part (i) of the Lemma shows that the information-sensitivity of a single bond, backed by the sum of the cash flows of the N projects, is lower than the information-sensitivity of a portfolio of bonds, where each bond is backed by a single project, which in turn is lower than the sum of information-sensitivities of the N individual bonds. This provides an explanation for securitization. This analysis suggests that securitization is not (primarily) about diversification of risks. If this were the case, then issuing equity shares on the pool would be optimal.[11]

4. Optimal Security Design when the Buyer is Potentially Privately Informed (at t=2)

Section 3 introduced some terminology and intermediate results. In order to solve for the equilibrium of the full game, we proceed in three steps. This section analyzes optimal security design by agent B when he faces a potentially informed buyer (agent C) in the secondary market at t=2 but there is no public information. In Section 5 we analyze optimal security design by agent B when he faces a potentially informed seller (agent A) in the primary market at t=1, anticipating best responses at t=2. In Section 6 we analyze optimal security design at t=1 when there is public information at t=2 but no private information production. Finally, we assemble these results in Section 7 to characterize the equilibrium of the full game with public information at t=2 and where agents A and C can both choose whether or not to produce private information.

In this section we assume that agent A cannot produce information at t=1 so as to focus on the strategic interaction between agent B (seller) and agent C (buyer) in the secondary market at t=2. So agent B does not face the information acquisition constraint of agent A and agent A is willing to sell any contract s(x) with E[s(x)]=w in the primary market at t=1. To simplify notation, we can thus think of agent B as owning X and designing s(x) for trade with agent C at t=2 who has a t=2 endowment of w. We first analyze the case where agent C is privately informed (γ=0); and then the general case where agent C has to pay $\gamma \geq 0$ to become privately informed.

For the case where agent C is informed (γ=0), agent B solves the following optimization problem:

$$\max_{p,s(x)} EU_B = \int_{Q_<} x \cdot f(x)dx + \int_{Q_>} (\alpha p + (x - s(x)) \cdot f(x)dx \qquad (*)$$

where $p \in R_+$, {s(x): s(x)≤x}, and $Q_< = \{x | s(x) < p\}$, $Q_> = \{x | s(x) \geq p\}$.

The first term in (*) says that in states x such that s(x)<p, the informed agent C does not buy and there is no trade, so that agent B consumes x at t=3. The second term states if there is trade, then agent B consumes p at t=2 and x-s(x) at t=3. So agent B needs to choose a real number p and a function s(x) in the functional space {s(x): s(x)≤x} to maximize his expected utility.

[11] The most information-sensitive residual equity tranche is typically kept by the issuer or sometimes bought by the most sophisticated banks. Part (ii) shows that more debt can be issued if the debt portfolio is larger. See Gorton and Souleles (2006) and Gorton and Pennacchi (1993).

Lemma 4: Suppose agent C is privately informed (γ=0). The optimal contract that agent B offers to sell to agent C is a debt contract $s^D(x)$=[x,D] with price p=D where D maximizes $(1-F(D))D$.

Proof: The proof is in two steps. (1) We first show that debt maximizes the probability that agent B obtains any (desired) amount, p, of goods from agent C, as well as avoids any repayment larger than p (i.e., p=D). Then, (2), we derive the optimal p (price) and face value D.

Step 1: Suppose agent B offers to sell $s^D(x) = \min[x, D]$ for the price p=D. Since agent A knows the true value of x, he buys $s^D(x)$ only if x≥D. The set of states with no trade is $Q^D = \{x | x < D\}$. The probability of trade is $1-F(D)$. If trade occurs, then agent B repays D to agent A at t=2. Now, note:

(i) Since $s^D(x) = x$ for x≤D, there exists no other contract s where the set of states with no trade is smaller than Q^D, i.e. $Q^D \subseteq Q^S = \{x | s(x) < D\}$ and $1-F(D) \geq prob(trade \text{ under contract s})$ for all s∈S.

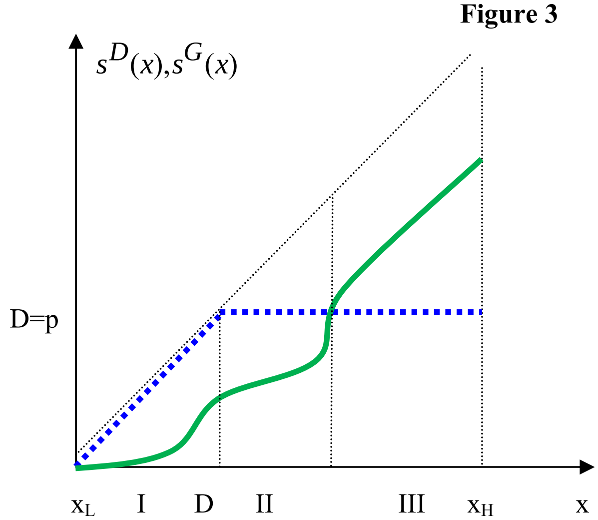
(ii) Consider a contract s where s(x)=x for x≤D and s(x)>D for some x. If trade occurs in these states, then agent B repays s(x)>D to agent C.

Step 2: Substituting p=D and s(x)=min[x,D] into (*) yields $EU_A = (1-F(D))(\alpha D - D) + E[X]$.

Agent B chooses D to maximize EU_B and thus $(\alpha-1)(1-F(D))D$, i.e. $(1-F(D))D$. //

Lemma 4 shows that for γ=0 and any distribution F(x), debt is the optimal contract for an uninformed seller to sell when facing a privately informed buyer. In other words, (p,s(x)) with p=D, s(x)=min[x,D] and D solving $(1-F(D))D$, is the unique solution to (*). Figure 3 highlights the intuition which compares the (dotted blue) debt contract with the ("thick green") contract $s^G(x)$ where $p^D=p^G$=D and $E[s^D(x)]=E[s^G(x)]$. In region I, under both contracts, there is no trade and agent B consumes x at t=3. In region II, under debt contract, agent B consumes D at t=1 and x-D at t=3, while with contract $s^G(x)$ he consumes nothing at t=1 and x t=3. Since αD+(x-D)>x, integrating over the states x in region II implies EU(D)>EU(G). In region III, agent B consumes D at t=1 and x-D at t=3 under the debt contract, agent B consumes D at t=1 and $x-s^G(x)$<x-d at t=3. Again, in this region EU(D)>EU(G).

Figure 3



The next result shows that debt is an optimal security for any γ, i.e. debt maximizes the expected utility of agent B when facing a potentially privately informed buyer (agent C).

Proposition 3: For any $\{F, \alpha, \gamma\}$, the optimal contract that agent B offers to sell to agent C is a debt contract.

(a) If $\gamma \geq \pi$, then agent B sells debt with face value D and price $p=E[s^D(x)]=w$.

(b) If $\gamma<\pi$, depending on $\{\alpha, \gamma\}$ agent B either chooses:

(i) Strategy I (Down-sizing debt): Debt with (D^I, p^I) such that $D^I<D$ and $p^I=E[s^I(x)]<w$. Agent C does not produce information.

(ii) Strategy II (Debt with Information Acquisition): Debt with (D^{II}, p^{II}) such that $D^I<D^{II}\leq D$ and $p^I<p^{II}<E[s^{II}(x)]\leq w$. Agent C produces information and there is adverse selection.

Proof: In step 1 we derive optimal contracts without triggering information acquisition and in step 2 we derive optimal contracts with information acquisition.

Step 1: Agent B may not be able to trade the amount of debt so that he can consume w at t=2 without information acquisition. He may have to reduce or "down-size" the debt relative to the maximum amount that can be traded without triggering information production, given in Corollary 3. This is Strategy I below. If down-sizing of the efficient amount of debt results in a transaction that is very small, then agent A can consider how much could be transacted if he makes an offer that just induces agent C to produce information, i.e., just covers agent B's cost of information production. This is Strategy II below. We start with Strategy I.

Strategy I (Down-Sizing Debt): Agent B chooses $(p^I, s^{DI}(x))$ where the

price p^I solves $\int_{x_L}^{p^I} (p^I - x) f(x)dx = \gamma$, and a face value D^I solves

$$\int_{x_L}^{D^I} xf(x)dx + \int_{D^I}^{x_H} D^I f(x)dx = p^I ,$$

i.e., the expected payoff is: $E[s^{D^I}(x)] = p^I$.

Note, if agent C is uninformed, he does not accept any offer $(p,s(x))$ with $p>E[s(x)]$. On the other hand, any $(p,s(x))$ with $p\leq E[s(x)]$ and $p>p^I$, triggers information production. Suppose agent B sells the whole project (i.e. $s(x)=x$) for $p=p^I+\varepsilon<E[X]$. For any $\varepsilon>0$, $\int_{x_L}^{p}(p-x) f(x)dx > \int_{x_L}^{p^I} (p^I - x) f(x)dx = \gamma$. In other words, agent C produces information because a larger expected overpayment can be avoided. On the other hand, in states where agent C trades, i.e. for x such that $s(x)>p$, agent C is better off than under the debt contract which implies that agent B is worse off.

Corollary 3 shows that if γ is small, then p^I (the amount agent B can consume at t=2) is small. Therefore, if γ is sufficiently low and α is large, then avoiding information acquisition may not be optimal.

Step 2: Now we derive a strategy that maximizes the payoff of agent B with private information acquisition by agent C. The construction of Strategy II is similar in spirit to the proof of Lemma 4 which shows that debt is optimal if $\gamma=0$.

Strategy II (Debt with Information Acquisition): Suppose agent B wants to consume the amount p^{II}, which he will choose optimally. Then debt is optimal since it maximizes the probability of obtaining p^{II}. Facing an informed agent C the set of states with no trade is minimized since $s^D(x)=x$ for $x \le p^{II}$. Given the "desired" p^{II}, the optimal modified debt contract has the following form: price $p=p^{II}$ and a face value $D^{II}= p^{II} + \kappa$ such that $\pi(p^{II}, D^{II}) = \gamma$, i.e. when agent C acquires information he just covers his information cost. Note, without information acquisition agent C would not buy the contract $s^D(x)$ for the price p^{II} since $E[s^D(x)]<p^{II}$. This can be seen as follows: Suppose $E[s^D(x)]=p^{II}$. Since $p^{II}>p^{I}$ (Corollary 3), this implies that $\pi^D > \gamma$. Reducing the face value to D^{II} such that $\pi(p^{II}, D^{II}) = \gamma$, implies that $E[s^D(x)]<p^{II}$.

