Labour Market Dynamics with Sequential Auctions

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Preliminary and Incomplete

Abstract

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1 Introduction

2 The Basic Matching Model

We consider a standard Mortensen-Pissarides (MP) search-matching framework as in Shimer (2005), Mortensen and Nagypál (2007), Hagedorn and Manovskii (2008), Hall and Milgrom (2008) or Pissarides (forth.). Time is discrete and indexed by $t \in \mathbb{N}$. The global state of the economy is described by some ergodic Markov chain with transition probability matrix $\mathbf{\Pi} = (\ _{ij})$. All the jobs in the economy have the same productivity value $y_t \in \{y_1, ..., y_N\}$, with $y_i < y_{i+1}$ (with a slight abuse of notation, y_t denotes the stochastic process and y_i a realization).

2.1 Search-matching

Let u_t denote the number of unemployed workers at the end of period $t - 1$. At the beginning of period t, a new productivity state i is realized, a fraction $s(1 - u_t)$ of current matches is destroyed, and vacancies are endogenously created or destroyed. Let v_t be the new number of vacancies. A number $\ (u_t + \ (1 - s)(1 - u_t), v_t)$ of employer/employee meetings is then realized, where $\ \geq 0$ is the search efficiency of employees relative to unemployed. We define market tightness as the ratio of vacancies to total search intensity:

$$\ _t = \frac{v_t}{u_t + \ (1 - s)(1 - u_t)}. \tag{1}$$

The matching function is increasing, strictly concave and linearly homogeneous. The job finding rate of unemployed workers is $f(\ _t) \equiv \ (1, \ _t)$, with $f(0) = 0$, and the job offer arrival rate to employees is $\ f(\ _t)$.

2.2 Equilibrium

We postulate Postel-Vinay and Robin's (2002) sequential auctions as wage setting mechanism. In the basic version that we consider, employers have full monopsony power unless a credible separation threat is presented (as in Diamond's (1971) seminal equilibrium search model). Unemployed workers are thus offered their reservation wage, and they take it. Rent sharing accrues later via on-the-job search. As all matches produce the same output, on-the-job search and Bertrand competition eventually transfer all the match rent from the employer to the employee (unless exogenous match destruction happens before). Workers are indifferent between competing employers and we assume for simplicity that the tie is broken in favour of the poaching employer with probability [illegible].

For unemployed workers, the value of employment is only marginally better than the value of unemployment. It follows that the unemployment value only depends on the productivity index i and is otherwise stationary:

$${}_i = z_i + \frac{1}{1+r} \sum_j \pi_{ij} \quad {}_j$$

where z_i is the opportunity cost of employment in macroeconomic state i, or, in obvious matrix notations:

$$\mathbf{U} = \left[\mathbf{I} - \frac{1}{1+r} \mathbf{\Pi} \right]^{-1} \mathbf{z}. \tag{2}$$

Define the match surplus value S_t as the discounted sum of all future match output flows plus what the worker and the firm separately get after a separation, minus the value of unemployment and minus the value of a vacancy (which is equal to zero if free entry). The expected surplus flow as long as the match continues is $y_t - z_t$. On-the-job search and poaching results in the worker receiving the whole surplus whether she stays with her current employer or accepts the other job.

Match surplus S_t only depends on the current productivity value, say y_i. Write S_i,

$i = 1, ..., N$, for the surplus if the productivity state is i. It satisfies the Bellman equation:

$$S_i = y_i - z_i + \frac{1-s}{1+r} \sum_j \pi_{ij} S_j$$

that is solved as:

$$\mathbf{S} = \left[\mathbf{I} - \frac{1-s}{1+r} \mathbf{\Pi} \right]^{-1} (\mathbf{y} - \mathbf{z}). \tag{3}$$

