Interest on Cash with Endogenous Fiscal Policy

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Abstract

Monetary policy and the welfare cost of inflation cannot be studied without some specification of allowable fiscal instruments. Here, feasible policies are implied by the frictions imposed to get roles for money and credit. The model has extreme versions of an above-ground economy—people who are perfectly monitored and who, therefore, can be taxed—and a benign underground economy—people who are anonymous and, who, therefore, cannot be taxed. For examples, ex ante (representative-agent) optima are computed. For most examples for the outside-money version, it is not optimal to use taxes to raise the return on money held by anonymous people. {101 words}

Key words: interest on cash, inside-money, outside-money, matching, monitoring, mechanism-design

JEL classification: E52, E58

1 Introduction

It is well-known that monetary policy and the welfare cost of inflation cannot be studied without some specification of allowable fiscal instruments. Phelps [15] made that point and subsequent work has only reinforced it. Indeed, if the set of fiscal instruments is rich enough, then monetary policy is superfluous (see, for example, Wallace [20], Sargent and Smith [16], and Correia *et. al.* [6]). Those and related results suggest that substantial progress depends on having a *theory* of feasible policies. We provide one such theory here: roughly speaking, a policy is feasible if it is consistent with the frictions we impose to get roles for money and credit. In other words, we conduct a *mechanism-design* analysis given the constraints implied by the frictions we assume.

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According to recent work in monetary theory, the crucial friction that generates a role for *money* is imperfect monitoring—incomplete knowledge of the previous actions of at least some people (see, for example, Ostroy [14], Townsend [18], and Kocherlakota [10]). Not surprisingly, that friction has consequences for feasible policies—in particular, for taxation. That the consequences are likely to be important is suggested by even the most casual observations concerning the role of *currency* in actual economies—currency because the money in the model we study resembles currency.

A pervasive observation is that currency is intensively used to help evade taxes and in what is labeled the underground economy—the part of the economy in which, by definition, activities are somewhat hidden and, therefore, difficult to tax. Even if the underground economy is benign, as it is in the model we study, it would be surprising if the difficulty of taxing underground activities did not have important implications for the desirability of paying interest on currency.

We conduct our analysis within a variant of the model in Cavalcanti and Wallace [5], a model that has extreme versions of an above-ground economy and an underground economy. The former consists of people who are perfectly monitored, while the latter consists of people who are anonymous and not monitored at all. Our variant allows for various cross-section distributions of a one-time cost of becoming permanently monitored. For several examples, we compute the ex ante (representative-agent) optimum—the optimum before people realize their idiosyncratic costs of becoming monitored or their initial holdings of money. We do so for an inside- or private-money version of the model and for an outside-money version. (Inside money should be interpreted as checks made out to cash, cash cards, or payable-to-the-bearer trade credit instruments issued and redeemed by monitored people and used by them and nonmonitored people.)

We compare the average return on money for nonmonitored people—a return implied by the trades that occur, their frequencies because discounting is applied, and the model's analogue of inflation—across optima for examples that have different costs of becoming monitored. Sufficient discounting is assumed so that if the cost of becoming monitored is prohibitive for everyone, then payment of interest on money—although, not feasible—would be desirable. We use the average return in the optimum for that extreme example as a benchmark and ask whether economies with lower costs of becoming monitored—lower in such a way that some fraction of the population becomes monitored—have higher returns on money. In the inside-money version, the presence of monitored people is used to raise that return somewhat. In the outside-money version, that is not

the case for most of our examples.

We are not the first to consider payment of interest on cash and its connection to the underground economy (see Nicolini [13] and Koreshkova [11]). However, in those studies the set of fiscal instruments is only loosely connected to the specification of the underground economy and the role of money. Camera [4] studies money and an underground economy in a matching model, but uses a specification in which the only policy instrument is the distribution of money. Our model is closest in spirit to Antinolfi *et. al.* [2]. They study an economy with two sectors: one is a credit economy modeled as in Kehoe and Levine [9]; the other is a currency economy modeled as in Bewley [3]. However, in their model there is no contact in equilibrium between the currency and credit sectors; in our model, the contact between monitored people and nonmonitored people is central to the results. While there is a version of our model without contact between them, in that version the possible *policies* are so limited that the model is uninteresting.

2   The model

Time is discrete with two stages at each date. There is a nonatomic measure of people each of whom maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. The first stage at each date has pairwise meetings and the second stage has a centralized meeting. Just prior to the first stage, a person looks forward to being a consumer who meets a random producer with probability $\frac{1}{K}$, looks forward to being a producer who meets a random consumer with probability $\frac{1}{K}$, and looks forward to no pairwise meeting with probability $1 - \frac{2}{K}$, where $K \geq 2$. The period utility of someone who becomes a consumer and consumes $y \in \mathbb{R}_+$ is $u(y)$, where u is strictly increasing, strictly concave, differentiable, and satisfies $u(0) = 0$. The period utility of someone who becomes a producer and produces $y \in \mathbb{R}_+$ is $-c(y)$, where c is strictly increasing, convex, and differentiable and $c(0) = 0$. In addition, $y^* = \arg\max_{y \geq 0}[u(y) - c(y)]$ is positive. Production is perishable; it is either consumed or lost.[1] There is no utility associated with actions at the second stage, which is used only to make transfers of money.

People in the model are ex ante identical and make an initial and one-time choice between becoming monitored (an m person) or becoming nonmonitored (an n person). For m people, histories and money holdings are common

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[1] This formulation is borrowed from Trejos-Wright [19] and Shi [17]. If K is an integer that exceeds two, then, as is well-known, it can be interpreted as the number of goods and specialization types in those models.

knowledge; for n people, they are private. However, the monitored status and consumer-producer status (in a pairwise meeting) of each person are common knowledge. And, no one, except the planner, can commit to future actions.

People choose m or n status after receiving a private and independent draw from a distribution of an additively separable, one-time utility cost of becoming an m person. Let $F : \mathbb{R}_+ \rightarrow [0,1]$, where $F(x)$ is the probability of having a utility cost of becoming monitored no greater than x. We treat F as both the distribution from which individual draws are made and as the realized distribution of costs over the population.

We study two classes of F functions. Let B be a cost so high that a person who realizes cost B would never choose to become an m person. One class of F functions is that used in Cavalcanti and Wallace [5]; namely,

$$F_\alpha^e(x) = \left\{ \begin{array}{l} \alpha \text{ if } x < B \\ 1 \text{ if } x \geq B \end{array} \right. . \tag{1}$$

Here, α is the exogenous fraction who are m people. The other class has a smoother F. For it, if the allocation ends up with some m people and some n people, then there is an internal cut-off cost.