Under Strategy II agent B's payoff is: $\alpha \int_{p^{II}}^{x_H} p^{II} f(x)dx + \int_{D^{II}}^{x_H} (x - D^{II}) f(x)dx + \int_{x_L}^{p^{II}} xf(x)dx.$

The first two terms correspond to a transaction where agent B obtains the amount p^{II} at date 2 and consumes the residual at date 3. The last term is the case where there is no transaction and agent B just consumes the value s(x) at date 3. Agent B's payoff can be written (with simple algebra) as:

$$\alpha(1 - F(p^{II}))p^{II} + E[X] - R(p^{II}, D^{II}),$$

where R can be interpreted as the expected payment to agent C; it is given by:

$$R = \int_{p^{II}}^{D^{II}} xf(x)dx + \int_{D^{II}}^{x_H} D^{II} f(x)dx.$$

Formally, Strategy II is debt with price p^{II}, face value D^{II}, and an expected payoff $E[s^D(x)] < p^{II}$ where:

p^{II} and D^{II} maximize $\alpha(1 - F(p^{II}))p^{II} + E[X] - R(p^{II}, D^{II})$

where R is the expected payment to agent B and

$$D^{II} \text{ solves } \pi(p^{II}, D^{II}) = \gamma : \quad \int_{p^{II}}^{D^{II}} (x - p^{II}) f(x)dx + \int_{D^{II}}^{x_H} (D^{II} - p^{II}) f(x)dx = \gamma .$$

The set of debt-like contracts maximizing $\alpha(1 - F(p^{II}))p^{II} + E[x] - R$ is given by {s: $s(x)=x$, for $x \le p^{II}$, $E[s(x)]$ such that $\pi=\gamma$} and $p=p^{II}$. If $\gamma=0$, the optimal contract is unique and given by Proposition 3.

Agent B chooses the strategy, either Strategy I or Strategy II, with the highest expected utility. In any case, debt is issued. //

Proposition 3 says that agent B has two types of potential best responses when facing a potentially informed buyer (agent C) at t=2. Strategy I is writing down debt relative to the efficient level such that agent C buys without information production. In this case agent B consumes p^{I} at t=2. Agent B is not able to receive any higher price without triggering information production. Note, $p>p^{I}$ implies $\pi^D > \gamma$ since p^{I} is set such that $\pi^D = \gamma$. We will come back to this point in the next section. For γ small, p^{I} is small. Then the best response of agent B may be to induce adverse selection. Trade occurs with probability less than one but if trade occurs agent B can consume $p^{II} >p^{I}$.

Note, if $\gamma \geq \pi^D$, then it is clearly optimal for agent B to choose Strategy I with $p=E[s^D(x)]=w$ for any α. (Agent B consumes the maximum possible amount w at t=2 and agent C gets his outside option.) If $\gamma < \pi$, then depending on γ and α agent B may either choose Strategy I or Strategy II.[12]

Proposition3 describes the optimal security design by agent B, facing agent c who can potentially produce private information. With regard to agent B facing agent A at t=1, we have:

Corollary 4: Suppose agent A cannot produce information at t=1 and there is no public information at t=2. Agent B has two types of optimal debt strategies. (i) At t=1, agent B buys a debt contract $s^D(x)$ from agent A with $E[s^D(x)]=w$ and p=w. At t=2, when facing agent C, agent B possibly writes down debt according to Strategy I or II in Proposition 3 and consumes any unsold part of s(x) at t=3. (ii) At t=1 agent B buys debt from agent A according to Proposition 3 and sells it to agent C at t=2 without redesign and consumes any unspent amount w-p at t=1.

Corollary 4 is a building block for the equilibrium of the full game, analyzed in Section 7.

5. Optimal Security Design when the Seller is Potentially Informed (at t=1)

In this section we analyze optimal security design by agent B when he faces a potentially informed seller (of a security), namely, agent A in the primary market at t=1. To focus on the strategic interaction between agent B and A, we assume that agent C cannot produce information at t=2 and there is no public information. We first analyze the special case γ=0 (in subsection A) and then case of $\gamma \geq 0$ (in subsection B). Since actions are publicly observable and agent B and C have symmetric information, agent C is willing to buy s(x) from agent B for the price $p = E[s(x) | \text{agent A sells } s(x)]$.

A. Agent A is Private Informed (γ=0)

At t=1, agent B chooses the pair (p, s(x)), i.e. a price and a security (a function) s(x), to maximize his expected utility. For γ=0, i.e. agent A is informed, agent B solves the following optimization problem:

$$\max_{p,s(x)} EU_B = \int_{Q_<} (w - p + \alpha s(x)) \cdot f_k(x)dx + \int_{Q_>} w \cdot f_k(x)dx \qquad (**)$$

where $p \in R_+$, $\{s(x): s(x) \leq x\}$, $Q_< = \{x | s(x) \leq p\}$ and $Q_> = \{x | s(x) > p\}$.

The first term of (**) says that in states where agent A sells (i.e. $s(x) \leq p$), the value of the security is $E[s(x) | s(x) \leq p]$ which is the third term in the first integral. This is also the amount of goods that agent C is willing to pay for the security at t=2. Thus, in each of these states x, agent B consumes s(x) at t=2 and w-p at t=1. Integrating over the relevant set of states $Q_< = \{x | s(x) \leq p\}$ yields the expected utility of agent B conditional on that he has bought s(x) from agent A. The second term states that agent B consumes his endowment w at t=1 if there is no trade with agent A. (In this case he does not

[12] It is easy to see that Strategy I dominates Strategy II if γ is only slightly smaller than π. On the other hand, Strategy II (adverse selection) dominates Strategy I if $\alpha(1 - F(p^{II}))p^{II} + E[x] - R >$ $> \alpha p^I + E[x] - p^I \Leftrightarrow \alpha(1 - F(p^{II}))p^{II} - R > \alpha p^I - p^I$. This is the case if γ close to zero since $p^I \approx 0$ and $R \approx (1 - F(p^{II}))p^{II}$, which implies that $\alpha(1 - F(p^{II}))p^{II} - (1 - F(p^{II}))p^{II} > 0$ for $\alpha > 1$.

trade his t=1 goods for agent C's t=2 goods since agent B has a higher marginal valuation of consumption at t=1 than agent C.)

Lemma 5: Suppose agent A is privately informed at t=1 (γ=0). Then the optimal contract for agent B to buy from agent A is debt with price p=D and face value D, where D maximizes:

$$\int_{x_L}^{D} \alpha x f(x)dx + \int_{D}^{x_H} \alpha D f(x)dx - D .$$

Proof: Agent A only sells (the security) if s(x)≤p. For a given price p, it is a strictly dominated strategy for agent B to choose a contract s(x) where s(x)>p (for any x>p) since in these states agent A does not sell which reduces $E[s(x)|trade]$, i.e. the expected consumption of agent B at t=2 by trading $E[s(x)|trade]$ for agent C's goods. Thus an optimal contract must have s(x)≤p for all x. In words, agent A should not ask for a repayment larger than the price. Since $s^D(x)=\min[x,p]$ maximizes (pointwise) what agent A can consume at t=2 because $s^D(x)=x\geq s(x)$ for all x≤p and all s(x), debt is the optimal contract. Substituting p=D and s(x)=min[x,D] into (**) yields $EU_B = w + \int_{x_L}^{D} (\alpha x - D) f(x)dx + \int_{D}^{x_H} (\alpha D - D) f(x)dx = w + \int_{x_L}^{D} \alpha x f(x)dx + \int_{D}^{x_H} \alpha D f(x)dx - D$. (Note, suppose agent A offers to buy debt with price D and face value D' larger than D (price), then agent B does not sell in states x>p=D since he repays s(x)>D and $EU_B = w + \alpha E_k[s(x)|trade] - D = w + \int_{x_L}^{D} \alpha x f(x)dx - D$. With face value D', the expected value of the security conditional on trade is strictly smaller, i.e. $E_k[s^{D'}(x)|trade] < E_k[s^D(x)|trade] < p$.) //

We can use Figure 3, introduced previously, to highlight the intuition. Suppose $E[s^D(x)]=E[s(x)]$ and both contracts have the same price p=D. In regions I and II, there is trade under both contracts and agent B consumes w-p at t=1. Under the debt contract, agent B can sell $s^D(x)$ for the price $p^D = E[s^D(x)|\text{agent A sells}]$ to agent C and consume this amount at t=2. Under contract s(x), agent B can sell s(x) for the price $p^S = E[s(x)|\text{agent A sells}]$ to agent C and consume this amount at t=2. Since $s^D(x)\geq s(x)$ (point-wise) $p^D\geq p^S$, agent B can consume more under the debt contract at t=3. For x in region III, there is no trade under s(x) and agent B consumes w at t=1 and nothing at t=2. But under debt contract, agent C consumes w-D at t=1 and $p^D = E[s^D(x)|\text{Agent A sells}] = D$ at t=2. Since w-D+αD>w, debt also strictly dominates s(x) in region III. Now we turn to the general case where γ≥0. Note, $(p,s^D(x))$ is chooses to maximize EU_B such that EU_B>w (no trade at t=1).

Now we turn to the general case where γ≥0.

B. Agent A Faces Costly Information Production (γ≥0)

We now turn to the case where information production is costly.

Proposition 4: For any {F, α, γ}, the optimal contract that agent B offers to buy from agent A is a debt contract.

(a) If $\gamma \geq \pi$, then agent B buys debt with face value D and price $p=E[s^D(x)]=w$.

(b) If $\gamma < \pi$, depending on {α, γ} agent B either chooses:

(i) Strategy I (Down-sizing debt): Debt with (D^I, p^I) such that $D^I<D$ and $p^I=E[s^I(x)]<w$. Agent A does not produce information.

(ii) Strategy III (Surplus-sharing; bribe): Debt with (D^{III}, p^{III}) such that $D^I<D^{III}\leq D$ and $p^I<p^{III}<E[s^{III}(x)] \leq w$. Agent A does not produce information.