At the beginning of period t, would-be employers pay a fee c to participate in the lottery that generates a contact with a worker with probability $q(\theta_t) = f(\theta_t)/\theta_t$. A proportion $\frac{u_t}{u_t+\kappa(1-s)(1-u_t)}$ of contacts is with currently unemployed workers. They accept the job and the firm gets the whole surplus. A proportion $\frac{\kappa(1-s)(1-u_t)}{u_t+\kappa(1-s)(1-u_t)}$ of contacts is with employed workers who accept the job with probability . However, Bertrand competition gives all the rent to the employee. Hence, if free entry drives the value of a vacancy to zero, for a given value u_t of the unemployment rate, market tightness θ_t satisfies the equation:

$$c = q(\theta_t) \frac{u_t}{u_t + \quad (1-s)(1-u_t)} S_t \Leftrightarrow cv_t = f(\theta_t) u_t S_t. \tag{4}$$

where u_t follows the motion process:

$$u_{t+1} - u_t = s(1-u_t) - f(\theta_t) u_t. \tag{5}$$

The equilibrium is completely described by the fully recursive set of equations (3), (4) and (5) with unemployment u_t and productivity y_{t+1} as state variables. At the beginning of period t, a new productivity index is observed, say i. Equation (3) determines $S_t = S_i$, equation (4) determines θ_t given u_t and $S_t = S_i$, and equation (5) then determines u_{t+1} given u_t and θ_t. The equilibrium path is unique if $q(\theta_t)$ is one-to-one.

Steady-state tightness and unemployment. In a steady-state equilibrium where the economy remains in state i forever, (u_i, v_i, θ_i) are such that

$$
\begin{aligned}
\theta_i &= \frac{v_i}{u_i + \ (1-s)(1-u_i)}, \\
f(\theta_i)u_i &= s(1-u_i), \\
q(\theta_i) &= \frac{c}{S_i}\left(1 + \ (1-s)\frac{1-u_i}{u_i}\right).
\end{aligned}
$$

Let $f_i = \frac{s(1-u_i)}{u_i} = f(\theta_i) \in \mathbb{R}$. Then, f_i solves

$$
f\left(\frac{S_i}{c}\frac{f_i}{1+\frac{\kappa(1-s)}{s}f_i}\right) = f_i.
$$

The function $x \mapsto \frac{x}{1+\frac{\kappa(1-s)}{s}x}$ being increasing and concave, like f, it is easy to see that there is a unique solution to this equation if $\frac{S_i}{c}f'\left(\frac{S_i}{c}\right) = f\left(\frac{S_i}{c}\right) \quad 1$ and no solution otherwise.

2.2.1 Estimation/Calibration

Matching function. We follow standard practice and specify the job finding rate function as:

$$
f(\theta) = \ \theta^{\eta}.
$$

From the JOLTS data (Job Openings and Labor Turnover Survey) available from the BLS website, let V denote total job openings in the total nonfarm sector (seasonally adjusted), H the number of hires, Q the number of quits and L involuntary separations (layoffs and discharges). Since March 10, 2009, new series have been released which are consistent with total employment from the CES (Current Employment Statistics). Let E denote the CES total employment series. Lastly, let denote the number of unemployed (from CPS, Current Population Survey).

First, we note that voluntary quits and hires are procyclical whereas layoffs are countercyclical (see Figure 1, panel (a)). We therefore think of quits as revealing on-the-job search and take them out of total hires to calculate the job finding rate of unemployed workers. We thus write:

$$
\begin{aligned}
\theta &= \frac{V}{\quad + \quad E}, \\
\frac{H-Q}{} &= f(\theta), \\
\frac{Q}{E} &= \delta \quad f(\theta), \\
\Delta E &= E - E_{-1} = (H-Q) - L,
\end{aligned}
$$

where   is the relative search intensity of employees with respect to unemployed and δ is the fraction of employees' contacts with an alternative employers which result in a quit.