In order to allow for a discussion of inside money, people and the planner have printing presses capable of turning out identical, indivisible, and somewhat durable objects. Those turned out by the printing press of any one person are, however, distinguishable from those turned out by other peoples' printing presses.

Finally, each person's holding of money (issued by others) is restricted to be in $\{0,1\}$—both at the start of stage 1 and at the start of stage 2. Because of the assumed restriction on individual money holdings, we assume that the planner can choose a probability with which money disintegrates between stage 1 and stage 2 at each date—whether money is inside money or outside money. As explained further below, that is the model's analogue of inflation.

3   Implementable allocations and the optimum problem

We limit the search for an optimum to allocations that are constant—are steady states—and symmetric. By symmetry, we mean that all people in the same situation take the same action, where that action could be a lottery. (That is, there is no randomization.) We also limit allocations to ones in which all monies issued by m people who have not defected and money issued by the planner are

treated as perfect substitutes and in which all monies issued by n people are worthless.[2] (Hence, we simply assume that n people do not issue money.)

The planner chooses m or n status as a function of a person's realized cost of becoming an m person (the person's draw from F); the fraction of m people with a unit of money and the fraction of n people with a unit of money; trades in stage-1 meetings (as functions of the states of the producer and the consumer); the disintegration-of-money probability; and stage-2 transfers.

The sequence of actions at the first date is as follows. People are ex ante identical and the planner's objective is maximization of ex ante expected utility, a representative-agent criterion. First, the planner's choice is announced. Then each person gets a private draw from the monitoring-cost distribution, F, and chooses m or n status, a choice that is observed. Then, initial money holdings are distributed conditional on m or n status. Then, the two stages occur at the first date and all subsequent dates.

The planner's choice is subject to being a steady state and to self-selection constraints that follow from our specification of private information and of punishments. We assume that the only punishment is permanent banishment of an individual m-person to the set of n-people, which includes loss of the ability to issue money in the inside-money version of the model. Underlying this assumption is free exit at any time from the set of m-people and the ruling out of global punishments—like the shutting down of all trade in response to individual defections. We allow both individual and cooperative defection of those in a stage-1 meeting, but only individual defection from stage-2 transfers (because there are no static gains from trade at stage 2). The details follow.

3.1 Notation

Let $S = \{m, n\} \times \{0, 1\}$ be the set of individual states, where $s = (s_1, s_2)$ and $s' = (s'_1, s'_2)$ denote generic elements of S. The state of a stage-1 meeting is denoted (s, s'), where s is the state of producer and s' is the state of the consumer. Our main notation is summarized in the following table.

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[2] Given a bound on discrete money holdings, trade can be enhanced by distinguishing among monies—say, by color (see Aiyagari *et. al.* [1]). Here we ignore that possibility because the only role of the assumption that money holdings are in $\{0, 1\}$ is to limit the number of unknowns in our optimum problem.

Table 1. Notation	
$y^{s,s'}$	production by s and consumption by s' in a meeting
$\lambda_p^{s,s'}(i)$	prob. that end-of-stage-1 money is i for producer
$\lambda_c^{s,s'}(i)$	prob. that end-of-stage-1 money is i for consumer
ξ	prob. that end-of-stage-1 money disintegrates
$\phi^s(i)$	prob. that end-of-stage-2 money of s is i
θ^s	fraction in state s at the start of a date
v^s	discounted utility for s at the start of a date

The planner chooses the variables in the table subject to the constraints set out below.

3.2 Feasibility and steady state conditions

The λ's and ϕ's must, of course, be lotteries on $\{0,1\}$. Under outside money, money is not created in stage-1 meetings. Therefore, under outside money, if $s_2 + s'_2 = 0$, then $\lambda_j^{s,s'}(0) = 1$; while if $s_2 + s'_2 = 1$, then $\lambda_p^{s,s'}(1) \le \lambda_c^{s,s'}(0)$ and $\lambda_p^{s,s'}(0) \ge \lambda_c^{s,s'}(1)$. Under inside money, money can be created by m people. Therefore, under inside money these constraints apply only when $s_1 = s'_1 = n$. Also, these constraints permit disposal of money. This possibility is especially relevant in a meeting between an $(m,1)$ producer and an $(n,1)$ consumer. In this case, we do not require that $\lambda_p^{s,s'}(1) = \lambda_c^{s,s'}(1) = 1$. Instead, the allocation can require that the consumer give up money with some probability—even though this requires free disposal on the part of the $(m,1)$ producer.

Stage-1 and stage-2 actions imply the transition probabilities of a person's money holding from the start of one date to the start of the next date. The probability that a person in state $(s_1, i) \in S$ transits to state $(s_1, j) \in S$ is

$$t^{s_1}(i,j) = \frac{1}{K} \sum_{s' \in S} \theta(s') \left[(\lambda_p^{(s_1,i),s'} + \lambda_c^{s',(s_1,i)} + (K-2)\delta_i) \Psi \Phi^{s_1}(j) \right], \quad (2)$$

where $\lambda_p^{s,s'} = (\lambda_p^{s,s'}(0), \lambda_p^{s,s'}(1))$, $\lambda_c^{s,s'} = (\lambda_c^{s,s'}(0), \lambda_c^{s,s'}(1))$, δ_i is the two-element unit vector in direction $i+1$,

$$\Psi = \begin{bmatrix} 1 & 0 \\ \xi & 1-\xi \end{bmatrix}, \quad (3)$$

and $\Phi^{s_1}(j) = (\phi^{(s_1,0)}(j), \phi^{(s_1,1)}(j))'$. If T^{s_1} denotes the 2×2 matrix whose (i,j)-th component is $[t^{s_1}(i,j)]$, then the steady state requirements are

$$(\theta^{(s_1,0)}, \theta^{(s_1,1)}) T^{s_1} = (\theta^{(s_1,0)}, \theta^{(s_1,1)}), \quad (4)$$

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for $s_1 \in \{m, n\}$.[3]

3.3   Incentive constraints

It is convenient to first define discounted expected utility at the start of stage 1. For $s \in S$, we have

$$v^s = \frac{1}{K} \sum_{s' \in S} \theta^{s'} [\pi^p(s, s') + \pi^c(s', s) + (K - 2)\pi^0(s)], \tag{5}$$

where

$$\pi^p(s, s') = -c(y^{s,s'}) + \beta \lambda_p^{s,s'} \Psi \Phi^{s_1} v^{s_1}, \tag{6}$$

$$\pi^c(s, s') = u(y^{s,s'}) + \beta \lambda_c^{s,s'} \Psi \Phi^{s_1'} v^{s_1'}, \tag{7}$$

and

$$\pi^0(s) = \beta \delta_{s_2} \Psi \Phi^{s_1} v^{s_1}. \tag{8}$$

Here, δ_{s_2} is the 1×2 unit vector in direction $s_2 + 1$,

$$\Phi^{s_1} = \begin{bmatrix} \phi^{(s_1, 0)}(0), \phi^{(s_1, 0)}(1) \\ \phi^{(s_1, 1)}(0), \phi^{(s_1, 1)}(1) \end{bmatrix}, \tag{9}$$

and $v^{s_1} = (v^{(s_1, 0)}, v^{(s_1, 1)})'$. Given the variables in the first six rows of Table 1, Blackwell's sufficient conditions for contraction imply that v^s exists and is unique. We express the incentive constraints in terms of the v's. This is legitimate because the principle of one-shot deviations applies to this model.

