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Illiquidity and Interest Rate Policy

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The cheapest way for banks to finance long term illiquid projects is typically by borrowing short term from households. But when household needs for funds are high, interest rates will rise sharply, debtors will have to shut down illiquid projects, and in extremis, will face more damaging runs. Authorities may want to push down interest rates to maintain economic activity in the face of such illiquidity, but intervention may not always be feasible, and when feasible, could encourage banks to increase leverage or fund even more illiquid projects up front. Authorities may want to commit to a specific policy of interest rate intervention to restore appropriate incentives. For instance, to offset incentives for banks to make more illiquid loans, authorities may have to commit to raising rates when low, to counter the distortions created by lowering them when high. We draw implications for interest rate policy to combat illiquidity.

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There has been substantial recent debate on the role of central banks and interest rates, not so much in controlling inflation, but in dealing with episodes of illiquidity and financial fragility. For instance, Greenspan (2002) has argued that while the Federal Reserve cannot recognize or prevent asset price booms, it can "mitigate the fallout when it occurs and, hopefully, ease the transition to the next expansion." Others have responded that by following such an asymmetric interest rate policy – colloquially known as the Greenspan put -- a central bank can engender the kind of behavior that makes booms and busts more likely. In this paper, we ask whether a central bank can alleviate financial stress through untargeted lending at market interest rates. We then ask whether anticipation of such intervention gives banks the incentive to lever more or undertake more illiquid projects up front. Finally, we ask how such distortions to incentives can be rectified.

We start with a model where entrepreneurs borrow from banks to invest in long- term projects. Banks themselves borrow from risk-averse households, who receive endowments every period. Households deposit their initial endowment in banks in returns for demandable deposit claims (throughout the paper, we focus on demand deposits, though any form of overnight unsecured debt could be a close substitute). There is no uncertainty initially about the average quality of a bank's projects in our model, so the bank's asset side is not the source of the problem. However, there is uncertainty about household endowments (or equivalently, incomes) over time. This, coupled with the mismatch between the long gestation period for the projects and the demandable nature of deposits, is the source of banking sector difficulties.

Once households have deposited their initial endowment, and projects have been started, households may have an unexpectedly high need to withdraw deposits. One possibility is that they suffer an adverse shock to current endowment that causes them to want to run down financial assets to consume. But another is that they anticipate significantly higher income or endowments in the future and want to smooth consumption. Thus anticipated prosperity, as well as current adversity, can increase current household demand for consumption goods substantially. We focus on the former (which has antecedents in the Austrian School of Von Mises and Hayek – see later), though the results in this paper apply for the most part to the latter also.

As households withdraw deposits to satisfy consumption needs, banks will have to call in loans to long gestation projects in order to generate the resources to pay them. The real interest rate will rise to equate the household demand for consumption goods and the supply of these goods from terminated projects.[2] Thus greater consumption demand will lead to higher real rates and more projects being terminated, as well as lower bank net worth. This last effect is because the bank's loans pay off only in the long run, and thus fall in value as real interest rates rise, while the bank's liabilities, that is demandable deposits, do not fall in value. Eventually, if rates rise enough, the bank may have negative net worth and experience runs, which are destructive of value because all manner of projects, including those viable at prevailing interest rates, are terminated. Anticipated future prosperity, as well as current adversity, can thus induce fragility in the banking system, and slow the real economy.

How can this tendency towards banking sector fragility be mitigated? One obvious possibility is to alter the structure of banks. If deposits were not demandable, a loss of net worth would not result in a destructive run. And if banks financed themselves with long term liabilities that fell in value as real interest rates rose, banks would be doubly stable. Even if deposits had to be demandable, banks would be more stable if they financed with lower levels of deposits, or if deposits could be made state contingent, so they fluctuated appropriately in value with the aggregate consumption demands in each state. So can the structure of banks be altered easily while they continue to intermediate efficiently?

The answer from our previous work is no. Bank debt is demandable because it allows bankers to commit to pay out the value they collect (Diamond and Rajan (2001)). Put differently, given the skills

[2] While we focus on project liquidations (or equivalently, a halt in new projects) as affecting the aggregate supply of goods available for consumption, it is also plausible to think of them as affecting aggregate demand, as people lose jobs. We do not model this, though the qualitative effects would be similar.

needed to make loans to entrepreneurs and recover payments, financing with demandable debt is the cheapest form of financing for banks – financing with long term liabilities would reduce the efficiency of intermediation substantially. Given this, competitive conditions determine how much banks lever up. Competitive banks will be willing to accept some probability of financial distress and runs in order to commit to pay depositors more, and thus attract funds. Thus the commitment value of a hard liability structure, coupled with ex ante competition for funds, can lead to highly levered banks.

If altering bank structure is not easy, what about government intervention? It is not sufficient to just keep the volume of lending up if consumption is not altered. In our model, given the demand for consumption, the only way the market clears is if interest rates rise, bank lending falls (because projects are no longer so attractive at the higher interest rate), projects are terminated, and goods released for consumption. Put differently, there is a goods market that has to clear and any intervention has to recognize this.

The government can always violate property rights and keep the banking system intact – for instance, taxing households and gifting the proceeds to banks (or equivalently, lending to banks at rates the private market would not lend at). This sort of bailout would reduce household consumption while limiting project termination. While recognizing that such interventions may be necessary in extremis (and is taking place even as we write), we want to focus on more routine central bank interventions – lending or borrowing from the market to alter interest rates – that seemingly do less violence to property rights.

Lending takes resources. Once we recognize the fact that these resources have to ultimately come from the households, we see there are a variety of circumstances in which attempts to alter the real interest rate could be ineffective. For instance, could authorities lower rates by taxing households' current endowments, lending them to banks at market rates, and then repaying the households in the future at the market interest rate so as to not violate their property rights? The problem is that households could respond to any tax on their current endowment by withdrawing an equivalent additional amount from banks so as to keep consumption constant. If so, interest rates would remain unchanged, a form of Ricardian Equivalence (Barro (1974)) that limits how much governments can do.[3]

However, some households may not participate in the banking system, either because they withdraw all their deposits to compensate for current taxes or because interest rates are too low, given their endowments, for them to deposit in the first place. A further tax on these households' current endowments will not be compensated for by equivalent withdrawals from the banking system, since these households already have no claims on the banking system. The consumption of these households will fall. The amount collected from this tax is then an incremental source of resources to the banking system, and by taxing and lending, the government can bring down the real interest rate, reducing project liquidation and thus bolstering the net worth of banks.

Such intervention, even if seemingly respectful of household property rights, has real effects only when it forcibly changes household consumption decisions. Of course, if it prevents runs, this change could be Pareto improving. But if the objective is not to head off runs, some households could be made worse off, even if they are "fully" compensated for taxes at market interest rates.

An equally important concern is the effect of anticipation of such intervention on ex ante actions. A low ex post interest rate offers a very low reward for maintaining liquidity. If the authorities are expected to reduce interest rates when liquidity is at a premium, banks will take on more short-term leverage or illiquid loans, thus bringing about the very states where intervention is needed. Indeed, the current financial turmoil in the United States could be thought of as being partially caused by lenders, anticipating a continued environment of low interest rates following the implosion of the "tech bubble" in early 2000 and the subsequent collapse of corporate investment, choosing to take on more illiquid

[3] It is well understood that government borrowing to finance lump sum tax and transfer programs have effect only when this Ricardian Equivalence does not hold. Government lending at market rates of interest (liquidity lending), using its taxation authority to raise resources, with the interest augmented principal transferred back to households in a lump sum manner is almost the same transaction in reverse, and should have no effect for similar reasons.

financial assets financed with short term debt. Anticipation of low interest rates may have been strengthened by the so-called Greenspan Put, whereby financial markets believed that if the markets ever came under strain because of excessive expansion, the Federal Reserve would cut interest rates.

What would optimal interest rate policy look like, taking into account the effects on bank incentives? For instance, if the purpose is to make banks make more liquid loans, we show the central bank should commit to pushing down interest rates even below the level strictly necessary to restore stability in fragile states, in order to reward those banks that have chosen to remain relatively liquid with additional net worth – in a sense, creating "negative" moral hazard. It should also push up interest rates in states where the interest rate would otherwise be low so as to penalize the net worth of banks that have chosen to be illiquid. While it may be politically difficult for the authorities to raise rates in environments where there is no obvious reason for doing so, our model suggests interest rate smoothing may be useful to increase systemic stability and limit regulatory arbitrage. More generally, intervention needs to consider long-term effects if it is to be beneficial in the short-term.

In sum then, this paper derives the sources of illiquidity, and thus of economic volatility from necessary contractual rigidities in lending, and from the behavior of banks, households, and governments. In doing so, we abstract from distortions in lending resulting from banker misperceptions or incentive problems which, while no doubt important, have been explored in other work. The rest of the paper is as follows. In section I, we lay out the basic model, in section II, we solve it, in section III we consider macroeconomic intervention on interest rates and its effect on ex ante choice of deposit level, and in section IV, its effects on the ex ante choice of assets. We then conclude.

I. The Framework

1.1. Agents, Endowments, Technology, Preferences.

Consider an economy with risk-neutral entrepreneurs and bankers, and risk-averse households. The economy has three dates, 0, 1, and 2. Each household is initially endowed with a unit of good at date 0. Households can invest their initial endowment in banks, which will lend the resources to entrepreneurs. At date 1, households will get endowment e_1. They also learn their date-2 endowment. Some households will get a high endowment, e_2^H at date 2 while the rest will get a low endowment e_2^L. Let there be two states of the world at date 1, the *exuberant* state where θ^E is the fraction of high endowment households and the *normal* state, where θ^N is the fraction, with $\theta^E > \theta^N$. Thus E is a state with greater anticipated prosperity, where more households expect a high endowment. The probability of state E is p^E. The state of nature and household types are not verifiable and in addition, household types are private information.