Proof: See Appendix A.

Proposition 4 states that agent B compares two different strategies. In Strategy I agent B computes the maximal amount of debt that can be traded without triggering information production and without giving agent A any surplus. Strategy III also avoids information production but agent A gets some surplus (a bribe). In this case, agent B offers to pay more than the expected value of the bond, but the bond has a higher face value than in Strategy I. Strategy III may dominate Strategy I because it achieves a larger amount that is traded, and hence is more efficient.

Proposition 4 states that it is never a best response for agent B to induce agent A to acquire information. The intuition is the following. Consider any debt contract ($p,s^D(x)$) such that agent A acquires information. This implies that $\pi>\gamma$ (and thus D>p) and in particular agent A does not sell in states x where $s^D(x)=D>p$. For any price p, agent B can strictly do better if he reduces the face value to D=p. Trade occurs with probability one while he pays the same price. The optimal (p,D) is given in the proof of Proposition 4.

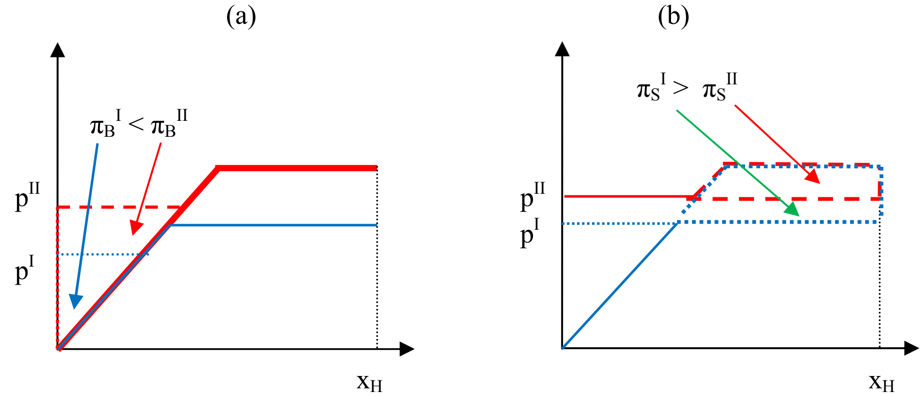
Another way of seeing why Strategy III (surplus sharing; bribe) strictly dominates a strategy that induces adverse selection is as follows. Lemma 5 shows that if agent A is informed the best response of agent B is to propose to buy a debt contract with price equals to face value. In other words, it is not optimal for an uninformed buyer to ask for more than he is paying, i.e. s(x)>p for x>p, because this reduces the probability of trade and the expected value of s(x) conditional on trade while paying the same price p. Consequently, if such a contract is to be traded as the optimal response of agent B when facing an informed agent A, then agent A does not produce information in the first place.

Comparing Proposition 4 with the earlier Proposition 3 reveals an interesting asymmetry in the strategies. If an uninformed buyer faces a potentially informed seller, his best response is to propose either Strategy I, the maximum debt write-down or, Strategy III, a combination of a debt write-down and a bribe for the seller not to produce information. Previously, if an uninformed seller faces a potentially informed buyer, his best response is to propose either Strategy I, maximally write-down debt, or Strategy II, a contract that induces the buyer to produce private information. Bribing the buyer not to produce information by offering a larger E[s(x)] is not a best response of the uninformed seller because it is ineffective.

These results formalize the notion that an uninformed agent on the sell side of the market may face more difficulty selling than an uninformed agent who wants to buy. If an uniformed seller (agent B at t=2) wants to sell a security, he cannot obtain (and thus consume) more than p^I (of Strategy I of Proposition 3) even if the expected payoff E[s(x)] is very large. If he wants to consume more than p^I, this triggers information production by the buyer and he faces the risk of not being able to sell at all. On the other hand, if an uninformed buyer (agent B at t=1) wants to buy and consume E[s(x)], then by offering a price high enough, he can always induce an (informed or potentially informed) seller to sell.

The asymmetry has to do with the information sensitivities of the buyer and the seller, π_B and π_S. Agent B (the uninformed seller) cannot raise the price above p^I because that *increases* the blue dotted-point triangle to the larger red dotted-line triangle in Figure 4 (a), raising π_C for agent C. However, an uninformed buyer (agent B) can offer to raise the price without this problem. In contrast, raising the price reduces π_A of the seller (agent A). In Figure 4 (b), the blue dotted point area *shrinks* to the red dotted line area. Therefore, it is "easier" for an uninformed agent to buy than to sell securities.

Figure 4



Proposition 4 also shows that the assumption that w<E[X] is not crucial. If w≥E[X] and γ is high, then trading the whole project or issuing a degenerate debt contract with face value $D=x_H$, is optimal, i.e. agent B buys s(x)=x for p=E[X]. If $\gamma<\gamma'$ (where γ' is the cost of information production such that exactly the efficient level of debt can be issued), instead of trading the whole project, buying debt with face value $D<x_H$ is optimal. As γ decreases, the best response of agent B is to reduce the face value.

6. Optimal Security Design When There is Public Information (at t=2)

Propositions 3 and 4 show that for γ sufficiently high and when there is no public information at t=2, any contract with expected payoff E[s(x)]=w is optimal. In this section we derive a *further* benefit of debt in a setting where γ is sufficiently high so that there are no adverse selection concerns, but where there is an interim public signal. We show that debt maximizes the amount of intertemporal trade that can be implemented. The amount of intertemporal trade that can be implemented we call a security's "trading capacity."

Here is a summary of the argument. Suppose no agent can acquire information about the true value of the underlying asset X at any date, i.e., $\gamma=\infty$. Since there are no adverse selection concerns, it is easy to see that an optimal security for agent B to buy at t=1 has p=w and payoff $E_m[s(x)]=w$. At t=2, it is efficient for agent B to consume by selling the security s(x) to agent C in exchange for goods. But at t=2, there is a public signal about the distribution of X. When the signal reveals that distribution k is the true distribution, then the market value of the security is $E_k[s(x)]$. This means that the resale price of security s fluctuates. In this section we ask: what date 1 security, s(x), maximizes the expected trading capacity between agents B and C and thus the expected consumption of agent B at t=2? Note, agent C owns w units of goods at t=2.

Proposition 5 (Debt maximizes trading capacity): Suppose {s(x): non-decreasing in x} and {F: each F_k has non-overlapping support}. And suppose no agent can acquire private information ($\gamma=\infty$). Then buying debt from agent A with $E[s^D(x)]=w$ and p=w at date 1 maximizes the expected utility of agent B.

Proof: See Appendix A.

To highlight the intuition, suppose K=2. Then we have $E_1[s(x)] < w < E_2[s(x)]$ and $E_1[s^D(x)] < w < E_2[s^D(x)]$. We show by contradiction that there exists no s(x) such that $E_1[s(x)] > E_1[s^D(x)]$. Suppose $E_1[s(x)] > E_1[s^D(x)]$. Then we must have s(x)>D for some x' where D<x'< x_L^1. Non-decreasing of s(x) implies that $s(x)>s^D(x)=D$ for all x>x'. In particular, we have $s(x)>s^D(x)=D$ for all $x \in [x_L^2, x_H^2]$. See Figure 5 (a). This implies that $E_2[s(x)] > E_2[s^D(x)]$. Since $\lambda_1 E_1[s(x)] + \lambda_2 E_2[s(x)] = w$ and $\lambda_1 E_1[s^D(x)] + \lambda_2 E_2[s^D(x)] = w$, this implies $E_1[s(x)] < E_1[s^D(x)]$.

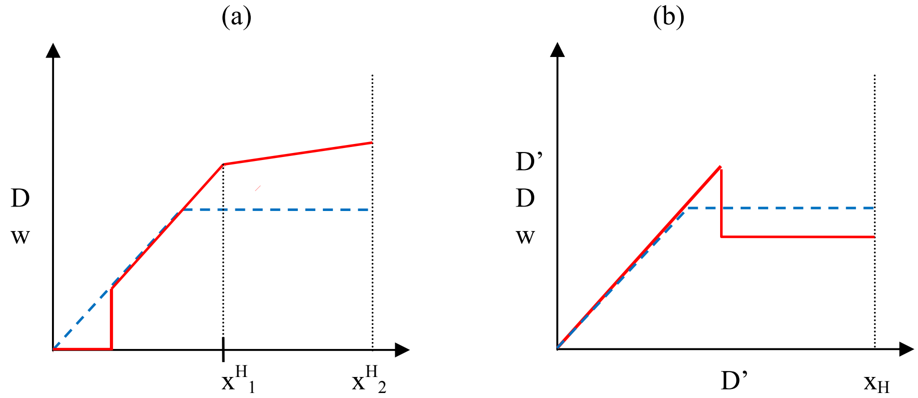
Proposition 5 shows that bad interim news reduces what agents B and C will trade, since they trade the amount $E_k[s(x)]<w$; and with good interim news agents B and C trade the amount w. Now suppose agent C has an endowment $w_C>w$ at t=2. As long as $w_C<E_K[s^D(x)]$ debt also (weakly) dominates any other security. On the other hand if $w_C>x_H$, then any security gives rise to the same expected trading capacity. In this case, from the ex ante (t=1) point of view the expected amount agent C can consume at t=3 is w, i.e. the expected payoff of the security (that agent A has issued at t=1). Note, $\sum_{k=1}^{K} \lambda_k E_k[s(x)] = E[s(x)] = w$.

Intuitively, Proposition 5 shows that even without any adverse selection concern, securities with a high variance of resale prices are less attractive than assets with low price fluctuations. Price fluctuation (payoff variance), however, is not the same as information-sensitivity as we have defined it. Note,

$\pi_B + \pi_S = \int_{x_L}^{x_H} |s(x)-p| f(x)dx = E[|s(x)-p|] \equiv \pi$. Although (total) information-sensitivity π of s(x) looks similar to the variance $E[(s(x)-p)^2]$ of s(x), we can show that they are not necessarily rank-correlated.

If we allow contracts to be non-monotonic, i.e. $\{s(x): E_m[s(x)]=w\}$, then the contract that maximizes agent B's expected consumption at t=2 is: s(x)=D' for x≤D' and s(x)=w for x>D' and $E_m[s(x)]=w$. See Figure 5 (b). This holds for {F} satisfying FOSD.