There are truth-telling constraints only for n people with money when they are consumers. They are

$$\pi^c(s, (n, 1)) \geq u(y^{s,(n,0)}) + \beta(\xi, 1 - \xi)\Phi^n v^n. \tag{10}$$

This potentially binds only when $s_1 = m$, when the producer is an m person.

The individual rationality constraints for stage 1 meetings are

$$\pi^p((s_1, 0), s') \geq \beta \phi^{(n,0)} v^n \text{ and } \pi^p((s_1, 1), s') \geq \beta(\xi, 1 - \xi)\Phi^n v^n, \tag{11}$$

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[3] We have not shown that a constant allocation is optimal. Nor have we shown that the best constant allocation is locally stable in the sense that an optimum starting from the vicinity of the distributions of the best constant allocation converges to it.

$$\pi^c(s, (s_1', 0)) \geq \beta \phi^{(n,0)} v^n \text{ and } \pi^c(s, (s_1', 1) \geq \beta(\xi, 1 - \xi)\Phi^n v^n, \quad (12)$$

and

$$\pi^0((s_1, 0), s') \geq \beta \phi^{(n,0)} v^n \text{ and } \pi^0((s_1, 1), s') \geq \beta(\xi, 1 - \xi)\Phi^n v^n. \quad (13)$$

We also have a constraint which says that m people prefer the stage-2 transfers intended for them to defecting to n-status just prior to those transfers; namely,

$$\phi^{(m,s_2)} v^m \geq \phi^{(n,s_2)} v^n. \quad (14)$$

There is also a constraint that transfers to n people at stage 2 are nonnegative.

We also allow cooperative defections for people in stage-1 meetings. However, because they turn out not to be relevant for the examples we present below, we describe the implied constraints in the appendix.

Finally, we have the self-selection constraint for the initial choice of m and n status. Let

$$D = (\theta_c^{(m,0)}, \theta_c^{(m,1)}) v^m - (\theta_c^{(n,0)}, \theta_c^{(n,1)}) v^n, \quad (15)$$

where $\theta_c^{(s_1,s_2)} = \theta^{(s_1,s_2)}/(\theta^{(s_1,0)} + \theta^{(s_1,1)})$, the probability of being in state (s_1, s_2) conditional on being in state s_1. For $F = F_\alpha^e$, we require only that $0 \leq D \leq B$, where the first inequality is implied by the individual rationality constraints and the second holds by the choice of the parameter B. If $(\theta^{(m,0)} + \theta^{(m,1)}) \in (0, 1)$ and $F^{-1}(\theta^{(m,0)} + \theta^{(m,1)})$ exists and is continuous at $(\theta^{(m,0)} + \theta^{(m,1)})$, then

$$F^{-1}(\theta^{(m,0)} + \theta^{(m,1)}) = D, \quad (16)$$

the usual cut-off property. If $(\theta^{(m,0)} + \theta^{(m,1)}) \in \{0, 1\}$, then the obvious inequalities must hold.

3.4 The planner's problem

The planner chooses the variables in Table 1 to maximize

$$W = \sum_{s \in S} \theta^s v^s - \int_{x=0}^{x=D} x dF(x) \quad (17)$$

subject to all the relevant constraints. It is well-known and easy to show that

$$\sum_{s \in S} \theta^s v^s = \frac{\sum \sum_{s \in S, s' \in S} \theta^s \theta^{s'} [u(y^{ss'}) - c(y^{ss'})]}{K(1 - \beta)}. \quad (18)$$

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That is, monitoring costs aside, ex ante welfare is proportional to the expected gains from trade in meetings.

There are two versions of this choice problem: one for outside money and one for inside money. Under outside money, no one except the planner issues money. Under the assumption that monies issued by different people are distinguishable, outside money satisfies all the constraints. In general, however, it leads to a worse outcome than inside money because under inside money, m people do not hold money and, therefore, have a defection payoff which is $v^{(n,0)}$. Under outside money, when m people hold money, their defection payoff is $v^{(n,1)}$, which is larger. We include the outside-money version because there is a government monopoly on currency-issue in most economies.[4]

4 The rate of return on money for n people

We measure the return as the average ratio of expected discounted goods consumed by spending money to goods produced to obtain money, all for n people. Our measure coincides with how an outside observer who knows and uses the discount factor β would measure that return.

There are three meetings in which an n person (without money) might produce to acquire money: with an $(n,1)$ consumer, with an $(m,1)$ consumer, and, under inside money, with an $(m,0)$ consumer. Therefore, for $s'' \in \{(n,1),(m,1),(m,0)\}$, let

$$R(s'') = \frac{\lambda_p^{(n,0),s''}(1)(1-\xi)\beta\tilde{v}^{(n,1)}}{y^{(n,0),s''}}, \tag{19}$$

where $\tilde{v}^{(n,1)}$, which will be defined in a moment, is the expected discounted quantity of goods obtained when an n person holds money. (That is, the denominator is the quantity of goods surrendered in an $((n,0),s'')$ meeting and the numerator is the discounted expected acquisition of goods by an n person with money.) Then the average return is

$$R = \frac{\sum_{s''} \theta^{s''} R(s'')}{\sum_{s''} \theta^{s''}}, \tag{20}$$

where the summations are over $s'' \in \{(n,1),(m,1),(m,0)\}$. Finally, $\tilde{v}^{(n,1)}$ is the (unique) solution to

$$\tilde{v}^{(n,1)} = \frac{1}{K}\sum_{s'\in S} \theta^{s'}[(K-1)(1-\xi)\phi^{(n,1)}(1)\beta\tilde{v}^{(n,1)} + \tilde{\pi}^c(s',(n,1))] \tag{21}$$

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[4] See Wallace [21] for an attempt to rationalize outside money by dropping the assumption that inside monies are perfectly distinguishable by issuer.

and

$$\tilde{\pi}^c(s', (n, 1)) = y^{s', (n, 1)} + \lambda_c^{s', (n, 1)}(1)(1 - \xi)\beta\tilde{v}^{(n, 1)}. \tag{22}$$

We use the notation $\tilde{v}^{(n, 1)}$ because $\tilde{v}^{(n, 1)}$ is similar to $v^{(n, 1)}$ (see (5)) except that $u(y)$ is replaced by y and $v^{(n, 0)}$ is replaced by 0.