Each entrepreneur has a project, which requires the investment of a unit of good at date 0. The project produces $\tilde{Y}_2$ in goods at date 2 if it is not liquidated, and X_1 at (or after) date 1 if the project is liquidated. At date 0, $\tilde{Y}_2$ is uncertain for each entrepreneur, with outcomes becoming known at date 1 and distributed uniformly over the range $\left[\underline{Y}_2, \overline{Y}_2\right]$. Entrepreneurs have no goods to begin with, and the demand from entrepreneurs looking for funds at date 0 is greater than the supply with households.

Households maximize expected utility of consumption given by $E(\log C_1 + \log C_2)$. Risk neutral entrepreneurs and bankers maximize $E(C_1 + C_2)$.

1.2. Financing Entrepreneurs

Anyone will lend only to the extent that they can coerce borrowers into repaying, either by independently being able to generate value by seizing the borrower's assets, and thus being able to obtain some "collateral" protection in case of default, or by finding ways to commit to inflict serious damage to the borrower. Specifically, since entrepreneurs have no endowments, they need to borrow to invest. Each entrepreneur can borrow from a banker who has, or can acquire during the course of lending, knowledge about an alternative, but less effective, way to run the project. The banker's specific knowledge allows him to collect γY_2 from an entrepreneur whose project just matures, with $\gamma < 1$. Once a banker has lent, no one else (including other bankers) has the knowledge to collect from the entrepreneur.[4]

Because there are more entrepreneurs than households, not all projects are funded. Banks can ask entrepreneurs to repay the maximum possible for a loan – they will lend only if the entrepreneur promises to pay $\overline{Y}_2$ on demand. If the entrepreneur fails to pay on demand, he can make a counter-offer to the bank. If that offer is rejected, the bank takes over the project and either harvests date-2 cash flow γY_2 or liquidates it. [5] Banks can store any unused assets at a gross interest rate of 1.

1.3. Financing Banks

Since bankers have no resources initially, they have to raise them from households. But households have no collection skills (and bank loans are worthless in their hands), so how do banks commit to repaying households? By issuing demand deposits! In our previous work (Diamond and Rajan (2001)), we argued that the demandable nature of deposit contracts introduces a collective action problem for depositors that makes them run to demand repayment whenever they anticipate the banker cannot, or will not, pay the promised amount. Because bankers will lose all rents when their bank is run, they will repay the promised amount on deposits whenever they can.

Deposit financing introduces rigidity into the bank's required repayments. Ex ante, this enables the banker to commit to repay if he can (that is, avoid strategic defaults by passing through whatever he collects to depositors), but it exposes the bank to destructive runs if he truly cannot pay (it makes non-

[4] For a relaxation of these assumptions, see Diamond and Rajan (2001).

[5] While we assume here that households lend via the bank, we show in Diamond and Rajan (2001) that banks and their fragile liability structures arise endogenously to facilitate the flow of credit from investors with uncertain consumption needs to entrepreneurs who have hard-to-pledge cash flows.

strategic default more costly): when depositors demand repayment before projects have matured and the bank does not have the means of payment, it will be forced to liquidate projects to get X_1 immediately instead of allowing them to mature and generate γY_2. In addition to making banks fragile, short-term funding leaves households exposed to interest rate risk when reinvesting from date 1 to date 2. This is unavoidable because a bank cannot commit to pay a return above the market return from date 1 to date 2 (due to the bank's ability to renegotiate).

Banks are competitive, and we assume that if it offers a competitive rate, each bank attracts enough entrepreneurs so that the distribution of $\tilde{Y}_2$ among entrepreneurs it finances matches the aggregate distribution of entrepreneurs. Because the date-0 endowment is scarce relative to projects, banks will compete to offer the most attractive deposit face value D to households per unit of endowment deposited (henceforth, all values will be per unit of endowment).

Assumption 1: $\dfrac{e_2^H}{e_1} > \dfrac{\gamma \overline{Y}}{X_1}$

Assumption 1 implies that at the highest interest rate payable by firms, the H household wants to withdraw at least some of its deposits. Through much of the paper, we will focus on the situation where the date -1 interest rate exceeds 1 so that storage is not in use, but storage is easily handled.

Timeline

Date 0	**Date 1**	**Date 2**
Banks offer deposit terms and entrepreneurs offer loan repayment terms. Households get endowment and invest in banks. Banks lend to entrepreneurs.	Uncertainty over states, household date-2 endowments, and project outcomes revealed. Households decide how much to withdraw and how much to consume. Banks decide what projects to liquidate.	Projects mature, loans repaid, and deposits fully withdrawn from banks. All agents consume.

II. Solving the basic model

In what follows, we will start by solving the bank's decision vis a vis entrepreneurs at date 1, then the households' consumption and withdrawal decisions, and finally, the bank's date 0 decision on what level of deposit repayment D to offer to maximize household willingness to deposit.

2.1. Bank decisions vis a vis entrepreneurs

Let us start our analysis at date 1, once uncertainty is revealed. Let the interest rate households demand in state *s* for re-depositing between dates 1 and 2 be r_{12}^S. The bank can get X_1 at date 1 if the project is liquidated. The maximum it can collect from the entrepreneur is γY_2 at date 2, which is the maximum the entrepreneur can commit to pay at date 1. So the bank liquidates projects with

$$Y_2 < Y_2(r_{12}^S) = \frac{r_{12}^S X_1}{\gamma} \qquad (1.1)$$

and continues projects with $Y_2 \geq Y_2(r_{12}^S)$ in return for a promised payment of γY_2. Liquidated entrepreneurs get nothing, while entrepreneurs who are continued retain $(1-\gamma)Y_2$. Note that the present value of the bank's assets at date 1 (before withdrawals) is $\frac{1}{\overline{Y}_2 - \underline{Y}_2} \int_{\underline{Y}_2}^{Y_2(r_{12}^S)} X_1 dY_2 + \frac{1}{\overline{Y}_2 - \underline{Y}_2} \int_{Y_2(r_{12}^S)}^{\overline{Y}_2} \frac{\gamma Y_2}{r_{12}^S} dY_2$,

which is easily shown to fall in r_{12}^S.

2.2. Household decisions

Once uncertainty is revealed, and households know both the state and their endowments, they decide how much they want to withdraw and consume so as to maximize their expected utility of consumption. Of course, if they anticipate the bank will not be able to meet its obligations, they will collectively run on the bank, in which case all projects will be liquidated to pay households. We assume households can coordinate on a Pareto preferred Nash equilibrium. Thus we rule out panic based runs, which would only add to the inefficiencies we document. We start by considering situations where the bank will meet its obligations.

When a household does not withdraw all its deposit, its utility is maximized when the marginal rate of substitution between consumption at dates 1 and 2 is equal to the prevailing deposit rate, r_{12}^S, that is, when $\frac{U'(C_1)}{U'(C_2)} = \frac{C_2}{C_1} = r_{12}^S$.[6]

If household H (with high date-2 endowment) withdraws amount $w_1^{H,S}$ at date 1 (where $w_1^{H,S} \leq D$), then $\frac{C_2^{H,S}}{C_1^{H,S}} = \frac{e_2^H + (D - w_1^{H,S})r_{12}^S}{e_1 + w_1^{H,S}}$. A similar expression can be derived for household L with low date-2 endowment. For both households to have an equal marginal rate of substitution (and deposit at a common rate r_{12}^S), it must be that H type households withdraw more so that $w_1^{H,S} > w_1^{L,S}$.

Lemma 1: *If* $r_{12}^S \geq \frac{e_2^H}{e_1 - D}$*, both households leave all their money in the bank at date 1. If* $\frac{e_2^H}{e_1 - D} > r_{12}^S > \frac{e_2^H}{e_1 + D}$*, neither household withdraws fully from the banking system, but the H household withdraws more than the L household. If* $\frac{e_2^H}{e_1 + D} \geq r_{12}^S > \frac{e_2^L}{e_1 + D}$*, the H household withdraws fully, while the L household maintains some deposits. If* $\frac{e_2^L}{e_1 + D} \geq r_{12}^S$*, both households withdraw fully.*

2.3. Equilibrium

Assume first the bank can repay depositors without failing. In equilibrium, markets for goods at date 1 and date 2 have to clear. At date 1, goods are produced when banks liquidate projects. Because all banks have the same distribution of projects and will liquidate projects with $Y_2 < Y_2(r_{12}^S) = \frac{r_{12}^S X_1}{\gamma}$, date-1

[6] The obvious intuition is that if the interest is lower, it could increase expected utility by withdrawing more at date 1 and consuming more, while if the interest rate is higher, it could increase expected utility by withdrawing less at date 1 and consuming more at date 2.

liquidation proceeds are $\frac{1}{\overline{Y}_2-\underline{Y}_2}\int_{\underline{Y}_2}^{Y_2(r_{12}^S)} X_1 dY_2 = \left[\frac{\frac{r_{12}^S X_1}{\gamma}-\underline{Y}_2}{\overline{Y}_2-\underline{Y}_2}\right] X_1$. Because this should equal the goods consumed by withdrawing households, it must be that (on simplifying)

$$\theta^S w_1^{H,S} + (1-\theta^S) w_1^{L,S} = \frac{r_{12}^S X_1{}^2 - \gamma X_1 \underline{Y}_2}{\gamma\left(\overline{Y}_2-\underline{Y}_2\right)} \tag{1.2}$$

where θ^S is the fraction of H type households in state s.