Figure 5



Proposition 5 highlights an interesting point. Even if agents have linear utility function, welfare under ignorance (IG) is higher than the welfare in a setting where agents have symmetric and partial or perfect information. Suppose that the interim information perfectly reveals the true realization of X at t=2 and $w\in(x_L,x_H)$ and arbitrary F(x). Note, $U_B=\alpha c_{B2}+c_{B3}$ and $U_C=c_{C2}+c_{C3}$. To highlight the intuition, suppose w=1.5 and X is binary and either 1 or 2 with equal probability. Consider the allocation that maximizes agent B's utility subject to agent C getting his reservation utility U_C=w=1.5. If agents are uninformed they trade w for X and $EU_B(IG)=1.5\alpha$. Under perfect information, if x=1, agent B consumes one unit at t=2 since agent C is not willing to trade w=1.5 for x=1. If x=2, agent B can consume at most w=1.5 at t=1. Thus $EU_A(PI)=0.5\cdot 1\alpha+0.5\cdot(1.5\alpha+0.5)=1.25\alpha+0.25<1.5\alpha=EU_A(IG)$.

The reason for this observation is that the utility function of agent B has a kink at the endowment level w of agent C. Therefore, if x>w, agent B is must consume some x at t=3. Thus for x<w, the utility function of agent B has slope α and for x>w, the slope is 1. So although agent B' intertemporal utility function is linear in consumption, the fact that $w\in(x_L,x_H)$ induces concavity in agent B's utility function. Thus ignorance at the date of trade strictly dominates perfect information or partial information if $E[x|I]<w$ for some information I. In the example above, the information I reveals the true x. This is reminiscent of Hirshleifer (1971).

7. Equilibrium Analysis of the Full Game

As a prelude to the equilibrium analysis, in subsection A we first examine how the interim public signal changes the information-sensitivity of debt and alters the strategies of the agents. Then we will be in a position to analyze the equilibrium of the full game. In subsection B we characterize the equilibrium of the whole game with public information at t=2 and where agents A and C can produce private information. Subsection C briefly discusses the role of agent B in the model.

A. Information Acquisition, Information-Sensitivity, and Trading at the Interim Date

In this subsection we first characterize how public information changes the information-sensitivity of s(x), and thus the incentive of agent C to produce information at t=2 if the whole s(x) is traded. Whether trading the security s(x) at t=2 triggers information acquisition or not depends on the date 2 information-sensitivity, π(k), of that asset relative to the information cost γ. Since prices $p(k)=E_k[s(x)]$ fluctuate with the public signal k, π(k) (i.e., the information-sensitivity after F_k has been revealed publicly) also changes with the public signal since:

$$\pi(k) = \int_{x_L}^{x_H} \max[p_k - s(x),0] \cdot f_k(x)dx \text{ where } p_k = \int_{x_L}^{x_H} s(x) \cdot f_k(x)dx.$$

If s(x) is non-decreasing, prices are monotonic in k given the assumption of partitional information (or First Order Stochastic Dominance). But the information-sensitivity, π(k), of a security is a complicated object. In particular, even with the assumption of partitional information (or First Order Stochastic Dominance), π(k) is typically non-monotonic in k. See Appendix C.

Suppose agent B bought a security s(x) from agent A at t=1. Now at t=2, agent B makes a take-it-or-leave-it contract offer to agent C who can produce private information. We allow for complete contracting, i.e. agent B can take s(x) as the underlying asset to create a new contract $\hat{s}(y)$, where y=s(x). Agent B can then sell the redesigned contract to agent C for agent C's t=2 good. For example, (i) if $\hat{s}(y)$=y than agent B proposes to sell the whole s(x) that he bought from agent A to agent C; (ii) if $\hat{s}(y) = \min[y, \hat{D}]$, then agent B proposes to sell a debt contract with face value $\hat{D}$ taking s(x) as collateral. If the initial contract s(x) is debt with face value D, then agent B writes down debt to $\hat{D}$<D. More generally, agent B chooses two elements (p, $\hat{s}(y)$), a price and a security where {$\hat{s}(y)$: $\hat{s}(y)$≤y , y=s(x)} to maximize his expected payoff.

Lemma 6: Suppose agent B purchased debt with E[s^D(x)]=w from agent A at t=1. If, (a), $E_k[s^D(x)] \geq E_m[s^D(x)]$ (i.e., good news), or (b), $E_k[s^D(x)] < E_m[s^D(x)]$ and $\pi^D(k) \leq \gamma$, (i.e., bad news, but not so bad as to trigger information production), then trading at t=2 results in an efficient consumption allocation between agents B and C.

Proof: (a) In this case, there was good news. There are two sub-cases: (i) If $\pi^D(k) \leq \gamma$, i.e., information production is not profitable, then *one* best response of agent B is to sell the fraction $\kappa = \frac{w}{E_k[s^D(x)]}$ of his debt for the price w (a vertical strip). This is because the good news has caused the bond to rise in value so that it is worth more than w. Agent B offers to sell a strip that is just worth w. Agent C buys without information acquisition since $\kappa\pi^D(k) < \pi^D(k) \leq \gamma$. (ii) If $\pi^D(k) > \gamma$, then information production is profitable. As a result, agent B will offer to sell a new debt contract with face value $\hat{D} < D$ and price w, taking the original debt contract as the underlying collateral (a horizontal slice). Agent B wants to design the new bond so as not to trigger information production by agent C. Agent C buys the horizontal slice without information acquisition since $\pi^{\hat{D}}(k) < \pi^D(m) \leq \gamma$. Note, $p^{\hat{D}}(k) = p^D(m) = w$ and $F_k \underset{SD}{\succ} F_m$ (i.e., stochastically dominates) imply that $\pi^{\hat{D}}(k) < \pi^D(m)$. In both these sub-cases, Agent B consumes w at t=2 and has an expected consumption of $E_k[s^D(x)] - w$ at t=3; and at t=2, agent C has no consumption at t=2 and an expected consumption of w at t=3.

(b) In this case there was bad news, but not so bad that information production is triggered. Agent B sells the whole debt for the price $E_k[s^D(x)]$. Agent C buys without information acquisition since $\pi^D(k) \le \gamma$. Agent B consumes the amount $E_k[s^D(x)]$ of goods at t=2; agent C consumes $w - E_k[s^D(x)]$ at t=2 and $E_k[s^D(x)]$ at t=3. //

Lemma 6 (a) part (i) of the proof shows how a pro rata reduction (a vertical strip) in the amount of debt traded can keep agent C from having an incentive to produce information. The cost of information production is a fixed amount, so reducing the amount of debt traded can eliminate the incentive to produce information. Part (ii) of the proof is different in that trading a vertical strip may not suffice to prevent information acquisition. The agent B needs to issue a new debt contract, where agent B retains a junior equity tranche relative to what agent C is willing to accept — the senior tranche. Horizontal slicing is more powerful because it introduces seniority: agent B retains the equity piece of the newly issued (redesigned) bond. Any information produced would be wasted because it mostly concerns the residual (the junior equity tranche) which agent B will keep in any case.

B. Equilibrium Debt Issuance and the Possible Collapse of Debt Trading

Now we are in the position to characterize the set of Perfect Bayesian Nash equilibria (BNE) in the full game with interim public news arrival about the distribution of x and the possibility of information acquisition by agents A and C.

***Proposition 6 (Debt Equilibrium)*:** Consider the economy $\{\alpha, \gamma, w, \{F\}_{i=1}^{K}\}$ and suppose $\gamma \ge \pi(m)$. All BNE are outcome equivalent in terms of consumption and have the following properties:

At t=1, agent B buys debt with $p=E[s^D(x)]=w$ from agent A who does not produce information.

At t=2, there is efficient trade between agents B and C,, if:

(a) $E_k[s^D(x)] \ge E_m[s^D(x)]$ or (b) $E_k[s^D(x)] < E_m[s^D(x)]$ and $\pi(k) \le \gamma$.

(b) If $E_k[s^D(x)] < E_m[s^D(x)]$ and $\pi(k) > \gamma$ then depending on the revealed distribution F_k, the best response of agent C is to choose either:

(i) Strategy I (maximum write down; agent C does not produce information);

(ii) Strategy II (adverse selection; agent C produces information).

There is insufficient trade.

At t=3, agents who own a claim on X consume the goods delivered by the claim.

Proof: See Appendix A.

Proposition 6 has the interpretation that at t=2 if there is good news $E_k[s^D(x)] \ge E_m[s^D(x)]$, then there is efficient debt trading between agents B and C. With bad news that causes the information-sensitivity of the original debt contract to rise, there is insufficient debt trading. For example, there is a collapse of debt trading in the sense that agents B and C trade less than the (new) market value of agent B's debt. In the numerical example below, if there is bad news agents B and C trade a senior tranche of 20% of the market value of agent B's debt, i.e. there is a 80% write-down of the original debt contract. This corresponds to "systemic risk" because the outcome is worse than that caused only by the fundamentals. The "fundamentals" corresponds to the bad shock k. Instead of trading at the new expected value of the debt, agents trade much less than they could or even not at all. In this sense there is a collapse of trade.

Proposition 6 is perhaps best understood with an example. Suppose F_1 ~u[0, 0.8], F_2~u[0.8, 1.2], F_3~u[1.2, 2] and $\lambda_1 = \lambda_2 = \varepsilon$, and $\lambda_3 = 1 - 2\varepsilon$. Then: f_m=5ε/4 for x∈[0, 0.8], f_m=5ε/2 for x∈[0.8, 1.2],

f_m=5(1−ε)/4 for x∈[1.2, 2]and f_m=0 else. That is, these are the prior densities (for the mixture distribution) over the different intervals corresponding to the k-distributions. Suppose $\varepsilon = 0.00001$, $w = 1$, $\gamma = 0.001$, and $\alpha = 1.1$. The subsequent numbers are exact up to the fourth decimal.

In this example, agent B buys debt with face value $D = 1$ and price $p_m^D = 1$. Agent A sells without producing information.[13] Equilibrium outcomes at t=2 are as follows.