Because future quantities of goods are discounted, we should think of R as being measured relative to β^{-1}. Thus, for example, if producing y implied consuming y/β with probability one at the next date (an outcome consistent with the Friedman rule), then R would be unity.

The rate of return, R, is affected by three features of an allocation: the distributions (the θ's), the trades in meetings, and the disintegration rate ξ. The distributions matter because they affect the frequency with which a holder of money gets to trade. As regards trades in meetings, those between n and m people are especially pertinent because n people might get more or less goods per unit of money spent with m producers than they produce in meetings with m consumers. Loosely speaking, in a steady state trades between n producers and consumers matter less because the same trades appear in both the numerator and denominator of R. And, of course, ξ matters exactly as would inflation. It makes it less attractive to expend effort to acquire money in a stage-1 meeting.

If individual money holdings were allowed to be larger or if money were divisible, then we would measure the rate of return in the same way and it would be influenced by exactly those features. Therefore, although restricting money holdings to $\{0, 1\}$ exacerbates the dependence of current trades on recent earnings and expenditures for n people, there is no reason to think that it gives misleading results for the return on money.[5]

5   Examples

In order to learn a bit about the properties of optima, we compute optima for some examples. Throughout, we fix u, c, and K as follows: $u(y) = 1 - e^{-10y}$, $c(y) = y$, and $K = 3$. This form for u has $u'(0) = 10$.[6]

We select the discount factor in accord with the comparisons we want to make. In this economy, the best ex ante outcome subject only to physical feasibility is production and consumption at each date in each stage-1 meeting

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[5] The restriction to $\{0, 1\}$ eliminates one possible motivation for inflation and transfers of money. With richer money holdings, inflation and transfers can change the distribution of money holdings in such a way as to facilitate trade (see Levine [12], Green and Zhou [8], and Deviatov [7]).

[6] We chose this functional form for u because the optimization program works more quickly with a finite marginal utility at zero.

equal to $y^* = \arg\max_y[u(c) - c(y)]$, the output that maximizes *surplus* in a meeting. *(*Our specification for u and c implies that $y^* = \ln 10/10 \approx .23$.*)*

If everyone were an m person (if monitoring costs were zero for everyone), then an output level y in every meeting is implementable if it satisfies

$$\frac{u(y)}{c(y)} \geq 1 + K(1 - \beta)/\beta. \tag{23}$$

(This assures that a producer in a meeting weakly prefers producing y when others will do so in future meetings to permanent autarky.) Let β^* denote the β for which (23) holds at equality with $y = y^*$. Because we want to focus on the role of money, a role which arises only in the presence of n people, we impose $\beta \geq \beta^*$. In other words, we assume that only the presence of n people prevents the first-best outcome from being attained in meetings. Below, all expected discounted utilities are reported relative to that implied by $y = y^*$ in every meeting—namely, relative to $[u(y^*) - c(y^*)]/K(1 - \beta)$, which we denote by W^*.

Now suppose that there are only n people—say, because monitoring costs are prohibitively high for everyone. Then, with $\{0, 1\}$ money holdings, trade occurs only in *trade* meetings—meetings in which the producer has no money and the consumer has money. The trade is some amount of production in exchange for a probability of a transfer of money from the consumer to the producer. The optimum with only n people and $\{0, 1\}$ money holdings is easy to describe. There is only one relevant constraint,

$$\frac{u(y)}{c(y)} \geq 1 + \frac{K(1 - \beta)/\beta}{1 - \theta}, \tag{24}$$

where θ is the fraction with a unit of money and y is the amount produced in a trade meeting. (This says that a producer in a meeting weakly prefers producing y and acquiring money with probability 1—given that others will do that in future meetings—to permanent autarky.) Let $\bar{\beta}$ be the value of β for which (24) holds at equality when $y = y^*$ and $\theta = 1/2$. (Obviously, $\bar{\beta} > \beta^*$.) As is well-known, if $\beta \geq \bar{\beta}$, then the optimum is $y = y^*$ and $\theta = 1/2$ with ex nate utility equal to $W^*/4$. If $\beta < \bar{\beta}$, then the optimum has $\theta < 1/2$, $y < y^*$, and (24) at equality. (With only n people, having $\theta < 1/2$ is the only way to loosen constraint (24). It does so by reducing the expected number of periods during which money is held before a trading opportunity occurs and comes at the expense of a reduction in the fraction of meetings that are trade meetings.) It follows that $\beta < \bar{\beta}$ is the condition under which it would be desirable to pay interest on money if it were costless to do so. Because we are interested in that case, we restrict attention to $\beta \leq \bar{\beta}$. In particular, we report results for $\beta \in \{\beta^*, (\beta^* + \bar{\beta})/2, \bar{\beta}\}$, where for the above, u, c, and K,

$$\beta^* = \frac{1}{1 + \frac{(9/\ln 10) - 1}{3}} \approx 0.51 \text{ and } \bar{\beta} = \frac{1}{1 + \frac{(9/\ln 10) - 1}{6}} \approx 0.67. \quad (25)$$

For the version with $F = F_\alpha^e$, for which the fraction of people with zero cost of becoming monitored is α (and for which the rest have a prohibitively high cost B), we compute optima for $\alpha \in \{0, 1/4, 1/2, 3/4\}$. For the version with an endogenous set of m people, we choose F's so as to facilitate a comparison between the implied optima and those for a *comparable* F_α^e, an F that implies that α is the exogenous fraction who are monitored. Let $D(\alpha)$ denote the optimal D (see (15)) implied by F_α^e and the other parameters. Then, for given $\mu \geq 0$, let $F_{(\alpha,\mu)}(x)$ be given by $F_{(\alpha,\mu)}(B) = 1$ and

$$F_{(\alpha,\mu)}(x) = \begin{cases} 1 \text{ if } \alpha + \mu[x - D(\alpha)] > 1 \\ 0 \text{ if } \alpha + \mu[x - D(\alpha)] < 0 \\ \alpha + \mu[x - D(\alpha)] \text{ otherwise} \end{cases} \quad (26)$$

for $x \in [0, B)$ (see figures 1-3 in the Appendix). This piecewise linear specification facilitates a comparsion between it and F_α^e in the following sense. For any μ, the optimal allocation for F_α^e is implementable for $F = F_{(\alpha,\mu)}$. Therefore, we can discuss the optimum for $F_{(\alpha,\mu)}$ in terms of how it deviates from the optimum for F_α^e. In particular, we focus on whether the former has more or fewer m-people than the latter.