Theorem 1:

(i) *An equilibrium at date 1 in state s is an interest rate r_{12}^S and withdrawals by the H and L households, $w_1^{H,S}, w_1^{L,S}$ such that the date 1 supply of goods equals the date 1 consumption, banks liquidate enough projects to meet withdrawals, and households do not want to, or cannot, withdraw more.*

(ii) *If it exists, the equilibrium is unique.*

(iii) *If there is a set $\{r_{12}^S, w_1^{H,S}, w_1^{L,S}\}$ that solves $\frac{e_2^H}{e_1+2w_1^{H,S}-D} = r_{12}^S$, $\frac{e_2^L}{e_1+2w_1^{L,S}-D} = r_{12}^S$, and (1.2), with $r_{12}^S > 0$, $w_1^{H,S} \in [0,D)$, $w_1^{L,S} < D$, then that is the equilibrium, else if there are r_{12}^S, $w_1^{H,S} = D$ and $w_1^{L,S}$ that solve $\frac{e_2^L}{e_1+2w_1^{L,S}-D} = r_{12}^S$, and (1.2), with $r_{12}^S > 0$, and $w_1^{L,S} \in [0,D)$, then $\{r_{12}^S, D, w_1^{L,S}\}$ is the equilibrium, else $\{\left[\frac{\gamma D(\overline{Y}_2-\underline{Y}_2)+\gamma \underline{Y}_2 X_1}{(X_1)^2}\right], D, D\}$ is the equilibrium.*[7]

[7] Note that we have assumed the household marginal rates of substitution are not so low that interest rates would fall below 1 in the absence of the bank's storage technology. If that were not the case, we would have to examine an interest rate of 1 as a candidate, with any excess deposits being stored by the bank. The three equations would now

Proof: See Appendix.

Corollary 1:

(i) *H households with a higher date-2 endowment always withdraw (weakly) more than L households.*

(ii) *The interest rate* r_{12}^{S}*, total withdrawals,* $\theta^{S} w_{1}^{H,S} + (1-\theta^{S}) w_{1}^{L,S}$*, and the fraction of projects liquidated all (weakly) increase in the fraction* θ^{S} *of high endowment households and in the face value of deposits, D.*

(iii) *The net worth of banks (date 1 value of assets less deposits) decreases in the fraction* θ^{S} *of high endowment households and in the face value of deposits, D.*

Proof: See appendix.

Since H households have more date-2 endowment than L households, at any given interest rate they will consume (weakly) more at date 1, and hence will withdraw more. This means total withdrawals will (weakly) increase in the fraction θ^{S} of high endowment households, which means the interest rate and liquidation will have to increase to equilibrate the market. Given the present value of the bank's assets decrease with r_{12}^{S}, while the value of its deposit liabilities do not, its net worth decreases with θ^{S}. Turning to the effect of D, a higher face value of deposit claims increases the wealth of the depositor (provided the bank is not run), and increases his desire to consume immediately by withdrawing. The other implications follow.

Note that all these implications would also hold if households differed, not in their date-2 endowments, but in their date-1 endowments, with H households receiving a *lower* date 1 endowment (poor current conditions) and, again, higher growth in marginal utility. Much of past analysis has followed Diamond-Dybvig (1983) and focused on liquidity shocks that are equivalent to poor current conditions, but it is useful to remember that exuberant views of the future could be equally problematic

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solve for the withdrawal by high households, the withdrawal by low households, and the amount invested by banks in the storage technology. This is straightforward.

from the perspective of demands for liquidity. We will focus on this latter aspect through the paper, but it is useful to bear in mind that because interest rates depend on anticipated consumption growth, pressures can stem both from anticipated future prosperity or current adversity.

2.4. Bank Fragility and the Ex Ante Choice of Deposit Rate

Let us now examine each bank's choice at date 0 of the face value D to offer for a deposit of a unit of good. Since the market is competitive and household endowment is scarce relative to entrepreneurs projects, banks will have to offer a D that maximizes household utility. Clearly, so long as the bank is not run in any state, a higher D makes households wealthier and better off. But the bank's net worth also falls by Corollary 1, and for a given D, is lower in state E because the fraction of H households is higher there than in state N (that is, $\theta^E > \theta^N$). When D is high enough that the bank's net worth is completely eroded, the bank is run – which means all projects are liquidated to generate funds to pay depositors, regardless of whether liquidation is value maximizing at the prevailing interest rate.[8] Each running household gets X_1 immediately after a run.[9]

Let $D^{E,\max}$ be the deposit level beyond which a run will be precipitated in the E state, while $D^{N,\max}$ be the corresponding level in the N state. From our discussion above, $D^{E,\max} < D^{N,\max}$, so $D^{E,\max}$ is the highest safe level of deposits, where no bank runs are experienced in either state, while with deposits set at $D^{N,\max}$, runs are experienced in the E state. If deposit repayments could be state contingent, then at date 0, the bank would offer to pay $D^{E,\max}$ in state E and $D^{N,\max}$ in state N. But state-contingent deposit contracts may not be possible (as we discuss later).

[8] This captures the idea that depositors simply want their money back before each bank runs out of resources. The results depend only on bank runs being inefficient, which could also stem from a fraction of projects being liquidated at fire-sale values, that is, at less than X_1.

[9] We assume that each household's deposits are evenly spread across banks and it joins enough lines to get its share of the proceeds with certainty. A run would be more problematic if it contributed to uncertainty about who gets what, with some households lucky enough to be at the front of the line getting more than households at the back. This would add to the cost of the run and complicate the algebra, but not change the results qualitatively.

What would be the D that competitive banks would set if they could not offer state contingent deposits? It is obvious that the only two candidates for D are $D^{E,\max}$ and $D^{N,\max}$.[10] $D^{E,\max}$ would mean the bank would not pay out the maximum it could in the N state. In contrast, setting the deposit level $D^{N,\max}$ would ensure the bank would be run in the E state, which would reduce its value significantly below what would obtain if the deposit level were $D^{E,\max}$ (since all projects are liquidated in a run even though at the prevailing interest rate, some deserve to be continued – the desire to withdraw money independent of consumption needs is what distinguishes a run from a normal withdrawal). This then is the basic trade-off banks face is setting deposits at $D^{N,\max}$ rather than $D^{E,\max}$ – commit to paying more in the N state but have a run in the E state. The trade-off turns on the cost of a run relative to its probability.

Lemma 2: Ceteris paribus, the lower the probability p^E of the exuberant state, the lower the expected cost of bank runs relative to the benefit of paying out more in state N, and the more attractive is the higher face value $D^{N,\max}$ than $D^{E,\max}$ to households.

Proof: See appendix.

Intuitively, since $D^{N,\max}$ and $D^{E,\max}$ maximize payouts for their specific states, the lower the probability of state E, the more attractive does $D^{N,\max}$ become as the ex ante choice.

2.5. Example With Comparative Statics

Let p^E =0.5, $e_1 = 1.2$, $e_2^L = 0.6$, e_2^H =3.6, X_1 =0.95, $\underline{Y}_2$ =0.5 , $\overline{Y}_2$ =2.5, γ =0.9, $\theta^E = 0.6$, θ^N =0.3. First, assume that the deposit level is set at $D^{E,Max}$, the maximum debt level with no run in either state, which is 1.016. In the E state, the interest rate r_{12}^E is 1.70, H households withdraw 0.97, while L households withdraw 0.08. In the N state, the interest rate r_{12}^N is 1.28, the H households withdraw their

[10] We know a higher D, provided it does not precipitate a run, makes depositors wealthier at date-1, and thus is preferred. This means one candidate for the equilibrium D offered is the maximum no-run D, $D^{E,\max}$. Once a run is precipitated in the E state, a higher value of D makes no difference to outcomes or payouts in that state. Which means a second candidate equilibrium D is $D^{N,\max}$, which maximizes payout in the N state. Of course, going beyond $D^{N,\max}$ will reduce value because the bank will be run in all states.

entire deposit, while L households withdraw 0.14. Now let the deposit level be set at $D^{N,\max}$ =1.12. Now the bank is run in the E state, while in the N state, the interest rate is 1.39, the H households withdraws all their deposit while the L households withdraw only 0.18.

In Figure 1, we plot the optimal ex ante choice of D for different ex ante probabilities (blue line). The bank will set D high at 1.12 until the probability of the exuberant state exceeds 0.24, at which point households are better off having safe banks with D set at 1.016. Note from Figure 2 that the utility of the banks and entrepreneurs shifts up substantially when the bank moves to setting a lower safe level of deposits – more rents are preserved for the banker/entrepreneur when deposit claims are lower.

2.6. Discussion

The point, thus far, is straightforward. In a competitive environment, the banking system can lever up to the point where it will fail with some probability when a significant fraction of households become exuberant about the future (or pessimistic about the present). Exuberance creates more pressure for current consumption, which the economy may be too illiquid to provide. Consequently, real interest rates rise to restore equilibrium, and projects are curbed. The more levered the banking system is to begin with, the more projects are liquidated, and higher the likelihood of systemic bank failures.

Ex post, in the exuberant state, the banking system looks over-levered. But ex ante, the extent of short-term leverage could be a competitive outcome, where banks know that it is the only way they can commit to paying depositors more. If banks could write state contingent deposit contracts, they would not have to resort to levels of deposits that risked runs. The problem really is that the state is hard to observe or verify in real time, and its correlation with the value of illiquid bank assets hard to determine precisely.[11] Moreover, making demandable deposits contingent on the state could lead to more instability, as depositors attempt to guess the state and attempt to front run any diminution of the value of their deposits.[12] Hellwig (1994) shows how to optimally set the value of state-contingent deposits to avoid

[11] Banks are thus unlike mutual funds holding traded assets, where there is a precise market value of assets.
[12] The problems caused by the need for short-term finance are not limited only to banks, although we do not separately analyze this here. When bank monitoring is not needed but the legal enforcement of debt contracts is

runs when full state-contingency is possible. We assume that such state-contingency is not possible and examine the consequences.

Of course, if banks could finance substantially with long term renegotiable debt or equity, they would be able to withstand significant variations in household consumption needs without failing. Long term debt or equity would not be a competitive mode of finance for banks, given how central short term demandable debt is to reducing banker rents (see Diamond and Rajan (2001)). The rigidity of bank financing may therefore be an essential feature of the environment.