(i) If F_1 is the true distribution, then $p^D(1) = 0.4$, $\pi(1) = 0.1$. (I) If agent B chooses to avoid information acquisition and does not give agent C any surplus (Strategy I), then $p^I(1) = E[s^I(x)] = 0.04$ and $D^I(1) = 0.0411$, and $EU_C = (1 - p^I) + \alpha p^I = 1.004$. (II) If agent B chooses to sell at a higher price and induces information production by agent C, then the best offer is $p_1^{II} = 0.4$. Agent C buys when $x \geq p^{II}(1)$ and $EU_B = w + prob(x \geq p^{II}) \cdot (\alpha p^{II} - E[s(x) | x \geq p^{II}]) = 1 + 0.5(1.1 \cdot 0.4 - 0.407)$ =1.217. All other p_1^{II}-type prices yield lower utilities. Thus agent B chooses Strategy II (adverse selection). There is no trade if x<0.4.

(ii) If F_2 is the true distribution, then $p^D(2) = 0.95$, $\pi^D(2) = 0.0281$. If agent B chooses Strategy I, then $p^I(2) = 0.8283$ and $EU_C = (1 - p^I) + \alpha p^I = 1.0828$. If agent B chooses Strategy II, then $p^{II}(2) = 0.95$ and $D^{II}(1) = 0.98$. Agent C buys if $x \geq p^{II}(2)$ and $EU_B = 1 + \frac{5}{8} \cdot (1.1 \cdot 0.95 - 0.97) = 1.0467$. Consequently, the best response of agent B is also to propose maximum write-down of debt. Although agent C has enough endowment agent B chooses not to sell the whole debt. Instead agent B sells a new debt contract, i.e. $\hat{\kappa} = \frac{\hat{p}_k^D}{p_k^D} = \frac{0.8283}{0..95} = 0.872$ or 87.2% percent of expected cash flow as a senior tranche.

(iii) If F_3 is the true distribution, then $p_3^D = 1$, $\pi(3) = 0$. Agent C buys the (whole) debt of agent B for the price 1.

To summarize this example, if there is good news (i.e., F=F_3), there is efficient trade between agents B and C at t=2. If there is bad news (i.e., F=F_2), then the market price of debt drops from 1 to 0.95 and agent C buys a senior tranche of 87.2% of agent B's debt. This can be interpreted as a haircut of 11.6%. If there is very bad news (i.e., F=F_1), then the market price of debt is 0.4. Agent B offers to sell debt for that price. If x<0.4, there is no trade.

A financial crisis is an event where the outcome (in terms of the amount traded, and hence utility) is worse than what would happen purely based on the "fundamentals", i.e. agents trade less than the expected value of the bond conditional on the new public information. Bad news can arrive and the

[13] Note, π(m)=0. If equity with $\beta \approx \frac{5}{8}$ and $p_m^E = 1$ is issued, then $\pi^E(m) \approx 0.0625$ and this triggers information acquisition at t=1.

information-sensitivity of the bond can increase. In the numerical example, the amount traded and consumed drops dramatically. In the worse case news is so bad that agent B chooses Strategy II and there is adverse selection.

In general, public news that triggers a reduction in trade and consumption is a signal that results in $\pi(k) > \gamma$. We assume that the cost of producing information is a fixed amount, γ. Once the threshold $\pi(k) > \gamma$, is crossed agents are concerned about potential adverse selection. This is the "loss of confidence" and the source of the suddenness of the financial crisis when information-insensitive debt becomes information-sensitive.

Proposition 7: Suppose {F} satisfies the assumption of a partition. The debt equilibrium is a second best outcome, i.e., it is constrained efficient.

Proof: Propositions 5 and 6 state that debt is optimal for facilitating trade between agents B and C. We have to prove that for $E_k[s^D(x)] < E_m[s^D(x)]$ and $\pi(k) > \gamma$, from the point of view of agents B and C, issuing debt at t=1 also weakly dominates any other contract s.

Suppose contract s has been issued at t=1. Note, Proposition 5 states that $E_k[s^D(x)] \geq E_k[s(x)]$. There are two cases to consider, (a) and (b).

(a) Suppose $\pi^S(k) \leq \gamma$, i.e., trading the whole contract s does not trigger information acquisition. Here is a "replication strategy" given debt has been issued: Agent C can propose to buy a new debt contract with face value $\hat{D} < D$ (taking the original debt contract as the underlying collateral) and where the price equals the market value of contract s, i.e. $p^{\hat{D}} = E_k[s^{\hat{D}}(x)] = E_k[s(x)]$. In this case $\pi^{\hat{D}}(k) < \pi^S(k) \leq \gamma$. (Proposition 1 states that debt is a least information-sensitive security given two securities with the same expected payoff.) Debt is at least as good as any contract s.

(b) Suppose $\pi^S(k) > \gamma$, i.e., trading the whole contract s triggers information acquisition. Since $E_k[s^D(x)] \geq E_k[s(x)]$, with a date 1 debt contract agent C can replicate any new contract s' that takes contract s as the underlying collateral. Thus $p^{\hat{D}} = E_k[s^{\hat{D}}(x)] = E_k[s'(x)]$ and $\pi^{\hat{D}}(k) \leq \pi^{S'}(k)$. More precisely, for x≤D, by redesigning the original debt contract, $s^D(x)=x$, agent C can replicate the payoff of any contract s(x) or any redesign contract s'(y) with y=s(x). For x>D, contract s(x) may generates a higher repayment in states where $s(x)>s^D(x)=D$. But in these states, the privately informed agent B only sells if agent C offers at least p=s(x)>D. But under the debt contract $s^D(x)$, if agent C proposes the price p=D, agent B sells in all states and agent C's expected consumption at t=3 is $E_k[s^D(x)]$. Under the contract s, agent C does not always sell, if s(x)>D for some x. Thus issuing debt at t=1 is optimal. //

Proposition 7 states that issuing a debt contract at date 1 is as good as any contract if $E_k[s^D(x)] \geq E_m[s^D(x)]$, and allows agents B to *replicate* the payoff of any contract at date 2 when $E_k[s^D(x)] < E_m[s^D(x)]$. In other words, debt issuance at t=1 maximizes the flexibility to redesign contracts at t=2. A graphical illustration is given in Figure C1 (b) in Appendix C, that compares debt and equity.

C. Model Assumptions

We assume that agent B cannot produce information. This assumption is meant to capture the idea that not all agents are equally sophisticated and able to produce private information which is realistic in financial markets, in particular in the market for securitized assets. This paper highlights how the design of security can avoid adverse selection to facilitate trade and efficient consumption and how public information about fundamentals and the (mere) concern of uninformed agents about potential adverse selection can cause a collapse of trade.

From a technical point of view, if agent B can also produce information, then we have to analyze a much more complicated game since the proposer can be informed and in that case he can signal both with prices and contracts. In a standard signaling game, the informed agent is endowed with an asset X and only chooses a price p to signal his type x (i.e. the realization of X). Here both the price p as well as the function s(x) are endogenous variables and X is a continuous random variable. This is a complicated signaling problem and to our knowledge an unexplored one. For example, DeMarzo and Duffie (1999) and Biais and Mariotti (2005) assume that the issuer designs a security before he obtains private information but at the date of sale there is asymmetric information.

Conceptually, agent A can calculate whether it would pay for agent B as the proposer to produce information at t=1. If it pays for agent B to learn then there is no pure strategy equilibrium. Seeing a low price and some s(x), equilibrium requires agent A to first randomize his information production decision. If he does not produce information then he will also randomize his acceptance decision. See Dang (2008) for a discussion of this issue. In any mixed strategy equilibrium there is a (strictly) positive probability that no trade occurs. Therefore, if all agents can produce information, the adverse welfare implications are more severe.

8. Discussion and Extensions

A. Security Design and Complexity

In the financial crisis, many bonds of securitized assets (asset-backed securities) were used as collateral for repo. These bonds are complicated. The internal workings of the cash flows from the underlying portfolios of loans are allocated in complicated ways, and the underlying loans themselves are complicated. See Gorton (2008). These asset-backed securities were also used as the assets in other structures, such as collateralized debt obligations and structured investment vehicles.

Why such were complicated structures used? Our model sheds light on this issue. If complexity raises the cost of producing information, raises γ, this can be welfare improving. Suppose that agent B could choose a level of complexity for the security designed at t=1. This corresponds to choosing some γ less than a given maximum. For large w, agent B would always choose to issue the most complex security, the one with the maximum γ because this maximizes the amount of debt.

Asset complexity can facilitate trade as long as uninformed agents commonly and correctly believe that this makes information production by sophisticated agents unprofitable. But if public information about fundamentals makes the assets information-sensitive and thus information production profitable, then we argue that uninformed agents face difficulty in reselling these assets and this has a negative feedback effect on trade even between two agents that are known not to able to produce any information. There is a trade-off between creating liquidity for a sequence of trades and a sudden collapse of trade in a financial crisis. In our model assets are designed to minimize adverse selection concerns so as to facilitate intertemporal trade, but when these assets become information-sensitive less sophisticated agents are only willing to buy at very low prices or have no demand at all.

B. Rating Agencies

Rating agencies are a puzzle. Why do they exist? Equities are not rated. The standard version of "efficient markets" in equities has agents becoming privately informed and trading on their information. Prices are informative and there is no need for rating agencies. Why are debt markets different? Also, why do rating agencies only produce coarse signals, when as the critics have pointed out, risk is multi-dimensional? Our model can address these questions.

One of the possible equilibrium outcomes is the possibility that agent B produces information and trade is reduced. A rating agency can minimize this welfare-reducing outcome, possibly by enough to justify the fee of γ charged by the rating agency for information production. In this subsection we sketch how this would work, but for brevity we do not present formal results.

The rating agency is a firm which commits to announce ratings just after the realization of the interim aggregate signal. For each possible distribution k that could be realized, the rating agency commits at date 1 to a set of partitions $\{I(k)\}$ of the support of distribution F_k. These are the ratings. Upon the realization of distribution k, the agency truthfully announces the rating (partition that contains x).

How could this help? Imagine that the distribution that is realized is one for which agent B would choose to produce information. If the agency has chosen its partitions correctly, then conditional on the announcement of the partition/rating, the value of information to agent B can decline sufficiently so that he does not find it optimal to produce information; welfare is improved. This is the mechanism by which the ratings can help.