6   Results

We start by describing a lower-bound benchmark in which everyone is treated as an n person. This is always a feasible choice for the planner and is the optimum for an economy with prohibitively high costs of becoming monitored, one with $F(x) = 0$ for all $x < B$—an optimum that is the same under inside and outside money. Table 2 describes this benchmark, which, obviously, does not depend on F.[7]

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[7]The optimum problem is solved using the General Algebraic Modeling System (GAMS), which is designed for the solution of large linear, nonlinear, and mixed integer optimization problems. It consists of a language compiler and a large menu of stable integrated high-performance solvers. The solvers are divided into two groups: local solvers (which are fast, but do not guarantee that the global solution is located) and global solvers (which are slow, but are very likely to find the global optimum). The global solver used is a Branch-And-Reduce Optimization Navigator (BARON) solver. BARON uses a deterministic algorithm of the branch-and-bound type, which is guaranteed to find the global optimum under very general conditions. These conditions include bounds on variables and the functions of them that appear in the nonlinear programming problem to be solved.

Table 2. Lower bound: no m people					
β	θ	y/y^*	λ	R_0	W/W^*
β^*	0.38	0.55	1.0	0.18	0.21
$\frac{\beta^*+\bar{\beta}}{2}$	0.45	0.76	1.0	0.21	0.24
$\bar{\beta}$	0.50	1.00	1.0	0.26	0.25

Here, θ is the fraction of agents with a unit of money, y is output in a trade meeting, λ is the probability that the consumer surrenders money, R_0 is the return on money, and W is ex ante welfare. (The optimal disintegration rate is zero.) These results are in accord with our qualitative discussion. In particular, both the fraction with money and output are increasing in β. Notice that the return on money is also increasing in β, which may be special to this example. Below, we report magnitudes for R relative to R_0—in accord with asking whether the presence of m people is used to raise the return on money for n people.

6.1 Inside money

Under inside money, a very general proposition implies that the search for optima can be limited to allocations in which $\theta^{(m,1)} = 0$ (see Wallace [21]). The logic is that money held by m people serves only to raise their defection payoffs.

We begin with the model with $F = F_\alpha^e$, the model with an exogenous fraction who are monitored. The rate of return on money holdings of n people relative to R_0 is given in the following table. Here, the presence of m people is used to raise the return on money for n people relative to that in the benchmark.

Table 3. R/R_0: inside-money, $F = F_\alpha^e$				
$\beta \setminus \alpha$	0	1/4	1/2	3/4
β^*	1	1.085	1.186	1.317
$\frac{\beta^*+\bar{\beta}}{2}$	1	1.155	1.269	1.403
$\bar{\beta}$	1	1.184	1.312	1.453

To see how these higher returns are accomplished, we describe the optimum in detail for $(\alpha, \beta) = (1/4, \frac{\beta^*+\bar{\beta}}{2})$. Table 4 describes aggregate features and Table 5 describes the trades in meetings.

Table 4. Aggregates: inside-money, $(\alpha, \beta) = (\frac{1}{4}, \frac{\beta^*+\bar{\beta}}{2})$							
W/W^*	Ev^m/W^*	$v^{(n0)}/W^*$	$v^{(n1)}/W^*$	$\theta^{(m0)}$	$\theta^{(n0)}$	$\theta^{(n1)}$	ξ
0.43	0.70	0.18	0.66	.25	.506	.244	.08

As we expect, ex ante welfare is higher than in the benchmark. And average welfare is higher for both m people and for n people, although the latter is only

about half the former. No m person holds money and only about one-third of n people hold money. Both the distribution of money and the disintegration rate, ξ, are best discussed together with the trades in meetings in Table 5.

Table 5. Trades: inside-money, $(\alpha, \beta) = (\frac{1}{4}, \frac{\beta^* + \beta}{2})$			
stage-1 meeting (producer)(consumer)	$y^{s,s'}/y^*$	$\lambda_p^{s,s'}(1)$	$\lambda_c^{s,s'}(1)$
$(n0)(n1)^*$	.606	1	0
$(n0)(m0)^*$	.606	1	-
$(m0)(n0)$	.296	-	0
$(m0)(n1)^*$	.717	-	0
$(m0)(m0)^*$	.717	-	-

In the table, the second column reports output relative to the first best; the third reports the end-of-meeting money holding of the producer; and the fourth that of the consumer. Also, a star (*) denotes a binding producer participation (IR) constraint and a dash ($-$) indicates that the variable is not identified. In the case of dashes, the planner has surplus instruments—state transitions in the meeting and transfers at stage 2. In particular, although m people acquire money when they produce for an $n1$ consumer (see the fourth row), they either destroy it or transfer it to the planner at stage 2. (Output is zero whenever the producer is an $n1$ type and in $(n0)(n0)$ meetings.) Only two kinds of constraints play a role in this optimum: producer IR constraints and the steady state condition that requires equality between the inflow into and outflow from money holdings of n people.

There is a binding producer IR constraint on the part of $n0$ producers in the first two rows—namely, $y^{(n0),s'} = \beta(1 - \xi)[v^{n1} - v^{n0}]$—and one for $m0$ producers in the last two rows—namely, $y^{(m0),s'} = \beta[v^{m0} - v^{n0}]$. (It is tempting to interpret all production by m people as taxation because it is supported entirely by the threat of banishment to $n0$ status.) Although no constraint is binding in the third row, a larger output in that meeting would lower v^{m0} and raise v^{n0}, and, therefore, lead to a violation of the IR constraints for all the other meetings. (Although the strict concavity of $u - c$ contributes to making additional output in the third-row meeting more valuable than output in any other meeting, the weight of the third row meeting, $\theta^{m0}\theta^{n0}$, is only $1/8$, while the sum of the weights of the other meetings with trade is approximately $7/16$.)

As regards the steady state condition, an inflow into money holdings by n people arises from spending by m people (see the second row in table 5). That is balanced by two outflows: the trades in the fourth-row meeting and the disintegration rate. Because the distributions are such that the flows from trade

give rise to a net inflow, there is a positive disintegration rate, even though it tightens the *IR* constraint for $n0$ producers.

Finally, we can summarize the roles of the three factors that influence the return on money for n people. Relative to the benchmark, the probability of being able to spend money is higher than in the benchmark because it is spent whenever an m producer is encountered and because fewer n people have money. Also, the trades between m and n people contribute to raising that return: n people produce less when they acquire money from m consumers (see the second row) than they consume when they spend money on m producers (the fourth row). The positive disintegration rate is a partial offset.