We are certainly not the first to place the emphasis for contraction and crises on the mismatch between the long duration before investment produces consumption goods, and the temporal pattern of consumption in an expansion. This dates back at least to Von Mises (1912) and the Austrian School. Von Mises placed the emphasis, though, on an artificially low initial rate of interest, induced by bank credit expansion, that makes the process of creating new goods excessively long compared with the tolerance of consumers to postpone consumption. While it is difficult to map his theory precisely to a rational expectations general equilibrium model, it would appear that Von Mises (1912) places the blame for crises squarely on the heads of overly optimistic, excessively aggressive, bankers (and on central bankers who encourage aggressive credit expansion). We will examine banker reactions to likely central bank interest rate interventions shortly. Thus far, though, our emphasis has been on the frictions stemming from the lack of perfect foresight about changes in the consumption patterns of depositors, and the intrinsic illiquidity of banking.

III. Macroeconomic Intervention

3.1. The Purpose of Intervention

costly, borrowers must issue short-term debt to fully exploit their debt capacity (Diamond (2004, 2008)), and leave their future fate subject to refinancing risk. Consequently, results similar to ours hold in an economy where debt is short-term, regardless of whether it is bank debt or not. Corporate debt maturity cannot be adjusted to facilitate stability without reducing access to capital.

As we have just argued, the inability of banks to offer state-contingent deposit contracts means households have to either accept too little value in one state or a destructive run in another. Moreover, the inability of banks to distinguish between H type households and L type households, and the inability to offer either type an above market return in the future, makes it difficult for banks to achieve better risk sharing between the types. One goal of intervention, regardless of whose interests the government serves, may simply be to avoid Pareto inefficient outcomes such as runs. More controversial, is to ensure that not only are runs prevented, but also to cause a transfer to some agents at the expense of others. Let us now examine intervention and the reactions it engenders in banks.

3.2. Limitations on Intervention

Clearly, if the government could identify states, types, and implement any kind of tax and transfer scheme, it could achieve first-best outcomes. Yet such an omniscient, omnipotent, and error-free government authority is clearly implausible. Instead, we will assume

(i) The government can identify the date-1 state of the world and modulate its actions accordingly.

(ii) The government cannot distinguish between households based on their future endowments.

Also, if the government can alter property rights, there is a lot they can do to enhance the welfare of a favored group – for instance, to rescue banks they can simply write down deposits, or tax households and transfer to banks. In extremis, this is what the authorities will do (as they are currently doing in these extraordinary times). But in more normal times, if the authorities have to return what they take from an agent, and they want to attempt a "liquidity" infusion simply through borrowing and lending at market rates (albeit forced), the scope for intervention is more limited. This is what we will focus on.

(iii) The government can tax current household endowments and lend the proceeds to the banks (by depositing in them), or borrow from the banks (issue government bonds) and transfer the proceeds to households, but it cannot otherwise alter property rights. So what it taxes (or transfers) today has to be transferred (or taxed) back in the future as a lump sum augmented at the market interest rate. Similarly, what it borrows or lends has to be repaid at the market interest rate. It cannot directly write-down the face value of deposits.

(iv) The government cannot store goods between dates. Its net intake of goods at any date has to be zero.

The interventions we have described are typically thought of as being undertaken by different organs of the government – e.g., the central bank and the Treasury. In reality, they may be connected. For instance, we will describe an intervention which involves taxing and lending. In the real world, expansionary monetary policy by the central bank could be thought of as a combination of a seigniorage tax and lending to banks (offset eventually by contractionary monetary policy). At any rate, in what follows, we will refer to a single government entity, the central bank, performing both fiscal and monetary interventions.

3.3 When Interventions Do Not Work

The government can use its taxation power to intervene. One candidate intervention could be to tax households at date 1 and lend the proceeds to the bank, transferring the repayment on the loan back to households at date 2. It turns out that a small intervention, when neither household withdraws entirely from the bank at date 1, will have no effect on interest rates or bank solvency. To see why, let Δt be the small tax on all households, which is lent to the bank at the post intervention rate $r_{12}^{S,i}$. The bank repayment, $\Delta t\ r_{12}^{S,i}$ is transferred back to households at date 2.

Post-intervention, it must be that the marginal rates of substitution for household H equals the common interest rate, so $\frac{e_2^{H,S} + \Delta t\ r_{12}^{S,i} + (D - w_1^{H,S,i}) r_{12}^{S,i}}{e_1 - \Delta t + w_1^{H,S,i}} = r_{12}^{S,i}$ where $w_1^{H,S,i}$ is the post-intervention withdrawal. Simplifying, we get $\frac{e_2^H}{e_1 + 2(w_1^{H,S,i} - \Delta t) - D} = r_{12}^{S,i}$. We get a similar expression for household L. Finally, since the bank gets a loan of Δt , its date-1 resource constraint now is $\theta^S w_1^{H,S,i} + (1-\theta^S) w_1^{L,S,i} = \frac{r_{12}^{S,i}\left(X_1\right)^2 + X_1 X_2 - \gamma X_1 \underline{Y}_2}{\gamma\left(\overline{Y}_2 - \underline{Y}_2\right)} + \Delta t$. Comparing to the equations in Lemma 2 (i), we see that if $\{r_{12}^S, w_1^{H,S}, w_1^{L,S}\}$ is the equilibrium before intervention, the equilibrium post-intervention is $\{r_{12}^{S,i} = r_{12}^S, w_1^{H,S,i} = w_1^{H,S} + \Delta t, w_1^{L,S,i} = w_1^{L,S} + \Delta t\}$.

Intuitively, households have no reason to change their current consumption if they can withdraw an additional amount equivalent to the amount they are taxed today while maintaining their future consumption. With unchanged consumption for all, the interest rate does not change, nor does the amount the bank has to liquidate since the additional loan it receives from the government is completely exhausted by additional withdrawals. In short, because households have access to the capital market, government intervention has no effect on the household budget set, consumption, or the interest rate -- household choices perfectly offset government actions. Furthermore, because the interest rate does not change, the liquidity infusion has no effect on the net worth of the banking sector. To restate this, because households can undo government taxation and lending through their deposit withdrawals, a form of Ricardian equivalence (see Barro (1974)) holds.

3.4. Interventions that work.

There is, however, a way of breaking out of this zone of neutrality. Let us now consider a larger tax (and accompanying loan to banks) so that H households withdraw their deposits fully without being able to compensate for the tax, that is $\Delta t > D - w_1^{H,S}$. This means H households' date 1 consumption will fall by $\Delta t - (D - w_1^{H,S})$ relative to the no-intervention case, and their marginal rate of substitution will go up. Prima facie, this seems to go in the wrong direction. But these households no longer participate in the banking system, cannot borrow against their future endowment, and do not determine interest rates. The L households do participate, and their date-1 consumption will have to go up (to absorb some of the consumption given up by the H households). In the new equilibrium, their marginal rate of substitution falls, the interest rate falls, and L households do not make up entirely for the fall in consumption of H households. Overall date-1 consumption falls, and the required liquidation to meet consumption needs falls, and bank net worth rises.

Lemma 3: (i) If the prevailing no-intervention equilibrium has $w_1^{H,S} < D$ *, then government lending to the market financed by a tax of* $\Delta t \le D - w_1^{H,S}$ *has no effect on the interest rate, on consumption, or bank net worth. (ii) Government lending financed by a tax* $\Delta t > D - w_1^{H,S}$ *will reduce the interest rate the bank faces, increasing bank net worth and reducing project liquidation.*

Proof: See appendix.

A very small intervention can have no effect. And there are limits to how much effect the government can have. First, it cannot tax more than the endowment, e_1. Second, once it pushes gross interest rates down to 1, it cannot push them down any further since the banks will simply store any funds they obtain at a lower interest rate.

What if the authorities wanted to raise rates? Here again, the authorities would have an effect only if at least the H type household withdrew its deposits entirely from the banking system in the absence of intervention. If the interest rate is so low to begin with that this is the case, authorities can reverse the above process to raise rates. They would borrow from the open market – that is from banks -- at date 1, and give the collected money back as a date-1 subsidy Δs to households. Date-2 taxes would be increased by $\Delta s\, r_{12}^{S,i}$.

Here is why this would raise the market interest rate. The marginal rate of substitution of the H household, prior to intervention, is higher than the pre-intervention market interest rate (which is why H households withdraw all deposits). H households would consume a small subsidy entirely, increasing their consumption. By contrast, L households would not change consumption if the interest rate did not change. But this would mean overall consumption would be higher than pre-intervention, which would require a higher market interest rate to clear the date-1 goods market and draw forth the necessary amount of liquidation. So it must be that in the new post-intervention equilibrium, the interest rate is higher, type L households consume less at date 1, while type H households consume more.

Lemma 4: If the prevailing no-intervention equilibrium has $w_1^{H,S} = D$ *, then government borrowing from the market and paying the collected resources as a subsidy* Δs *to households (with date-2 repayment of government debt financed by date-2 taxes on households) will increase the interest rate the bank faces and depositors get, decreasing bank net worth and increasing project liquidation*

Proof: See appendix.

3.5. Discussion

Government intervention has effect because one set of participants no longer uses the capital market at market interest rates. For example, when we tax type H households after they have withdrawn fully from the banking system, they would like to withdraw more to offset the tax at prevailing interest rates, but they cannot. Compensation at market interest rates at date-2 does not fully make up their loss (because their marginal rate of substitution is higher). More generally, a number of households do not participate in the financial system. Interventions "work" by effectively taxing them more heavily, and offering the proceeds to participants in the financial system.

Note that the interventions that we are referring to could well be thought of as monetary policy interventions (with either seigniorage or a fiscal component attached), that are not targeted at specific banks, and are meant to bring down the real interest rate. To have effect, they must "penalize" one set of households – those who do not participate as strongly in financial markets -- in order to benefit the system. Note that this differs from direct lender-of-last-resort lending, which is targeted, and undirected lending against illiquid collateral, which is not. In the former, the central bank lends to banks the market would not lend to, while in the latter, the central bank lends against collateral the market will not touch. While the market's aversion to lending may be due to a coordination problem or other friction, viewed at prevailing market prices the central bank's actions in both cases have an element of subsidy. By contrast, there is no explicit subsidized lending in our framework – the central bank picks a rate it will lend at and taxes enough to make sure that it has the resources to meet the market demand at that rate.