The rating agency's optimization problem, however, is very complicated. On the one hand, partitions cannot be too fine because information destroys trade. On the other hand, partitions cannot be too coarse or else agent B will still have an incentive to produce information when we would prefer that he not produce information.

C. Lender-of-Last-Resort

What exactly is the role of the lender-of-last resort? In our set-up this is clear. The lender-of-last-resort's role is to exchange information-sensitive debt for information-insensitive debt, possibly at a subsidized price to prevent information production, or, to make the private debt, which has become information-sensitive, information-insensitive. This prevents the crisis from being worse than the shock k. A lender-of-last-resort can prevent the deleterious effects of the switch to adverse selection. If the lender-last-resort were to purchase agent B's bond by issuing a riskless bond to agent B in exchange, such that there was no incentive to produce private information, adverse selection could be avoided. Or, if the central bank simply guaranteed the bond at a value such that agent C did not produce information then the same goal would be accomplished. In any case, the central bank would have to have some ability to tax at the final date as the proceeds from agent A's project might not cover the central bank's debt or guarantee. But, as presently constituted the model has no agents to tax at the final date.

9. Conclusion

In the stock market it is clear that the securities are information-sensitive, so there are many analysts producing information, and trade is centralized in a stock market. Debt is different. Even before deposit insurance, checks changed hands without due diligence be participants about the banks' assets backing them. Today billions of dollars are traded in sale and repurchase (repo) markets overnight, very quickly, every day, without extensive due diligence (i.e., information production) on the bonds used as collateral. Much corporate debt is purchased and traded based only on ratings. Trade in decentralized debt markets is facilitated by a lack of information, in fact, by symmetric ignorance.

Debt is the optimal contract for providing liquidity. It is optimal in three senses. First, with respect to public signals, it retains the most value and so produces the most intertemporal carrying capacity. Second, when costly private information can be produced, causing adverse selection, debt minimizes the incentive to produce private information and so reduces the adverse selection. Finally, when there is adverse selection, debt is optimal in maximizing the amount of consumption that can be achieved via trade. In the first two cases, debt is optimal because it is least information-sensitive. In the third case, debt is optimal because it maximizes the amount traded.

We propose a measure of information-sensitivity which is a kind of measure of tail risk. For a buyer, it is defined as the expected overpayment in "bad" states, i.e. the expected sum of overpayment in all states where $s(x)<p$. Analogously, for a seller information-sensitivity is the expected loss due to charging too little in "good states, i.e. the expected total loss in all states where $s(x)>p$. By taking this ex ante interpretation of potential ex post realized losses, we can use this definition as measure of liquidity. With respect to this measure debt is the optimal security for liquidity provision.

Systemic crises concern debt. The crisis that can occur with debt is due to the fact that the debt is not riskless. A bad enough shock can cause information insensitive debt to become information-sensitive, causing a collapse of trade as agents seek to avoid adverse selection. Instead of trading at the new and lower expected value of the debt given the shock, agents trade much less than they could or even not at all. There is a collapse of trade.

Appendix A: Proofs

Proof of Lemma 3 (Portfolio Information-sensitivity)

We prove part (ii) first. Note that p<E[x]=0.5 and for X uniformly distributed on [0,1], f(x)=1.

For N=2, we have $\overline{\Pi}_2^{DP} < \overline{\Pi}_1^{DP} = \Pi_1^{\Sigma} = \pi$ since

$$\Leftrightarrow \quad \tfrac{1}{2}\int_0^{2p}(2p-y)\cdot f_Y(y)dy < \int_0^{p}(p-x)f(x)dx$$

$$\Leftrightarrow \quad \int_0^{2p}(2p-y)\cdot ydy < 2\int_0^{p}(p-x)dx$$

$$\Leftrightarrow \quad p < \tfrac{3}{4}$$

For N>2, we have:

$$\overline{\Pi}_N^{DP} = \tfrac{1}{N}\int_0^{N\cdot p}(Np-y)\cdot f_{Y_N}(y)dy \quad \text{where} \quad f_{Y_N}(y) = \frac{1}{(N-1)!}\sum_{k=0}^{N}(-1)^k\binom{N}{k}[(y-k)]_+^{N-1}$$

Numerically we show that $\overline{\Pi}_{N+1}^{DP} < \overline{\Pi}_N^{DP}$ for all N.

(i) Part (ii) shows that $\Pi_N^{DP} < \Pi_N^{\Sigma}$. And $\Pi_N^{DP} = \int_0^{NpY}(Np-y)\cdot f_{Y_N}(y)dy$ $< \Pi_N^{PD} = \int_0^{Np}(Np-s(y))\cdot f_{Y_N}(y)dy$ since $y = \sum_{n=1}^{N}x_n \geq s(y) = \sum_{n=1}^{N}\min[x_i, D]$. //

Proof of Proposition 4 There are three types of potential best responses. (i) Strategy I, agent B avoids information acquisition by reducing the face value and chooses p=E[s(x)]. (ii) Strategy II, agent B avoids information acquisition by reducing the face value and giving agent A some surplus so as to reduce his incentive to become informed, i.e. p>E[s(x)]. (iii) Strategy III, agent B triggers information acquisition by agent A.

(i) Corollary 3 states that the maximal amount that agent B can buy at t=1 where p=E[s(x)] *and* that does not trigger information acquisition is given by p^I which solves:

$$\int_{x_L}^{p^I}(p^I - x)f(x)dx = \gamma ,$$

and the associated face value D^I solves:

$$p^I = \int_{x_L}^{D^I} xf(x)dx + \int_{D^I}^{x_H} D^I f(x)dx.$$

With this strategy agent B consumes $p^I = E[s^I(x)]$ at t=2 and has an expected consumption of $w - E[s^I(x)]$ at t=3. Thus $EU_B(I) = w - E[s^I(x)] + \alpha E[s^I(x)]$.

(ii) A second potential best response is for agent B to propose a contract such that agent A acquires information. If $p^{II} = p^I + \Phi$ (where $\Phi > 0$) and D^{II} such that $p^{II} = \int_{x_L}^{D^{II}} xf(x)dx + \int_{D^{II}}^{x_H} D^{II} f(x)dx.$ (i.e. $D^{II} > D^I$) agent A acquires information and there is no trade for $x > p^{II}$ and $E[s^{II}(x)|trade] = \int_{x_L}^{p^{II}} xf(x)dx$. However, a strategy that induces information acquisition by agent A is strictly dominated by the following strategy: For any given p^{II}, agent B reduces the face value from D^{II} to $\tilde{D}^{II}$ (i.e. $p^{II} = \tilde{p}^{II} > E[\tilde{s}^{II}(x)]$) such that agent A does not acquire information. In this case $E\left[\tilde{s}^{II}(x)|trade\right] = \int_{x_L}^{\tilde{D}^{II}} xf(x)dx + \int_{\tilde{D}^{II}}^{x_H} \tilde{D}^{II} f(x)dx > E\left[s^{II}(x)|trade\right]$ while agent B pays the same price for both contracts. This shows that a contract that induces information acquisition by agent A is strictly dominated by Strategy II. Note, Lemma 5 shows that if agent A is informed, the best response of agent B is to ask for a contract with price equals to face value. But if such a contract is to be traded, agent A does not acquire information.

(iii) So a third potential best response of agent B is to choose a surplus sharing offer, i.e. an offer that gives agent A some of the trading surplus by proposing a price $p^{III} > E[s^{III}(x)]$. This is another strategy that can avoid information acquisition by agent A. Consider any two contracts with the same expected payoff. Proposition 1 shows that debt is least information-sensitive, i.e. has the lowest π, thus it is least costly to bribe agent A with a debt contract. Suppose agent B offers the price $p^{III} = E[s^{III}(x)] + (\pi^{III} - \gamma)$ to buy this debt contract, then agent A does not acquire information and $EU_A = w + \pi^{III} - \gamma$.

This strategy yields $EU_B(III) = w - (E[s^{III}(x)] + \pi^{III} - \gamma) + \alpha E[s^{III}(x)]$. The optimal surplus sharing offer has a face value D^{III} and price p^{III} that maximizes:

$$(\alpha - 1)\int_{x_L}^{x_H} \min[x, D^{III}] f(x)dx - \int_{x_L}^{x_H} \max[x - p^{III}, 0] \cdot f(x)dx$$

and $\pi^{III} = \gamma : \int_{x_L}^{x_H} \max[x - p^{III}, 0] \cdot f(x)dx = \gamma$.

Consequently, only Strategies I and III are potential best responses. Agent B compares $EU_B(I) = w - E[s^I(x)] + \alpha E[s^I(x)]$ and $EU_B(III) = w - (E[s^{III}(x)] + \pi^{III} - \gamma) + \alpha E[s^{III}(x)]$, where $E[s^{III}(x)] > E[s^I(x)]$ and chooses the one with the higher expected utility. The following arguments show that Strategy I is not necessarily dominated by Strategy III. Note, if the face value $D^{III}=D^I$, then $EU_B(III) = EU_B(I)$ since $\pi^{III} = \pi^I = \gamma$. Increasing d at d^I, increases

$\Delta\pi = \pi^{III} - \pi^{I}$ as well as $\Delta E[s^{III}(x)] = E[s^{III}(x)] - E[s^{I}(x)]$. For α small (and there is a lot of probability mass on the left tail), then $\alpha\Delta E[s^{III}(x)] < \Delta\pi^{III}$ and thus Strategy I dominates Strategy III. Alternatively, Strategy I is not dominated by Strategy III if for all $D^{III} > D^{I}$,we have:

$$-E_k[s^I(x)] + \alpha E_k[s^I(x)] \underbrace{-\pi^I + \gamma}_{0} \geq -E_k[s^{III}(x)] - \pi^{III} + \gamma + \alpha E_k[s^{III}(x)]$$

$$\Leftrightarrow \quad E_k[s^{III}(x)] - E_k[s^I(x)] + \pi^{III} - \pi^I \geq \alpha E_k[s^{III}(x)] - \alpha E_k[s^I(x)]$$

$$\Leftrightarrow \quad \frac{E_k[s^{III}(x)] - E_k[s^I(x)] + \pi^{III} - \pi^I}{E_k[s^{III}(x)] - E_k[s^I(x)]} \geq \alpha$$

$$\Leftrightarrow \quad 1 + \frac{\pi^{III} - \pi^I}{E_k[s^{III}(x)] - E[s^I(x)]} \equiv \alpha' \geq \alpha \ .//$$

Proof of Proposition5: Suppose that at t=1 agent B buys debt or a security s, where $E_m[s^D(x)] = E_m[s(x)] = w$.