Now we turn to the version with a smooth F function. For $(\alpha, \beta) = (\frac{1}{4}, \frac{\beta^* + \bar{\beta}}{2})$, we compute optima for $F = F_{(\alpha,\mu)}(x)$ as described in (26) for $\mu \in \{.2, .4, .6\}$.

	Table 6. Aggregates: inside-money, $\beta = \frac{\beta^*+\bar{\beta}}{2}$, $F = F_{(1/4,\mu)}(x)$							
μ	W/W^*	Ev^m/W^*	Ev^n/W^*	$\theta^{(m0)}$	$\theta^{(n0)}$	$\theta^{(n1)}$	R/R_0	ξ
0	0.428	0.697	0.338	.250	.506	.244	1.155	.082
.2	0.421	0.647	0.334	.252	.512	.236	1.158	.089
.4	0.415	0.726	0.330	.258	.513	.229	1.162	.097
.6	0.409	0.741	0.326	.268	.511	.221	1.167	.105

To facilitate the comparison with the model with an exogenous fraction who are monitored ($\mu = 0$), that result is repeated in the first row. Over this range, the higher is μ, the larger is the fraction who choose to be monitored. That is, the planner is choosing to expend more total one-time costs of becoming monitored in order to have more monitored people. In other respects, the results are similar to the version with $\mu = 0$.

6.2 Outside money

We begin again with the model with $F = F_\alpha^e$, the model with an exogenous fraction who are monitored. The rate of return on money holdings of n people relative to R_0 is given in the following table.

Table 7. R/R_0: outside-money, $F = F_\alpha^e$				
$\beta \setminus \alpha$	0	1/4	1/2	3/4
β^*	1	0.84	0.81	undefined
$\frac{\beta^*+\bar{\beta}}{2}$	1	0.91	0.88	undefined
$\bar{\beta}$	1	0.95	0.95	1.04

Here "undefined" means that money is not traded, an outcome we describe in more detail below. In only one of the cells is the presence of m people used to raise the return on money holding of n people.

We first describe in detail the optimum for $(\alpha, \beta) = (1/4, \frac{\beta^* + \bar{\beta}}{2})$. Table 8 describes aggregate features and Table 9 describes the trades in meetings.

Table 8. Aggregates: outside-money, $(\alpha, \beta) = (\frac{1}{4}, \frac{\beta^* + \bar{\beta}}{2})$							
W/W^*	Ev^m/W^*	$v^{(n0)}/W^*$	$v^{(n1)}/W^*$	$\theta^{(m1)}$	$\theta^{(n0)}$	$\theta^{(n1)}$	ξ
0.34	0.77	0.09	0.57	.25	.57	.18	.16

Ex ante welfare is higher than in the benchmark (but lower, as it must be, than in the inside-money version). And here, those who turn out to be n people do worse than in the benchmark in which everyone is treated as an n person. Here, all the m people start with money—presumably, so that they can can spend when they meet $n0$ producers. (This is accomplished through transfers at stage 2 to those m people who spend money at stage 1 or who lose money through the 16% disintegration rate.) And relative to inside money, even fewer n people hold money.

Table 9. Trades: outside-money, $(\alpha, \beta) = (\frac{1}{4}, \frac{\beta^* + \bar{\beta}}{2})$			
stage-1 meeting	$y^{(s,s')}/y^*$	$\lambda_p^{(s,s')}(1)$	$\lambda_c^{(s,s')}(1)$
$(n0)(n1)^*$	0.573	1	0
$(n0)(m1)^*$	0.573	1	-
$(m1)(n0)$	0.113	-	0
$(m1)(n1)^*$	0.381	-	0
$(m1)(m1)^*$	0.381	-	-

Again, only two kinds of constraints play a role in this optimum: producer IR constraints and the steady-state condition that requires equality between the inflow into and outflow from money holdings of n people. There is a binding producer IR constraint on the part of $n0$ producers in the first two rows and one for $m1$ producers in the last two rows—namely, $y^{m1,s'} = \beta[v^{m1} - v^{n1}]$. Again, it is tempting to interpret all production by m people as taxation because it is supported entirely by the threat of banishment, but now to $n1$ status. That accounts for the lower output by m producers in the last two rows. As regards the steady-state condition, an inflow into money holdings by n people arises from spending by m people (see the second row). That is balanced by two outflows: the trades in the fourth row and the disintegration rate.

The rate of return on money for n people is lower than in the benchmark. Relative to the benchmark and similar to what we saw for inside money, the

distribution of trading opportunities changes in a way that raises that return. But now, there are two offsets. The trades between m and n people contribute to lowering that return: n people produce substantially more when they acquire money from m consumers (see the second row) than they consume when they spend money on m producers (the fourth row). And the positive disintegration rate contributes to lowering that return.

The results in Table 7 when the return on money is not defined are easy to describe. In those optima, none of the m people holds money and all the n people hold money. As a consequence, there is production in only two kinds of meetings: the $(m0)(m0)$ meeting, in which there is a binding producer IR constraint, and the $(m0)(n1)$ meeting. Money is not traded and $y^* > y^{(m0)(m0)} > y^{(m0)(n1)} > 0$. In this allocation, n people never produce. Their holding of money plays the role of a label and m people who defect do not get that label. An m person defects to being an $n0$ person—the value of which is permanent autarky, zero. In that respect, the allocation resembles optima under inside money.

Despite never being traded, money is important. If there is no money, then any production by m people for n people raises the defection payoff for m people, and, therefore, reduces production in the $(m0)(m0)$ meeting. A similar logic explains why this allocation cannot be improved by having a small fraction of n people with no money. That would allow for production by those people when they meet n people with money, which would tend to raise welfare. However, there would be an offset if $m0$ producers do not produce for $n0$ people. The allocation is telling us that the gain does not offset the loss—in part, because the frequency of $(n0)(n1)$ meetings is less than one-third the frequency of $(m0)(n0)$ meetings. And, of course, if the $m0$ producers were to produce for the $n0$ people, then that would make v^{n0} positive and increase the defection payoff for $m0$ people.

Now we turn to the version with a smooth F function. For $(\alpha, \beta) = (\frac{1}{4}, \frac{\beta^* + \bar{\beta}}{2})$, we again compute optima for $F = F_{(\alpha,\mu)}(x)$ as described in (26) for $\mu \in \{.2, .4, .6\}$.