Notice, however, that although colinear to a large extent, the quit rate $\frac{Q}{E}$ seems to increases slightly in proportion to the job reaccession rate $\frac{H-Q}{U}$ (see Figure 1, panel (b)). Using the results in Jolivet, Postel-Vinay, Robin (2006) who estimate a wage posting/equilibrium search model on PSID data, we estimate the proportion of employees' contacts with alternative employers resulting in actual mobility to 53%. We thus set $\delta = 0.5$. Then, we estimate   as the mean of $k \equiv \frac{Q}{E}\frac{U}{H-Q}$, i.e. $= 0.14$. We finally estimate   and   by regressing $\log \frac{H-Q}{U}$ on $\log \theta$. We estimate $= \exp(0.277) = 1.32$ and $= 0.78$. Figure 2 shows how the model fits the data.[1]

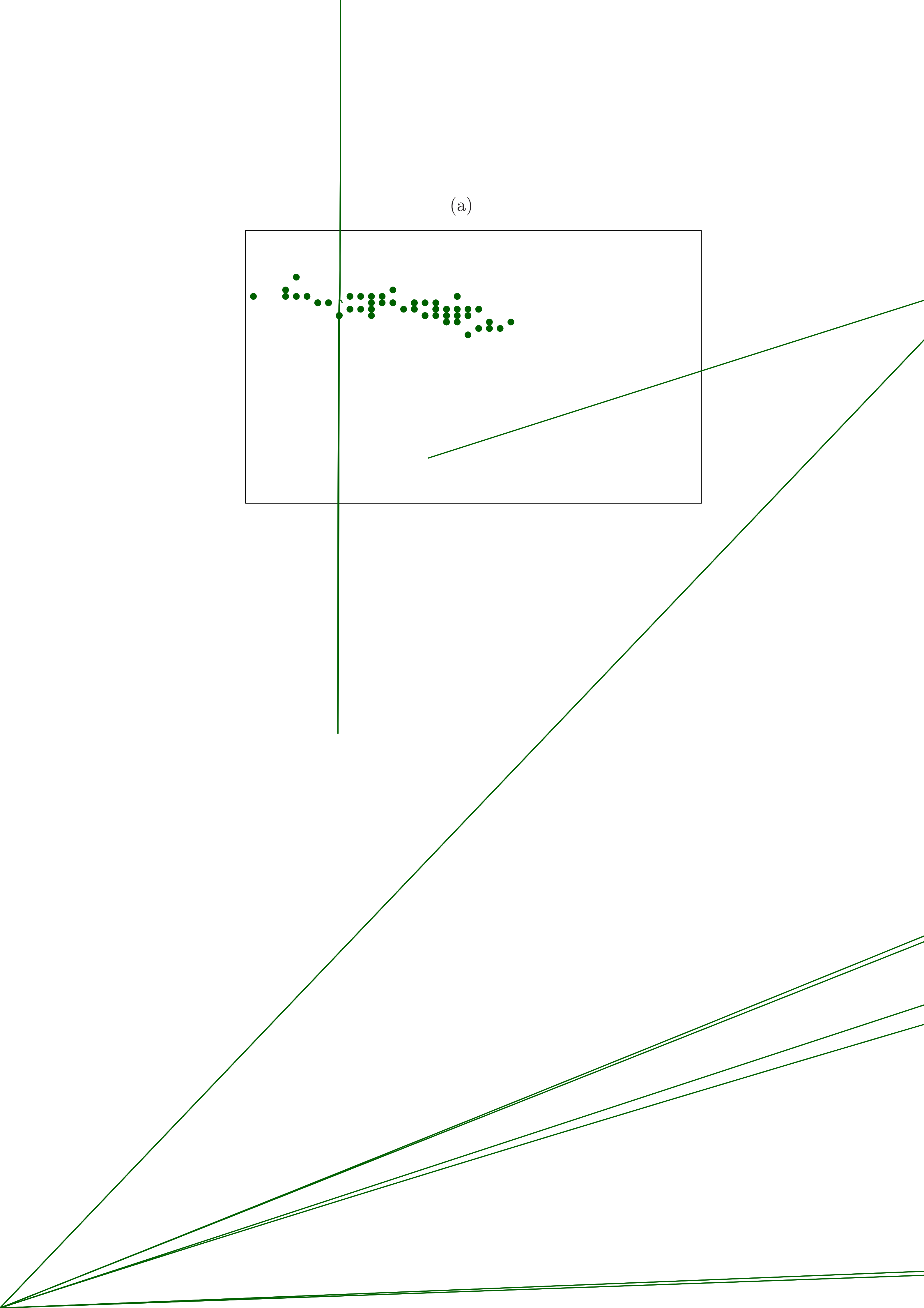
We shall also set the layoff rate s to the mean value of L/E, i.e. $s = 1.5\%$, for the period.