Finally, the government plays a similar role in our model as in Holmstrom and Tirole (1998) by making available for contracting household endowments that are inaccessible to the banking sector. However, the focus of the two models is very different, with the government providing new sources of collateral in their model, while it alters household consumption decisions in ours.

3.6. Objectives of Intervention

At date 1, let us assume the central bank always wants to avert a Pareto dominated run. Apart from this, a *household-friendly* central bank will want to undertake actions at date 1 that benefit both types of households. A *entrepreneur-friendly* central bank will undertake actions that benefit the entrepreneurs. We examine such interventions and their effect on the ex ante behavior of the banks in what follows.

3.7. Intervention at Date 0.

Given a level of deposits and an equilibrium interest rate, central banks will reduce interest rates to avoid a run. Beyond that, the interests of households and entrepreneurs diverge.

Lemma 5: An entrepreneur-friendly central bank will reduce interest rates as far as possible. A household friendly central bank will raise interest rates to the maximum possible consistent with the solvency of the banks.

Proof: See Appendix

Clearly, lower interest rates benefit entrepreneurs as fewer of them will be liquidated. Bankers too prefer lower interest rates as it increases their net worth (and thus their consumption). So an entrepreneur-friendly central bank is also banker-friendly.

By contrast, high type households (who have withdrawn all deposits in the range where the central bank can affect interest rates) obtain a lower interest rate only through a further reduction in their date-1 consumption through the tax on endowment. Even though they are compensated at date 2, it is at an interest rate that is even lower than their already high marginal rate of substitution, so they are worse off on net. Conversely, higher interest rates make them better off. Turning to Low type households, they use the financial system to save. For them, a reduction in interest rates reduces their investment opportunity set, making them worse off.

More generally, the corporate sector (firms and banks) is made better off through lower interest rates, while the households are made worse off. Now how will banks set deposits knowing the type of central bank they are dealing with?

3.8. Deposit Levels Chosen at Date 0 with Household-friendly Central Bank

Banks are forced by competition to choose deposit levels at date 0 that maximize the expected utility of households. From Lemma 5, the household-friendly central bank will raise interest rates at date 1 if it can, provided this does not cause banks to fail. It will lower interest rates only if banks would fail otherwise, and will not reduce interest rates below the rate at which banks are just solvent. Let us start with the deposit levels the banks would choose if the central bank were household-friendly. We have

Lemma 6: (i) In any state s where the central bank does not intervene, if D *and* D' *are two deposit levels where the bank is solvent, with* $D' > D$*, depositors will always be better off with deposit level* D' *(ii) If* D'' *is the highest deposit level in state s at which the bank is solvent without central bank intervention, then the low type household would not be better off if the deposit level were lower than* D'' *and the central bank intervened to raise interest rates to make the bank just solvent, or if the deposit level were higher than* D'' *and the central bank lowered interest rates to make the bank solvent. (iii) If the central bank is household friendly there is a pair* $\{\theta_1, \theta_2\}$ *where* $\theta_1 < 1$ *and* $\theta_2 < 1$ *such that if* $\theta^E < \theta_1$ *and* $\theta^N < \theta_2$*, the bank always chooses to set the deposit level at date 0 such that* $D \in [D^{E,\max}, D^{N,\max}]$.

Proof: See Appendix.

Lemma 6 (i) simply states that depositors are better off with a higher face value, provided the bank is solvent at that higher face value. However, we have to ask whether a bank may make its depositors better off by setting deposit rates low initially (e.g., even below $D^{E,\max}$) and have the household-friendly central bank raise interest rates by taxing future endowments of both types of households. Alternatively, the bank could set interest rates high (e.g., above $D^{N,\max}$) and have the household-friendly central bank reduce rates to avoid a run. Lemma 6 (ii) shows that the low type household is better off at date 1 if the deposit rates were set at $D^{E,\max}$ ($D^{N,\max}$) than if it were set lower (higher) and the central bank intervened to raise (lower) rates. If the central bank only intervenes to help both types of households or if its welfare weight on the high type household is not too high, banks will set date-0 deposit rates so as to not require unidirectional intervention by a household-friendly central bank for sure, that is they will not set deposit rates below $D^{E,\max}$ or above $D^{N,\max}$.

We can now examine the optimal ex-ante promised payments to depositors knowing that if the bank turns out to be insolvent at date 1, the central bank will reduce interest rates just enough to make it solvent, while if it is solvent, the central bank will increase rates as high as possible. For our earlier example, we plot the ex ante optimal deposit levels set in the expectation of household-friendly central bank intervention in Figure 1. The green line indicates that deposit levels are set at the higher $D^{N,\max}$ for a broader range of probabilities of the exuberant state, and deposit levels come down only when the probability is very high. If the central bank will intervene to reduce rates ex post if banks are insolvent, the cost to banks (and consequently depositors) of having too high leverage is greatly diminished. Competition forces banks to offer higher deposit levels that will necessitate central bank intervention if the state turns out to be exuberant. As Figure 2 suggests, the expected utility of the households is uniformly higher with intervention by a household-friendly central bank than with no intervention.

The interesting question is why the bank does not set deposits at $D^{N,\max}$ for all possible ex-ante probabilities of the exuberant state, including probability one, and allow the central bank to make payouts to depositors state contingent through intervention. The reason is that, as we have seen in Lemma 6, intervention that pushes down interest rates when D is unnecessarily high is distortionary and hurts depositors . As the probability of the exuberant state nears certainty, it makes more sense to set the deposit level at the maximum consistent with that state, $D^{E,\max}$, than to set it higher and wait for intervention to bring rates down.

3.9. Deposit Levels Chosen at Date 0 with Entrepreneur-friendly Central Bank

Now let us turn to an entrepreneur (and banker)-friendly central bank, which wants to bring rates down as low as possible. Knowing this, banks will offer as high a deposit level as they can up front. Beyond a certain level of deposit, the central bank will not be able to bring rates down to a level that keeps the bank solvent, even by taxing all date-1 endowments and lending them to the banks. This then determines the level of deposits set up front, which turns out to be 1.41 in the example.

In Figure 3, we plot the utility of entrepreneurs and bankers without and with the possibility of intervention by the central bank. Recall that without anticipated intervention, banks set deposit levels high at $D^{N,\max}$ =1.12 only if the probability of the exuberant state is low, for that level of deposits would set off costly runs in the exuberant state. Runs are particularly costly for bankers and entrepreneurs because all projects are liquidated to pay depositors so as the probability of the exuberant state increases, their utility falls. But if the probability of the exuberant exceeds 0.24, the bank switches to setting the level of deposits low at $D^{E,\max}$. This low level of deposits generates low date-1 interest rates, low liquidation, and thus high utility for bankers and entrepreneurs in the normal state, as well as moderate interest rates and liquidation in the exuberant state. Hence entrepreneur/banker utility shoots up (the blue dotted line in Figure 3) and comes down slowly as the probability of the exuberant state increases.

By contrast, if banks anticipate an interventionist central bank, competition forces them to set deposit levels so high that date-1 interest rates even after the central bank intervenes are still moderately high. The utility of the entrepreneur/banker is moderate (the red solid line in Figure 3) regardless of the probability of the exuberant state. Interestingly, therefore, the presence of a entrepreneur-friendly interventionist central bank can make entrepreneurs and bankers worse off for a range of intermediate probabilities because banks raise the promised payout on deposits in anticipation of intervention. Indeed, all agents are worse off over a range of parameters relative to a policy of no intervention.

3.10. Discussion.

In general, because central banks intervene to prevent runs, they raise the ex ante payment competitive banks will promise depositors. Thus a corollary effect of more intervention is more leverage. This can allow for a more state contingent, and thus beneficial contractual structure. However, certain kinds of intervention can be problematic, even for those they are intended to help. In particular, anticipated intervention by an entrepreneur-friendly central bank can make both entrepreneurs and households worse off than in a situation where no intervention was anticipated. This is because banks will adjust promised deposit payments up to compensate for the possible intervention, and this can make everyone worse off.

The more general point we are making is not just the obvious one that government intervention will prompt reaction from the regulated, but that reaction will take specific forms. What we have shown up above is a reaction that takes the form of higher up-front promised payments and thus effective leverage of the banking system. Governments may be better off committing not to intervene, or to intervene only to avoid runs, than to be perceived as entrepreneur-friendly, which can make everyone worse off under a variety of circumstances.

IV. Intervention and the Choice of Assets

Let us now examine a different ex ante bank reaction, which involves altering the liquidity of the bank loan portfolio. Clearly, the more a borrower can repay at date 1 (by liquidating his project), the more liquid a loan to him is.[13] Illiquid projects (that is, projects that liquidate for less at date 1) are dominated assets unless they have higher long-term payoffs. Bank portfolio choices can be sub-optimal in the environment we have modeled because date-1 interest rates do not always fully reflect the preferences of at least one set of households, the High type, who may have withdrawn entirely from the banking system. Let us now examine the effects of interest rate policy (and deposit levels) on the incentives of banks to alter the liquidity of the assets they hold.

4.1. Asset choices

Let each bank choose to lend either solely to liquid projects or to illiquid projects after issuing deposits at date 0: liQuid projects, with superscript *q*, have $X_1^q = X_1$ and $\tilde{Y}_2^q = \tilde{Y}_2$ and Illiquid projects, with superscript *i*, have $X_1^i < X_1$ and $\tilde{Y}_2^i = Z\tilde{Y}_2$ with Z>1. Let us start by assuming the choice is not observed until date 1, when the state of nature is known. We examine the conditions under which a bank

[13] One interpretation of a liquid project is that the bank monitors the borrower to make sure that the borrower maintains the investment is a way that allows easy unwinding or sale. An illiquid investment could be one that the bank knows it will have to refinance fully in the future (as when it sets up a special interest vehicle (SIV) with assets that are hard to sell). Illiquid projects could also require more capital injections before they become operational or will take a long time before they generate cash flows, and cannot be sold before then.

will choose to make only loans to liquid projects (we will call such a bank and such loans "liquid"). We will later explain why we are interested in such a choice.