The market value of a security under distribution k is $p_k = \int_{x_L}^{x_H} s(x) \cdot f_k(x)dx.$ The assumption of s(x) non-decreasing and information partition (FOSD) imply that $E_k[s(x)] \leq E_{k+1}[s(x)]$ for all k. Consider a debt contract. Define $\{F^<\}=\{F_1,.., F_{k'}\}$ to be the set of distributions where $E_k[s^D(x)] < E_m[s^D(x)] = w$. Analogously, define $\{F^>\}=\{F_{k'+1},.., F_N\}$ to be the set of distributions where $E_k[s^D(x)] \geq E_m[s^D(x)]$. At t=2, if $E_k[s^D(x)] \geq w$, agent B sells $\kappa E_k[s^D(x)]$ where $\kappa = \frac{w}{E_k[s^D(x)]}$ for w and consumes it. At t=3, agent B consumes s(x)-κs(x). If $E_k[s^D(x)] < w$, then agent B sells the whole for the price $E_k[s^D(x)]$ and consumes this amount and has no consumption at t=3. Now we show that $EU_B[c_B^D] \geq EU_B[c_B^S]$ since:

$$\sum_{k=1}^{k'} \lambda_k \alpha \underbrace{E_k[s^D(x)]}_{E[c2]} + \sum_{k=k'+1}^{K} \lambda_k (\alpha \underbrace{E_k[s^D(x)]}_{E[c2]} + \underbrace{w - E_k[s^D(x)]}_{E[cC3]})$$
$$\geq \sum_{k=1}^{k'} \lambda_k \alpha E_k[s(x)] + \sum_{k=k'+1}^{K} \lambda_k (\max\{w - E_k[s(x)], 0\} + \alpha \min\{w, E_k[s(x)]\}).$$

Note, $s^D(x) = x \geq s(x)$ for all x≤D, implies that $E_k[s^D(x)] \geq E_k[s(x)]$ for all k≤k'-1. For k', there are two cases to consider. (i) If $x_H^{k'} \leq D$, then $E_k[s^D(x)] \geq E_k[s(x)]$. (ii) If $x_H^{k'} > D$, then it is possible that $E_k[s^D(x)] < E_k[s(x)]$. However, for this to be true it must be that $s(x)>s^D(x)=D$ for some x>D. In this case s(x) non-decreasing implies $s(x)>s^D(x)=D$ for all k>k' and thus $E_k[s^D(x)] < E_k[s(x)]$ for all k>k'. From $EU_B[c_B^D] = EU_B[c_B^S] = w$ and $\underbrace{\sum_{k=1}^{k'} \lambda_k E_k[s^D(x)]}_{W_L^D} + \underbrace{\sum_{k=k'+1}^{K} \lambda_k E_k[s^D(x)]}_{W_H^D}$

$= \underbrace{\sum_{k=1}^{k'} \lambda_k E_k[s(x)]}_{W_L^S} + \underbrace{\sum_{k=k'+1}^{K} \lambda_k E_k[s(x)]}_{W_H^S}$, we have $W_H^S > W_H^D$ and thus $W_L^S < W_L^D$. //

Proof of Proposition 6: At t=1, issuing debt maximizes the payoff of agent A and thus is a best response. Agent B does not acquire information since $\gamma \geq \pi(m)$. At t=2, the following cases arise:

Case (i): There is efficient trade at t=2.

(a) Suppose $E_k[s^D(x)] \geq E_m[s^D(x)]$. Proposition 6(a) shows there is always efficient trade.

(b) Suppose $E_k[s^D(x)] < E_m[s^D(x)]$. There is efficient trade if

(bi) $\gamma \geq \pi^D(k)$, see Proposition 6(b); or

(bii) $EU_C(II) \geq EU_C(I)$, i.e. agent C chooses Strategy II with $p^{II} > E_k[s^{II}(x)] = E_k[s^D(x)]$ and $p^{II} \leq w$; (agent C buys the whole debt for a price premium). See Proposition 3(ii); or

(biii) $EU_C(II) \geq EU_C(I)$, i.e. agent C chooses Strategy II with $p^{II}=w$ and $p^{II} > E_k[s^{II}(x)]$ but $E_k[s^{II}(x)] < E_k[s^D(x)]$. (Agent C spends all his w to buy a new debt contract with $d^{II} < D$) See Proposition 3(ii).

Case (ii): There is inefficient trade, if agent C chooses strategy I or II and has positive consumption at t=2. //

Numerical Example

$$\int_{p^{II}}^{D^{II}} (x - p^{II}) f(x)dx + \int_{D^{II}}^{x_H} (D^{II} - p^{II}) f(x)dx = \gamma$$

Given p, solving cheapest adverse selection contract.

$$1.25 \int_{p}^{D} (x - p)dx + 1.25 \int_{D}^{0.8} (D - p)dx = \gamma$$

$$\tfrac{1}{2}D^2 - Dp - \tfrac{1}{2}p^2 + p^2 + 0.8D - 0.8p - D^2 + Dp = 0.8\gamma$$

$$-\tfrac{1}{2}D^2 + 0.8D + \tfrac{1}{2}p^2 - 0.8p - 0.8\gamma = 0$$

$$D^2 - 1.6D - p^2 + 1.6p + 1.6\gamma = 0$$

$$D = 0.8 - \sqrt{0.64 + p^2 - 1.6p - 1.6\gamma}$$

Appendix B: Optimality of Debt When Information Production Results in Partial Information

In the main text we consider the case where agents either obtain perfect information or are ignorant. Now we suppose that the information that agents learn does not reveal the true realization of X. Instead, the agents receive a signal that is informative, but it provides less than perfect information. We discuss two types of signals: (i) a mean preserving spread; that is, an agent receives a noisy signal of the type $\phi=x+\varepsilon$, where ε is a random with $E[\varepsilon]=0$; and (ii) an agent learns information about which distribution is relevant, where x initially can be drawn from more than one distribution. Debt remains a least information-sensitive security.

Proposition B1: If the agent receives a noisy signal, then Propositions 2 and 3 hold.

Proof: Upon observing ϕ, the expected payoff of the security is $E[s(x)\mid\phi]$. The buyer does not buy the security s(x), if he observes $E[s(x)\mid\phi]<w$. Since $E[x\mid\phi]=x$, the same arguments as given in the proof of Proposition 2 show that debt gives rise to the smallest set of states where information has value to the buyer and for any of these states $p-E[{}^{D}s(x)\mid\phi]\le p-E[s(x)\mid\phi]$. Consequently, Propositions 2 and 3 hold under this information structure. //

Proposition B2: Suppose the signal induces a posterior distribution where the support of each posterior distribution is a partition of the state space X. (See Section 4 for details.) Consider the feasible set of securities $S=\{s\colon s(x)\le x,\ p=E[s(x)]=w\}$. Then debt is the least information-sensitive security in the set S, i.e. $\pi_A^D\le\pi_A^S$ and $\pi_B^D\le\pi_B^S$ for all $s\in S$.

Proof: (i) Debt is a least information-sensitive security for agent B; $\pi_B^D\le\pi_B^S$. For a security s, define $\{F^{<}\}$ as the set of distributions where $E_k[s(x)]<E_m[s^D(x)]=w$. Analogously, define $\{F^{>}\}$ to be the set of distributions where $E_k[s(x)]\ge E_m[s(x)]$. Given a debt contract, agent B does not buy the bond for the price w if he observes $k\in\{F\}^{<}$. The value of information is $\pi_B^D=\sum_{k\in\{F^{D<}\}}\left(w-E_k[s^D(x)]\right)\cdot\lambda_k$. The value of information of another security s(x) is $\pi_B^S=\sum_{k\in\{F^{S<}\}}\left(w-E_k[s(x)]\right)\cdot\lambda_k$. Proposition ? shows that $\{F^{D<}\}\subseteq\{F^{S<}\}$ and $E_k[s^D(x)]\le E_k[s(x)]$ for all $k\in\{F^{D<}\}$. Thus $\pi_B^D\le\pi_B^S$ for all s.

(ii) Debt is a least information-sensitive security for agent A; i.e.

$$\pi_A^D=\sum_{k\in\{F^{D>}\}}\lambda_k\left(E_k[s^D(x)]-w\right)\le\sum_{k\in\{F^{S>}\}}\lambda_k\left(E_k[s(x)]-w\right)=\pi_A^S$$

For all s (including debt), we have

$$\sum_{k\in\{F^{<}\}}\lambda_k E_k[s(x)]+\sum_{k\in\{F^{>}\}}\lambda_k E_k[s(x)]=E_m[s(x)]$$

$$\Leftrightarrow\qquad\sum_{k\in\{F^{<}\}}\lambda_k(E_k[s(x)]-w)+\sum_{k\in\{F^{>}\}}\lambda_k(E_k[s(x)]-w)=E_m[s(x)]-w$$

$$\Leftrightarrow \qquad \sum_{k \in \{F^{>}\}} \lambda_k (E_k[s(x)] - w) = E_m[s(x)] - w + \sum_{k \in \{F^{<}\}} \lambda_k (w - E_k[s(x)])$$

Now compare the information-sensitivity of debt with a security s,

$$\pi_A^D = \sum_{k \in \{F^{D>}\}} \lambda_k (E_k[s^D(x)] - w) = E_m[s^D(x)] - w + \sum_{k \in \{F^{D<}\}} \lambda_k (E_k[s^D(x)] - w)$$

$$\pi_A^S = \sum_{k \in \{F^{S>}\}} \lambda_k (E_k[s(x)] - w) = E_m[s(x)] - w + \sum_{k \in \{F^{S<}\}} \lambda_k (E_k[s(x)] - w)$$

Case (i) shows that $\sum_{k \in \{F^{D<}\}} \lambda_k (w - E_k[s^D(x)]) \leq \sum_{k \in \{F^{S<}\}} \lambda_k (w - E_k[s(x)])$. Thus $\pi_A^D \leq \pi_A^S$.