Table 10. Aggregates: outside-money, $\beta = \frac{\beta^* + \bar{\beta}}{2}$, $F = F_{(1/4,\mu)}(x)$							
μ	W/W^*	Ev^m/W^*	Ev^n/W^*	$\theta^{(m1)}$	$\theta^{(n0)}$	$\theta^{(n1)}$	R/R_0	ξ
0	0.34	0.77	0.20	.250	.574	.176	0.909	.159
.2	0.32	0.76	0.20	.249	.574	.178	0.909	.156
.4	0.31	0.75	0.21	.244	.575	.181	0.911	.151
.6	0.29	0.73	0.21	.235	.579	.186	0.915	.143

Here, all the m people have money. Over this range, the higher is μ, the smaller is the fraction who choose to be monitored. That is, the planner is choosing to conserve on total one-time costs of becoming monitored at the expense of having fewer m people. In comparison with the inside-money version, having additional m people is less beneficial—perhaps, because their defection payoffs are higher.

7   Concluding remarks

Our analysis is intended to be illustrative. After all, we study essentially one numerical example. And the model is very special in several respects—people are either perfectly monitored or anonymous, and money holdings are in $\{0, 1\}$. Despite that, we think that the results deserve to be taken seriously.

Although we studied only one $u - c$ function and one magnitude for K, their role is mainly to determine the first-best level of output and the range for the discount factor—which is chosen so that the first best would be attained if there were no costs of becoming monitored and so that paying interest on cash would be desirable if no one were monitored. If that range for the discount factor is maintained, then the general thrust of the results should not be sensitive to different $u - c$ functions and different magnitudes for K. The extreme monitoring assumptions—people are either perfectly monitored or anonymous—play an important role. As discussed in Wallace [21], they account for why the only asset is a currency-like asset. Finally, the restriction to $\{0, 1\}$ money holdings serves only one role; it limits the number of unknowns. Because there are few unknowns, we can describe the results easily, and, thereby, learn a bit about how the constraints interact.

We have chosen to summarize the results according to whether the presence of monitored people, people who in a sense can be taxed, is used to raise the return on cash. Our examples show that even if the underground economy is benign and is given its proportional weight in the objective function and even if there is no public-good spending to be financed, the optimum does not display even qualitative uniformity. In the outside-money version, in most examples feasible taxation on the above-ground economy is not used to raise the return on currency. It doesn't because those in the above-ground have the option to defect (to the underground economy) and because, despite the absence of public good spending, there are rival uses for such taxation—namely, production by monitored people for nonmonitored people without money.

8   Appendix

8.1   Pairwise defection

We start with a definition of the pairwise core for meetings between n people who might trade.

Definition. For $(s, s') = ((n, 0), (n, 1))$, we say that (π^p, π^c) is in the pairwise core if it solves the problem,

$$\max_{y^{s,s'}, \lambda_p^{s,s'}, \lambda_c^{s,s'}} \gamma \pi^p(s, s') + (1 - \gamma)\pi^c(s, s') \tag{27}$$

subject to (11) and (12) for some $\gamma \in [0, 1]$.

The following lemma fully characterizes this pairwise core.

Lemma 1 *Let $a = \beta \phi^{(n,0)} v^n$ (the lowest possible payoff for the producer in the above problem) and let b be the solution for $\pi^p((n, 0), (n, 1))$ to the above problem for $\gamma = 1$ (the highest possible payoff to the producer in the above problem). Let $\psi : [a, b] \to \mathbb{R}$ be defined by*

$$\psi(x) \equiv \begin{cases} \varsigma - x & \text{if } x \in [a, \ \rho) \\ u[c(\beta(\xi, 1 - \xi)\Phi^n v^n - x)] + a & \text{if } x \in [\rho, \ b] \end{cases} , \tag{28}$$

where

$$\rho = \beta(\xi, 1 - \xi)\Phi^n v^n - c(y^*), \tag{29}$$

and

$$\varsigma = u(y^*) - c(y^*) + \beta[\phi^{(n,0)} + (\xi, 1 - \xi)\Phi^n]v^n. \tag{30}$$

(Notice that ρ is π^p if the producer acquires money with probability 1 and produces the surplus maximizing output, while ς is the sum of π^p and π^c if that output is produced.) For $(s, s') = ((n, 0), (n, 1))$, (π^p, π^c) is in the pairwise core iff

$$(\pi^p, \pi^c) \in \{(x, \psi(x)) : x \in [a, b]\}. \tag{31}$$

Proof. First we note that the preservation-of-money constraints for the $((n, 0), (n, 1))$ meeting imply that $\lambda_p^{s,s'} = (1 - \lambda, \lambda)$ and $\lambda_c^{s,s'} = (\lambda, 1 - \lambda)$ for some $\lambda \in [0, 1]$, where λ is the probability that the producer acquires money. Also, because the objective in the definition (see (27)) is concave in y, output, and

(linear in) λ and the constraint set is convex, the following first-order conditions are necessary and sufficient for that problem:

$$-c'(y)\left(\gamma + \sigma^s\right) + \left(1 - \gamma + \sigma^{s'}\right)u'(y)\left\{\begin{array}{l} = 0 \text{ if } y > 0 \\ \leq 0 \text{ if } y = 0 \end{array}\right. , \qquad (32)$$

$$[(\gamma + \sigma^s) - \left(1 - \gamma + \sigma^{s'}\right)]\beta(\varphi^{(n,1)} - \phi^{(n,0)})v^{(n,\cdot)}\left\{\begin{array}{l} \geq 0 \text{ if } \lambda = 1 \\ = 0 \text{ if } 0 < \lambda < 1 \\ \leq 0 \text{ if } \lambda = 0 \end{array}\right. , \qquad (33)$$

where $\varphi^{(n,1)} = (\xi, 1 - \xi)\Phi^n$ and where $\sigma^s \geq 0$ is the Lagrange multiplier associated with (11) and $\sigma^{s'} \geq 0$ is that associated with (12). Notice that if the second of these holds at equality, then $y = y^*$. And if $\lambda = 1$ and the second holds with strict inequality, then $y < y^*$. We assume that $(\varphi^{(n,1)} - \phi^{(n,0)})v^n > 0$ (valued money) because otherwise only $y = 0$ satisfies (11). And valued money and $u'(0) > c'(0)$ imply that any solution to the pairwise core problem satisfies $y > 0$ and $\lambda > 0$. We provide a proof for the case, $a < \rho < b$, which says that $(y, \lambda) = (y^*, 1)$ is interior with respect to constraints (11) and (12). Cases in which $\rho < a$ or $\rho > b$ are similar.

Necessity. There are three cases: $\gamma \in \left[0, \frac{1}{2}\right)$, $\gamma = \frac{1}{2}$, and $\gamma \in \left(\frac{1}{2}, 1\right]$.