4.1. Project Liquidation

At date 1 in state *s*, a solvent liquid bank chooses to liquidate projects with realization Y_2 that satisfies $Y_2 < Y_2^q(r_{12}^S) = \frac{r_{12}^S X_1}{\gamma}$. A solvent illiquid bank liquidates projects with $Y_2 < Y_2^i(r_{12}^S) = \frac{r_{12}^S X_1^i}{\gamma Z} < Y_2^q(r_{12}^S)$, so at any given interest rate, illiquid banks liquidate fewer projects. The value (at date 1) of the liquid bank's assets is $V_1^q(r_{12}^s) = \frac{1}{\overline{Y}_2 - \underline{Y}_2}\left[\int_{\underline{Y}_2}^{Y_2^q(r_{12}^s)} X_1 dY_2 + \frac{\gamma}{r_{12}^s}\int_{Y_2^q(r_{12}^s)}^{\overline{Y}_2} Y_2 dY_2\right]$ while that of an illiquid bank's assets is $V_1^i(r_{12}^s) = \frac{1}{\overline{Y}_2 - \underline{Y}_2}\left[\int_{\underline{Y}_2}^{Y_2^i(r_{12}^s)} X_1^i dY_2 + \frac{\gamma Z}{r_{12}^s}\int_{Y_2^i(r_{12}^s)}^{\overline{Y}_2} Y_2 dY_2\right]$. In what follows, we assume individual bankers do not think they will affect the date-1 rate through their actions.

4.3. Relative Asset Values

Given a deposit level D, a bank will choose to lend to liquid projects at date 0 if :

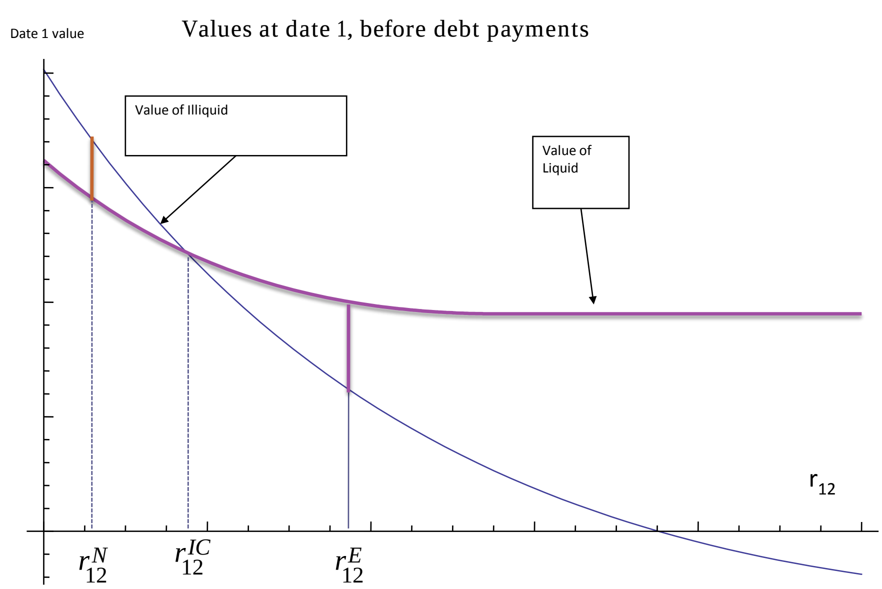
$$
\begin{aligned}
&p^E[r_{12}^E(\max\{0, V_1^q(r_{12}^E) - D\} - \max\{0, V_1^i(r_{12}^E) - D\})] + \\
&(1-p^E)[r_{12}^N p^E(\max\{0, V_1^q(r_{12}^N) - D\} - \max\{0, V_1^i(r_{12}^N) - D\})] \geq 0
\end{aligned}
\qquad \text{(ICq)}
$$

The ex-ante incentive to choose liquid projects (ICq) depends on the probability weighted average of the ex-post state-contingent differences in banker payoffs between liquid and illiquid loans. The incentive to choose liquid loans depends on the deposit level, *D*, and on date-1 interest rates. When *D* is low enough that the bank will not fail in any state for any type of loan (this ensures the truncation at 0 is not binding), differentiating the incentive constraint, state by state, with respect to r_{12}^S, we get

$$\frac{X_1 - X_1^i}{\overline{Y}_2 - \underline{Y}_2} \int_{\underline{Y}_2}^{Y_2^i(r_{12}^S)} 1dY + \frac{X_1}{\overline{Y}_2 - \underline{Y}_2} \int_{Y_2^i(r_{12}^S)}^{Y_2^q(r_{12}^S)} 1dY + \frac{dY_2^q(r_{12}^S)}{dr_{12}^S} \{r_{12}^S X_1 - \gamma Y_2^q(r_{12}^S)\} - \frac{dY_2^i(r_{12}^S)}{dr_{12}^S} \{r_{12}^S X_1^i - \gamma Z Y_2^i(r_{12}^S)\}$$

The last two expressions are zero because, $r_{12}^S X_1 - \gamma Y_2^q(r_{12}^S) = 0$ and $r_{12}^S X_1^i - \gamma Z Y_2^i(r_{12}^S) = 0$. So the expression simplifies to $P[\tilde{Y}_2 \le Y_2^i(r_{12}^s)](X_1 - X_1^i) + P[Y_2^i(r_{12}^s) \le \tilde{Y} \le Y_2^q(r_{12}^s)]X_1$, which is positive. Thus the loan to the liquid project becomes relatively more attractive as the interest rate expected to prevail at date 1, r_{12}^S, increases. Intuitively, a higher interim rate enhances the reinvestment opportunity rate (or, equivalently, the discount rate on long term flows), which enhances the relative value of being liquid.

Turning to date-1 bank loan values, we can show that $V_1^q(r_{12}^s), V_1^i(r_{12}^s)$ each decrease in r_{12}^S. Also, we can show $V_1^q(r_{12}^s) - V_1^i(r_{12}^s)$ increases in r_{12}^S, and at low date-1 interest rates $V_1^i(r_{12}^s) > V_1^q(r_{12}^s)$ while at high interest rates $V_1^i(r_{12}^s) < V_1^q(r_{12}^s)$. Therefore, there must be an interest rate r_{12}^{IC} such that $V_1^i(r_{12}^s) = V_1^q(r_{12}^s)$. In any state, liquid loans are more valuable if and only if the interest rate is above r_{12}^{IC}.



4.4. Bank value given the possibility of default

So long as the bank expects to be solvent in all states, its ex ante choice between the loans it makes (that is, between the assets it chooses to hold) depends only on their relative value in different states. However, if *D* exceeds the bank's asset value at the equilibrium interest rate in a state, the banker's limited liability ensures that his loss is capped at zero. The value of the bank's assets in that state to him is then zero. It is straightforward to incorporate the effects of these truncated payoffs into the analysis above. Clearly, the level of *D* determines whether a bank is solvent or not, given the choice of liquid or illiquid loans, in a particular state. Let us examine more carefully how *D* then influences the choice of liquid versus illiquid.

4.5. Deposit levels and interest rates

Let $r^S(D)$ be the date 1 general equilibrium interest rate from date 1 to 2, which is an increasing function of *D* (we drop r's subscripts "12" to denote a function). We know $r^E(D) > r^N(D)$. If $r^N(D) > r_{12}^{IC}$, then even at the lowest state contingent interest rate consistent with the deposit level, the illiquid project is unattractive. So the bank will choose liquid projects (at higher deposit levels, interest rates will be higher, so the preference for liquid projects will, weakly, increase). By contrast, if $r^E(D) < r_{12}^{IC}$, the bank prefers illiquid projects at the highest state contingent interest rate that can prevail for that project, so it will choose illiquid projects (and do so for all lower deposit levels). More interesting is when $r^E(D) \geq r_{12}^{IC} \geq r^N(D)$. The bank's ex ante preference for liquid vs illiquid turns on the probabilities of the states, and the bank's value in those states.

Given the discussion above, it is clear that D has to be above D', where $r^E(D') = r_{12}^{IC}$, for the banker to choose liquid over illiquid. However, the banker's incentive to choose liquid does not increase monotonically with D beyond that. The intuition is that a higher *D* has two effects. One the one hand, it increases equilibrium state contingent interest rates, which increases the preference for liquid. On the other hand, it reduces both liquid and illiquid asset values in the exuberant state. A higher D will improve incentives to choose liquid whenever both liquid and illiquid banks are solvent (because an increase in rates associate with the increase in deposits reduces the value of the illiquid bank's assets in the exuberant state by even more than the value of the liquid bank's assets). But once the illiquid bank becomes insolvent, further reductions in its asset value have no effect on incentives. Now increases in *D* can reduce incentives to choose liquid.

To see an example where a high *D* does not necessarily induce banks to choose liquid, consider $D^{E,Max}$, the maximum deposit level consistent with solvency in the exuberant state for a bank lending to liquid projects, and let $r^E(D^{E,Max}) \geq r_{12}^{IC} \geq r^N(D^{E,Max})$. It is clear that the banker's value net of deposits in the E state will be just zero, as would be his stake if he had chosen illiquid, since the illiquid asset's value is even lower in the E state. By contrast, in the N state, both liquid and illiquid banks have value net of deposits, with the latter higher than the former. So if the deposit level is set at $D^{E,Max}$, the bank will not choose liquid. More generally, we have

Lemma 7:

(i) If $r_{12}^{IC} > r^E(D^{E,Max})$, then there is no deposit level for which the bank will choose liquid.

(ii) If $r^N(D^{N,Max}) > r_{12}^{IC}$, then the bank will choose liquid if the deposit level is set at $D \in \left[D^{E,Max}, D^{N,Max}\right]$.

(iii) If $r^E(D^{E,Max}) \geq r_{12}^{IC} \geq r^N(D^{N,Max})$, the bank will not choose liquid if $D \geq D^{E,Max}$. For any *D* such that $r^E(D) > r_{12}^{IC}$, the bank's incentives to choose liquid increase with an increase in D, so long as the illiquid bank is solvent at *D* in the exuberant state.