Appendix C: The Non-Monotonicity of the Value of Information

The value of information is in general non-monotonic in the distribution k. Basically, the issue concerns the tail of the distributions, F_k. Stochastic dominance and even partitional information structures do not put enough structure on the (left) tail, but this is the relevant part of the distribution with regard to the information-sensitivity of debt. The value of information at t=2 is given by:

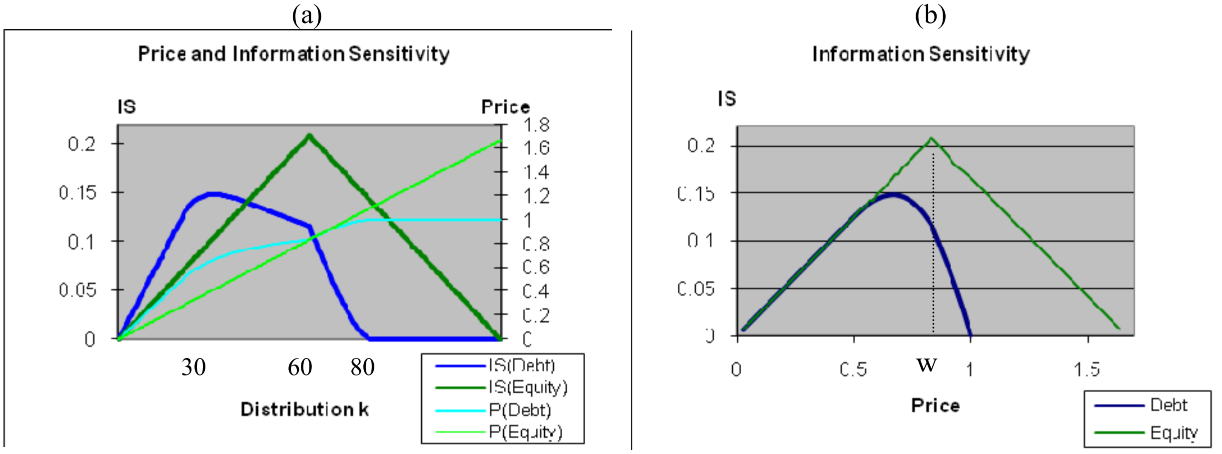
$$\pi(k) = \int_{x_L}^{x_H} \max[p_k - s(x), 0] \cdot f_k(x)dx \text{ where } p_k = \int_{x_L}^{x_H} s(x) \cdot f_k(x)dx..$$

The intuition is the following: Bad news (a distribution with more mass in the left tail) reduces the price of the security, and thus the “area” between price and s(x). But on the other hand that smaller area is evaluated with more probability mass. The overall effect is ambiguous. Similarly, good news increases the price but there is less probability mass on the left tail. For example, if we add an additional posterior distribution to the numerical example in section 5, such that F_1~u[0,0.2], F_2 ~u[0.2, 0.8], F_3 ~u[0.8, 1.2], F_4 ~u[1.2, 2], then prices are increasing in k but, $\pi(4)<\pi(1)<\pi(3)<\pi(2)$.

The following example (satisfying FOSD), which includes both debt and equity, illustrates that $\pi(k)$ is a complicated object. Suppose F_1 ~u[0, 0.05], F_2 ~u[0, 0.1], F_3~u[0, 0.15],.., F_{59} ~u[0, 2.95], F_{60} ~u[0, 3], F_{61}~u[0.05, 3],...., F_{119}~u[2.95, 3], and $\lambda_i = \frac{1}{119}$, $w = \frac{5}{6}$. Then: F_m ~u[0,3], f_m=1/3 for x∈[0, 3] and f_m=0 else. At t=1, if debt with face value D=1 is issued then $p_m^D = \frac{5}{6}$, $\pi^D(m) \approx 0.116$ and if equity($\beta = \frac{5}{9}$) with price $p_m^E = \frac{5}{6}$ is issued, then $\pi^E(m) \approx 0.2083$. At t=2: if F_k=F_{30}~u[0,1.5], then $p^D(k=30) = \frac{2}{3}$, $\pi^D(k=30) \approx 0.1482$ and $p^E(k=30) = \frac{5}{12}$, $\pi^E(k=30) \approx 0.1042$. Note that for F_{60}, for example, the value of information for equity (0.1042) is lower than the value of information for debt (0.1482) but so does the price of equity (and the amount that can be potentially traded). Furthermore, in each case the value of information is non-monotonic in k. Figure B1 (a) plots price and information-sensitivity as a function of the posterior distribution k. Figure C1 (b) plots the information-sensitivity in the (p,π) space.

Figure C1

(a) (b)



As Figure C1 (a) illustrates, depending on the distribution, the information-sensitivity of debt can be larger or smaller than the information-sensitivity of equity. Stochastic dominance does not imply an ordering for the date 2 information-sensitivity of a security as well as the information-sensitivity across securities.

Furthermore, information-sensitivity is non-monotonic in prices since the price function is weakly increasing in k. See Figure B1(b) which also shows that $\pi^D(p) \leq \pi^E(p)$ for any given price. Note for k<m, $p_k^D = p_{k'}^E$ only if k<k'. In this example, there exist prices such that $\pi^D(p) = \pi^E(p)$ since for k<20, the posterior distribution k only has positive support on [0,D] with D=1 and in this range debt has a slope of one and is "equity".

Figure B1 (a) also shows that for a debt contract with face value D, if distribution k has support such that $x_L^k \geq D$ (i.e. k≥80 where F_{80}~u[1, 3]), then $\pi^D(k) = 0$. Note, $x_L^k \geq D$ implies that debt is riskless, i.e. $s^D(x)$=D and $p_k^D = D$. This observation is one of the results in Gorton and Pennacchi (1990). In contrast to debt, the information-sensitivity of equity is $\pi^E(k) > 0$ for any distribution k where $x_L^k \neq x_H^k$. Note, for $x_L^k \neq x_H^k$, $p_k^E = \int_{x_L^k}^{x_H^k} \beta x \cdot f_k(x)dx = \beta E_k[x]$. There exists a set of x such that $s^E(x) < p_k^E$, since $\beta x < p_k^E = \beta E_k[x]$ and $x < E_k[x]$ for $x \in [x_L, E[x]]$. Thus $\pi_k^E > 0$.

References

Aghion, Philippe and Patrick Bolton (1992), "An Incomplete Contracts Approach to Financial Contracting," *Review of Economic Studies,* vol. 59, no. 3, 473-494.

Akerlof, George A. (1970) , "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," *The Quarterly Journal of Economics*, vol. 84, no. 3, 488-500.

Andolfatto, David (2009), "On Informationally Efficient Markets and Efficient Outcomes in Monetary Economies," Simon Fraser University, working paper.

Banerjee, Abhijit V. and Eric S. Maskin (1996), "A Walrasian Theory of Money and Barter," *The Quarterly Journal of Economics*, vol. 111, no. 4, 955-1005.

Biais, Bruno and Thomas Mariotti (2005), "Strategic Liquidity Supply and Security Design," *Review of Economic Studies*, vol. 72(3), 615-649.

Bolton, Patrick and David Scharfstein (1990), "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review* LXXX: 94-106.

Dang, Tri Vi (2008), "Bargaining with Endogenous Information," *Journal of Economic Theory 140*, 339-354.

Demarzo, Peter and Darrell Duffie (1999), "A Liquidity-Based Model of Security Design," *Econometrica* 67(1), 65-99.

Diamond, Douglas and Philip Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, vol. 91, 401-419.

Gale, Douglas and Martin Hellwig (1985), "Incentive-Compatible Debt Contracts: The One-Period Problem," *The Review of Economic Studies*, vol. 52, no. 4, 647-663.

Gorton, Gary (2009), "Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007," paper prepared for the 2009 Federal Reserve Bank of Atlanta Jekyll Island Conference.

Gorton, Gary and Andrew Metrick (2009a), "Securitized Banking and the Run on Repo," Yale School of Management, working paper.

Gorton, Gary and Andrew Metrick (2009b), "Haircuts," Yale working paper.

Gorton, Gary and George Pennacchi (1990), "Financial Intermediaries and Liquidity Creation," *Journal of Finance* 45:1 (March 1990): 49-72.

Gorton, Gary and George Pennacchi (1993), "Security Baskets and Index-Linked Securities," *Journal of Business*, Vol. 66, No. 1: 1-29.

Gorton, Gary and Nicholas Souleles (2006), "Special Purpose Vehicles and Securitization," chapter in The Risks of Financial Institutions, edited by Rene Stulz and Mark Carey (University of Chicago Press).

Hart, Oliver and John Moore (1995), "Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management," *American Economic Review* 85(3) (June): 567-585.

Hart, Oliver and John Moore (1998), “Default and Renegotiation: A Dynamic Model of Debt,” *Quarterly Journal of Economics* CXIII(1) (February): 1-41.

Hirshleifer, Jack (1971), “The Private and Social Value of Information and the Reward to Inventive Activity,” American Economic Review 61(4) (September): 561-574.

Holmström, Bengt (2008), "Discussion of ‘The Panic of 2007,‘ by Gary Gorton,“ in Maintaining Stability in a Changing Financial System, Proceedings of the 2008 Jackson Hole Conference, Federal Reserve Bank of Kansas City, 2008.

Jacklin, Charles (1987), “”Demand Deposits, Trading Restrictions, and Risk-Sharing,” in Contractual Arrangements for Intertemporal Trade, edited by Edward Prescott and Neil Wallace (University of Minnesota Press: Minneapolis, MN), 26-47.

Kaplan, Todd (2006), “Why Banks Should Keep Secrets,” *Economic Theory* 27, 341-357.

Kiyotaki, Nobuhiro and John Moore (1997), “Credit Cycles,” *Journal of Political Economy* 105(2), 211-248.

Myers, Stewart and Nicholal Majluf (1984), “Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have,” *Journal of Financial Economics* 13, 187-221.

Pagano, Marco and Paolo Volpin (2009), “Securitization, Transparency and Liquidity,” working paper.

Townsend, Robert (1979), “Optimal Contracts and Competitive Markets with Costly State Verification,” *Journal of Economic Theory*, vol. 21, no. 2 (October): 265-293.