If $\gamma \in \left[0, \frac{1}{2}\right)$, then (33) implies that (11) binds. Therefore, $\pi^p = a$. Moreover, $a < \rho$ implies $y = y^*$ and $\lambda \in (0, 1)$. The former implies $\pi^c = \varsigma - a = \psi(a)$, where, as noted above, ς is the sum of the payoffs implied by $y = y^*$.

If $\gamma = \frac{1}{2}$ and (11) is slack, then condition (32) implies that $y = y^*$. This yields,

$$\pi^p = -c(y^*) + \beta(\varphi^{(n,1)} - \phi^{(n,0)})v^{(n,\cdot)}\lambda + \beta\phi^{(n,0)}v^n \qquad (34)$$

and

$$\pi^c = u(y^*) - \beta(\varphi^{(n,1)} - \phi^{(n,0)})v^n\lambda + \beta\varphi^{(n,1)}v^n = \varsigma - \pi^p = \psi(\pi^p).$$

If (11) is not slack, then we have $\pi^p = a$ as in the first case.

If $\gamma \in \left(\frac{1}{2}, 1\right]$, then the condition (33) and $\rho < b$ imply that $\lambda = 1$. Therefore,

$$\pi^p = -c(y) + \beta\varphi^{(n,1)}v^n \qquad (35)$$

and

$$\pi^c = u(y) + \beta\phi^{(n,0)}v^n = u[c(\beta\varphi^{(n,1)}v^n - \pi^p)] + a = \psi(\pi^p),$$

where the second equality comes from substituting for y from (35).

Sufficiency. The proof proceeds by construction. In particular, for each $x \in [a, b]$, we show that the unique (y, λ) that supports $(x, \psi(x))$ and a proposed $(\sigma^s, \sigma^{s'}, \gamma)$ satisfy (32) and (33), which, as noted above, are sufficient to solve the problem that defines the pairwise core. We deal with two cases: $x \in [a, \rho]$, $x \in (\rho, b]$.

If $x \in [a, \rho]$, then, $y = y^*$ and

$$\lambda = \frac{x + c(y^*) - \beta \phi^{(n,0)} v^n}{\beta \left(\varphi^{(n,1)} - \phi^{(n,0)} \right) v^n} \in (0, 1].$$

uniquely supports $(x, \psi(x))$. We propose $(\sigma^s, \sigma^{s'}, \gamma) = (0, 0, 1/2)$ for all such x. Then, (32) and (33) hold (at equality).

If $x \in (\rho, b]$, then $\lambda = 1$ and

$$y = c(\beta \varphi^{(n,1)} v^n - x) \in (0, y^*)$$

uniquely supports $(x, \psi(x))$. We propose $\sigma^s = \sigma^{s'} = 0$ and γ such that (32) holds. Notice that this $\gamma \in (1/2, 1)$. Then (32) holds at equality by the choice of γ and (33) holds (at strict inequality). $\square$

Because defection by an m person converts the person to an n person, the payoffs in a stage-1 meeting in which one person is an m person or both are m people must satisfy

$$\pi^c(s, s') \geq \psi(\pi^p(s, s')). \tag{36}$$
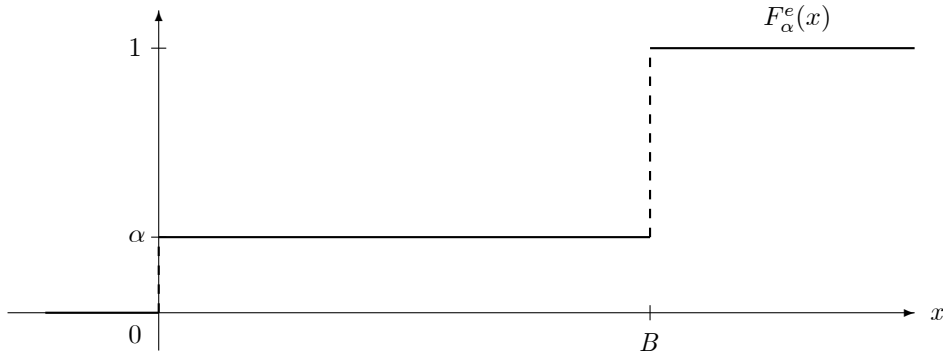
21

8.2 Figures

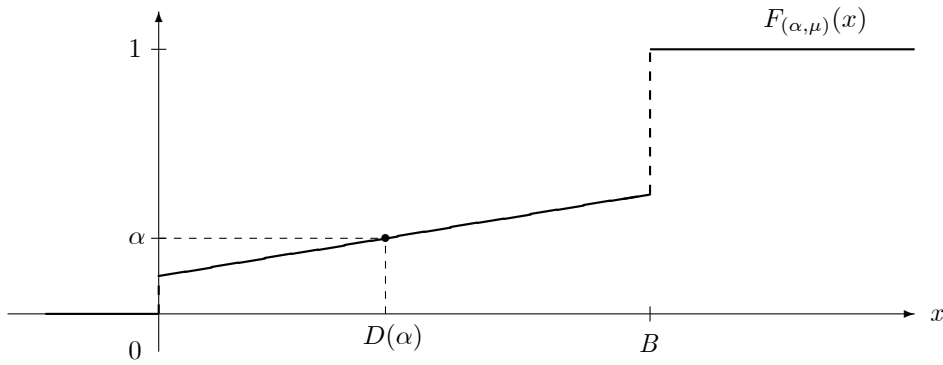Figure 1: Monitoring cost distribution: $F_\alpha^e(x)$.

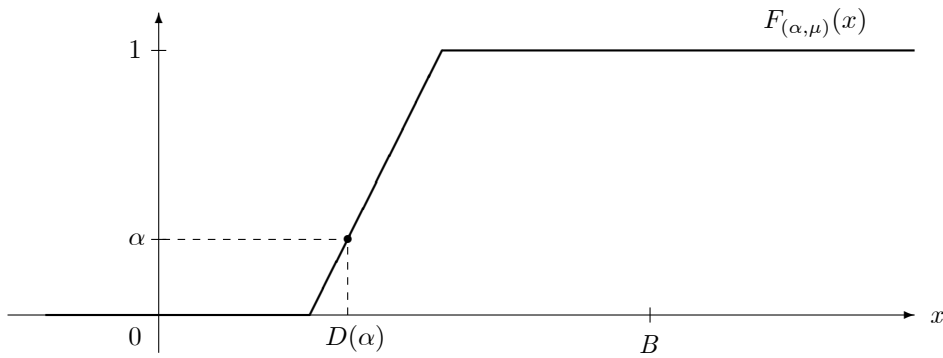Figure 2: Monitoring cost distribution: $F_{(\alpha,\mu)}(x)$ for small μ.

Figure 3: Monitoring cost distribution: $F_{(\alpha,\mu)}(x)$ for large μ.

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