Proof: See Appendix.

In summary, ex ante deposit levels will influence both date-1 interest rates and the choice between liquid and illiquid. Furthermore, the incentive to choose liquid is not monotonic in the deposit level, even though non-run state contingent interest rates rise (weakly) with an increase in the deposit level. Given that deposit levels may be chosen to be inconsistent with the choice of liquid, there may be an added reason for ex post intervention – to improve the ex ante preference for liquid.

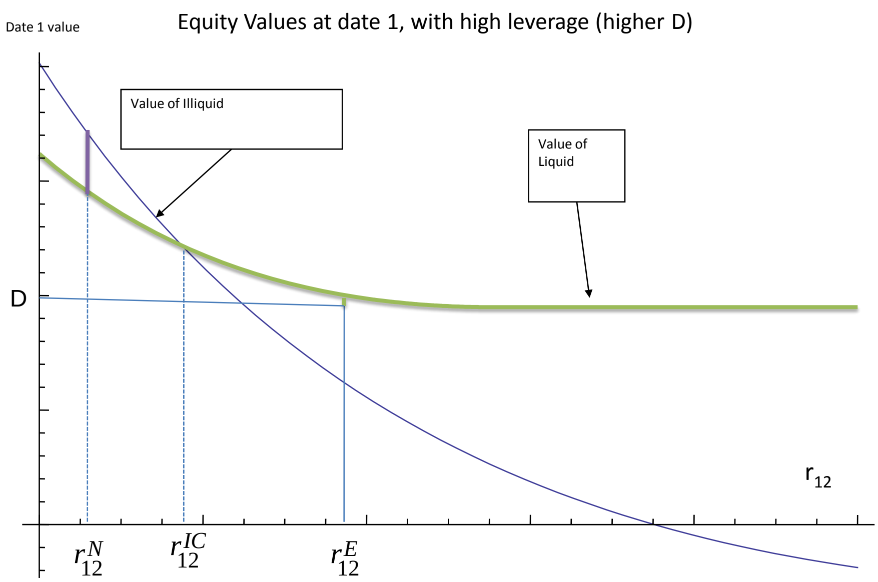
But why might authorities want banks to choose liquid projects? As we have seen, the incentive to choose more liquid projects rather than less liquid projects depends in large measure on the date-1 interest rate, which reflects the value from retaining liquidity. The limited participation of the liquidity-needy High type households in the financial sector at date-1 (and their inability to borrow against endowments) will imply that date-1 interest rates are typically too low from the ex ante perspective of determining liquidity choices. In addition, any ex post intervention to reduce interest rates to preserve bank solvency will again make them too low. Therefore, authorities may have a specific interest in creating stronger incentives for banks to choose liquidity.

So let us now examine the effects of date -1 intervention on the incentive to choose liquid, taking the deposit level set at date 0 as given. We will see that in order to induce more appropriate asset choices, the central bank may want to commit to smooth interest rates across states – in a sense to be more entrepreneur-friendly in times of naturally high interest rates, and be more household friendly in times of naturally low interest rates.

4.6. Intervention and its consequences

First, if $r_{12}^{S} < r_{12}^{IC}$ and $D > V_1^{i}(r_{12}^{s}) > V_1^{q}(r_{12}^{s})$ then clearly, the bank's payoffs with either loan is zero because it will have to default in either case. A small reduction in the equilibrium rate in the state would initially have no effect on relative bank values (because the bank would still be in default whether it was liquid or illiquid). Further reductions will eventually result in a positive value for the illiquid bank as its asset value rises above *D*. Thus when the interest rate is below r_{12}^{IC}, lowering it further (weakly) enhances the relative value of the illiquid loan in that state (and dissuades the ex ante choice of liquid), regardless of the value of *D*.

Suppose now the equilibrium interest rate is above r_{12}^{IC}. If $D > V_1^q(r_{12}^s) > V_1^i(r_{12}^s)$ and $V_1^q(r_{12}^{IC}) > D$, or if $V_1^q(r_{12}^s) > D > V_1^i(r_{12}^s)$, the liquid bank's value can be enhanced by reducing the interest rate. Intuitively, lowering the interest rate will raise both V_1^q and V_1^i. So long as $D > V_1^q(r_{12}^s) > V_1^i(r_{12}^s)$, neither bank will be solvent and hence changes in interest rate will not affect the relative value. But once $V_1^q(r_{12}^s) > D > V_1^i(r_{12}^s)$, lowering the interest rate further will enhance the value of the liquid bank net of deposits, while it will not affect the value of the (underwater) illiquid bank. It will then make sense to lower the interest rate until $V_1^i(r_{12}^s) = D$, when the difference in values is maximized. The idea here is to create a "negative moral hazard" – reduce interest rates more than necessary for solvency in order to benefit those who have maintained the desired (liquid) set of assets.

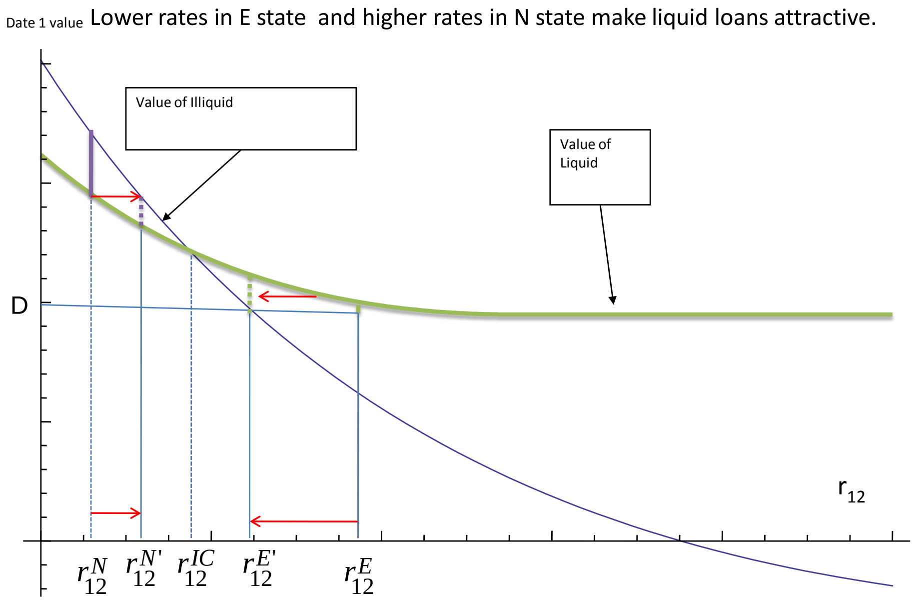


Finally, if $V_1^q(r_{12}^s) > V_1^i(r_{12}^s) \geq D$, the liquid bank will be made relatively more attractive if the interest rate is raised until it can be raised no more or $V_1^i(r_{12}^s) = D$, whichever comes first.

4.7. Ex post intervention given ex ante incentive constraints

We have seen the effects of intervention conditional on a state. Now consider the overall form interest rate intervention should take across states if the authorities want to encourage the choice of liquid, and they can commit to any interest rate policy that satisfies the no-bank-failure condition.

As we have seen, if $r_{12}^{IC} > r_{12}^E > r_{12}^N$, then no interest rate intervention can restore ex ante incentives to choose liquid – the authorities have to allow interest rates to go above r_{12}^{IC}, but at that interest rate all banks fail. If $r_{12}^E > r_{12}^N \geq r_{12}^{IC}$, ex ante incentives to choose liquid are well established. The interesting case is when the deposit level is chosen such that $r_{12}^E > r_{12}^{IC} > r_{12}^N$. Now interventions in both states could help generate incentives to choose liquid projects. In the E state, the interest rate should be brought down to the point that the illiquid bank is just solvent. In the N state, however, the interest rate should be raised – if possible to the same interest rate as in the E state, but failing that, to the highest interest rate possible.



In sum, policies on interest rates should not just take into account the possibility of bank failures, but also the potential for banks to choose excessively illiquid projects if they anticipate a low interest rate environment. Optimal interest rate policies may require committing to raising rates when relatively low and reducing them when high, in order to foster the right ex ante incentives.

4.8. Discussion

We have attempted to give a flavor of the forces at work through a set of partial analyses. In a full fledged general equilibrium, banks will set deposit levels taking into account interest rate policy, and then choose asset portfolios. Contingent on the state, the authorities will then carry out their committed interest rate intervention. The point, however, is that all these choices are linked.

For instance, if deposit levels are limited by regulation to be at a low level, they may give banks an incentive to increase the illiquidity of their portfolios. Similarly, if authorities are seen as too ready to reduce rates, banks will typically react by increasing the level of deposits accordingly (see previous section), as well as by reducing the liquidity of their assets. If the authorities want to rescue insolvent banks but also encourage greater asset liquidity because the market interest rate does not fully reflect the value of liquidity, they will have to commit to raising rates in some states. Such interest rate smoothing may strictly not be necessary if the authorities were solely focused on maintaining activity, indeed it may be counterproductive, but if it can be committed to, it may help improve stability and welfare.

4.9. Relation to literature

To be written

V. Conclusion

Our work has been done in very interesting times. Following a period of extremely benign financial conditions, we are in the midst of a financial panic caused by financial firms overloading on illiquid assets and taking on too much leverage. Why might we have arrived in this state and what can we do about it? While agency problems in banks and the breakdown in risk controls and compensation structures (see Kashyap, Rajan, and Stein (2008)) must be part of the explanation, our model offers another one; the anticipated benign environment, perhaps accentuated by hopes of central bank intervention if interest rates rose too high and asset prices fell substantially, must have also been an important causal explanation for both the illiquidity of assets and the excessive leverage.

Our analysis points to the dangers of one-sided interventions. For instance, reducing interest rates drastically when the financial sector is in trouble, but not raising them quickly as the sector recovers could create incentives for banks to seek out more illiquidity than good for the system. Such incentives may have to be offset by raising rates in normal times more than strictly warranted by macroeconomic conditions. While our entire model is couched in terms of real goods, we hope in future work to describe monetary implications, as well as explore the dynamic implications.

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Appendix 1: Proofs (to be written)

Appendix 2: Runs and observable but verifiable liquidity.

Finally, let us consider what happens if bank choice of liquidity is observed at date 0, after the deposit but before any news about the state of nature is known. A deposit of one unit can be withdrawn for one unit immediately or for D once the state of nature on date 1 is known. Because there are always unfunded projects, if a depositor withdraws at date 1, he can reinvest at D in another bank. We seek to determine if there is an equilibrium where all banks are willing to choose liquid loans when they are interest rate takers.

If a bank is run immediately after deviation to illiquid is observed, then the threat of the run would deter deviation to illiquid loans even if the incentive constraint with unobservable liquidity is violated. If the incentive constraint (ICq) is satisfied, then a bank will make liquid loans with or without the threat of a run.

If the incentive constraint is violated and, in addition, $r_{12}^{IC} \geq r_{12}^{E} \geq r_{12}^{N}$, then deviation to illiquid will not make the bank worse off (than the liquid bank) in either state and the threat of runs will not commit banks to be liquid. If $r_{12}^{E} > r_{12}^{IC} > r_{12}^{N}$, then the threat of a run can commit a bank to be liquid if D is sufficiently *high* such that the bank will be run in the E state if it deviates to illiquid (and not so high that it would be run if it chooses liquid). Because the payoff of a solvent bank is ex-post preferable to that of a run bank, deviation to illiquid will lead to a run if other banks will choose to be liquid. This requires that the bank be solvent in state E if and only if it chooses to be liquid, or

$$\int_{\underline{Y}_2}^{Y_2^q(r_{12}^E)} (r_{12}^E X_1) dY_2 + \gamma \int_{Y_2^q(r_{12}^E)}^{\overline{Y}_2} Y_2 dY_2 \geq r_{12}^E D > \int_{\underline{Y}_2}^{Y_2^i(r_{12}^E)} (r_1^E X_1^i) Y_2 dY_2 + \gamma Z \int_{Y_2^i(r_{12}^E)}^{\overline{Y}_2} Y_2 dY_2 \, .$$

If deviation to being illiquid is observed immediately, the threat of a run can allow the bank to have maximum leverage consistent with never failing; the bank chooses D to be just solvent in state E and will be run if it deviates to become illiquid. This only will work if the government can commit to allow these runs. If the government will intervene to reduce interest rates to stop runs, then high leverage will not serve as a commitment device and the only way to assure that banks remain liquid is to satisfy the incentive constraint, which requires that interest rates not be too low in state N.

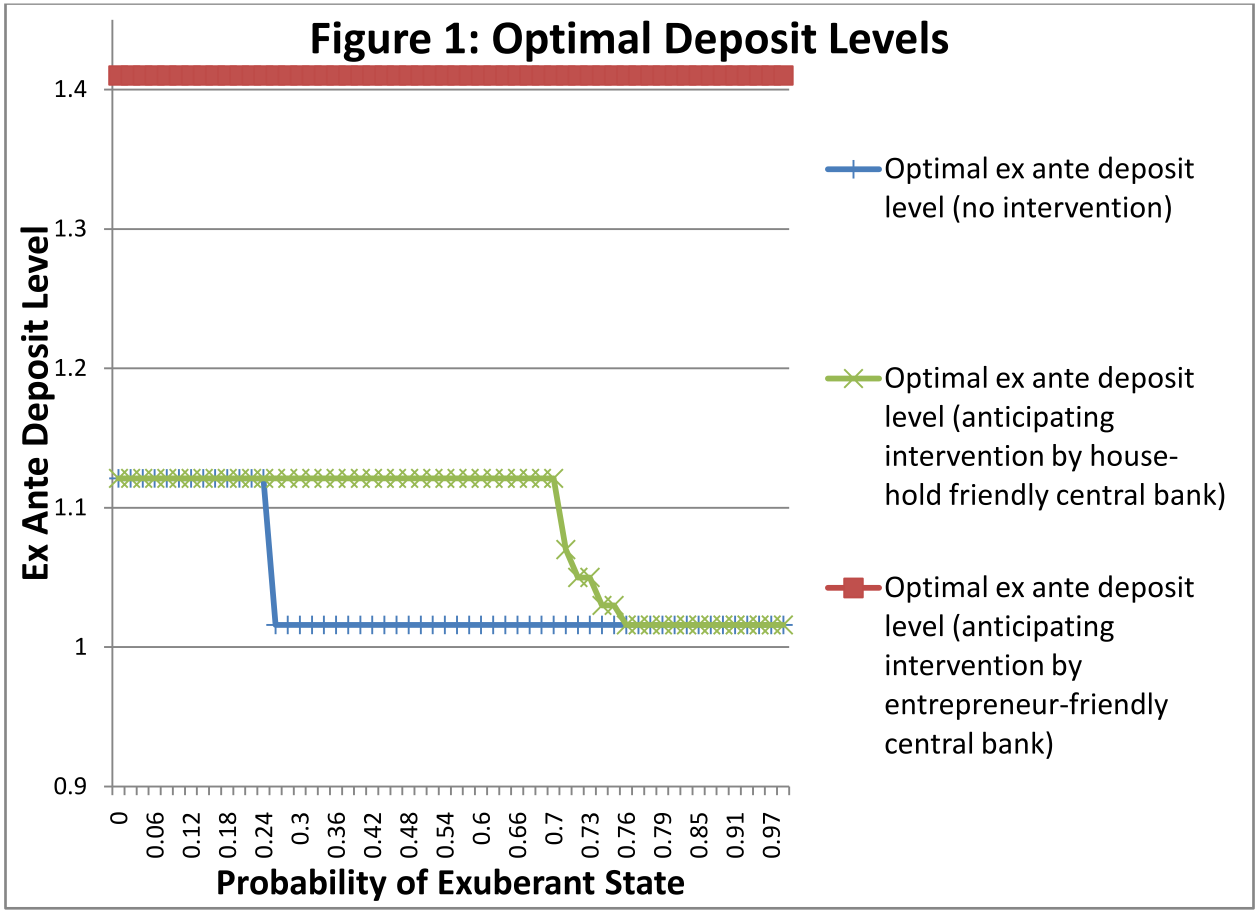
Figure 1: Optimal Deposit Levels
Ex Ante Deposit Level
1.4
1.3
1.2
1.1
1
0.9
0
0.06
0.12
0.18
0.24
0.3
0.36
0.42
0.48
0.54
0.6
0.66
0.7
0.73
0.76
0.79
0.85
0.91
0.97
Probability of Exuberant State
Optimal ex ante deposit level (no intervention)
Optimal ex ante deposit level (anticipating intervention by house-hold friendly central bank)
Optimal ex ante deposit level (anticipating intervention by entrepreneur-friendly central bank)

Figure 2: Expected Utility of Households with and without intervention

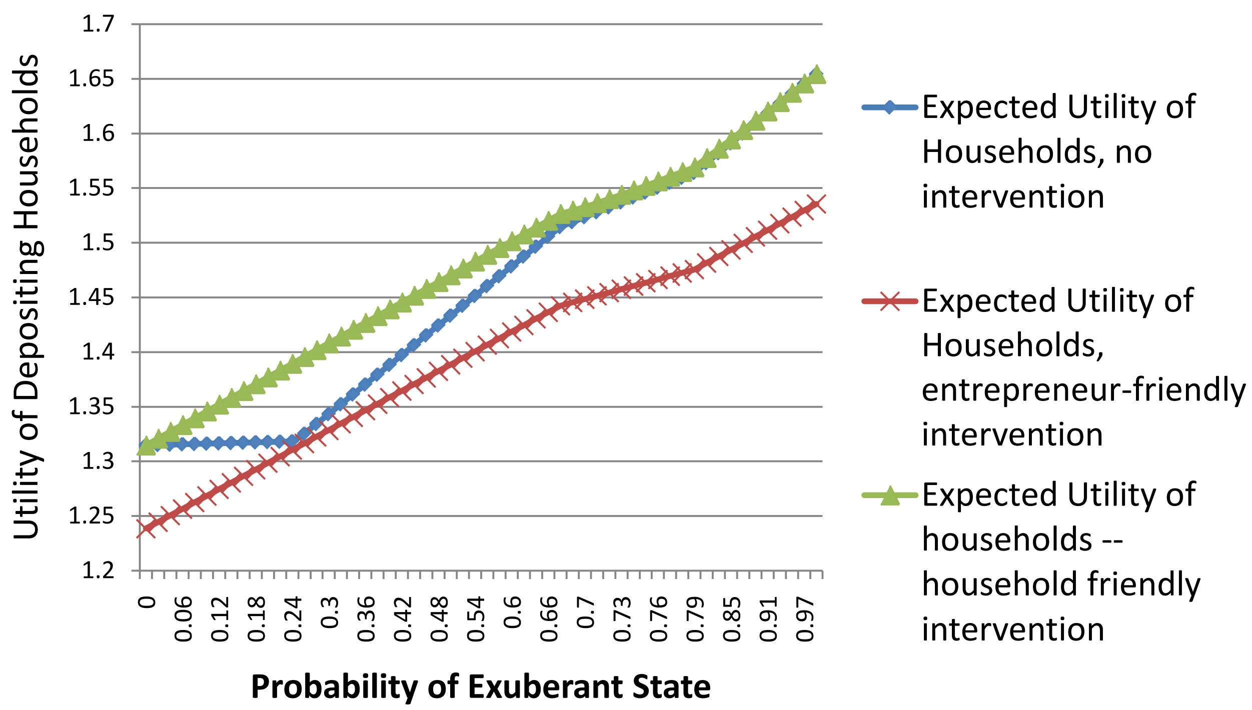

Figure 3: Expected Utility of Entrepreneurs/Bankers with and without intervention .