Productivity. We then turn to the estimation of the productivity process. The source of identification of S_t is the free entry condition (4) that we rewrite as:

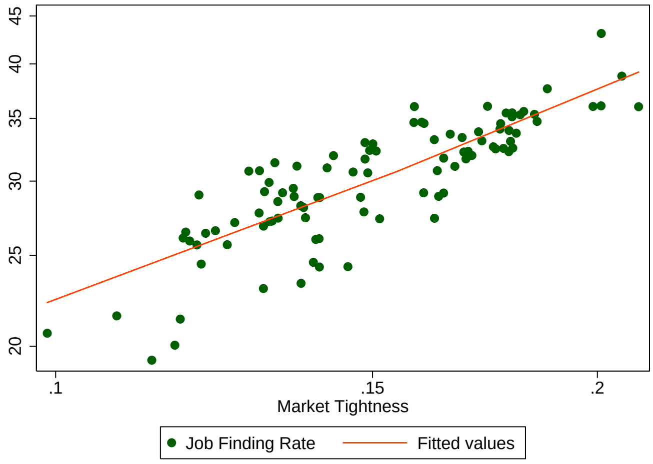
$$\frac{v_t}{u_t f(\theta_t)} = \frac{S_t}{c},$$

[1]Defining market tightness as V/U, we find $\phi = \exp(-0.863) = 0.422$ and $\eta = 0.503$.

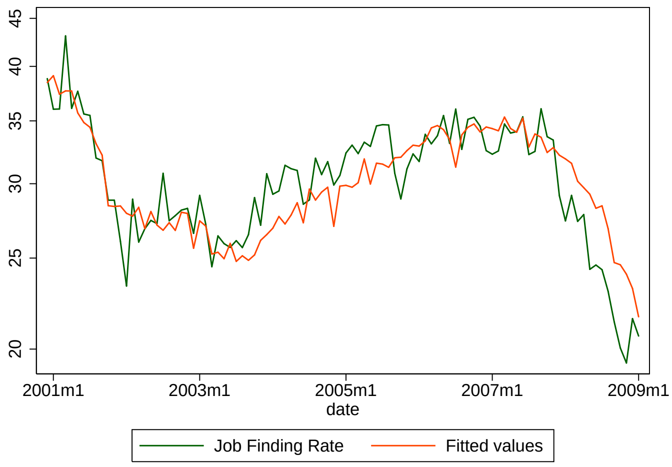
(a)



(a) Matching Function



(b) Exit Rate from Unemployment



(c) Quit Rate

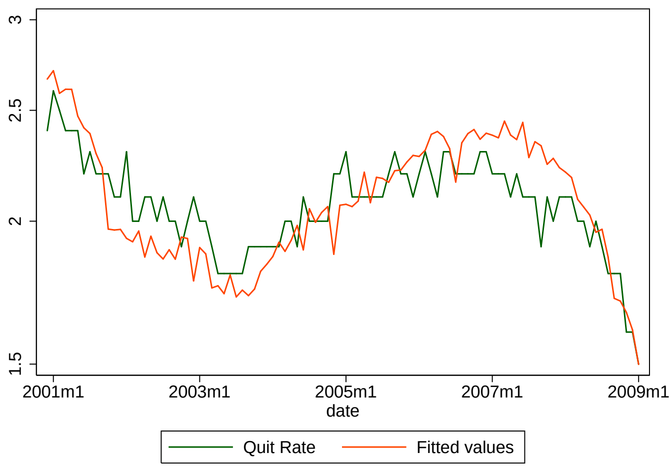


Figure 2: Fit of the Matching Function

where $\frac{v_t}{u_t f(\theta_t)}$ is easily calculated from the JOLTS data as $\frac{V}{H-Q}$(see figure 3, panel (a); the dotted line is a Markov chain approximation that we shall later use). Hence we know the series of surplus values up to a scale transformation $1/c$.

Equation (3) establishes a correspondence between the support of S_t and the support of $(y-z)_t$. We shall assume that matrix $\left[\mathbf{I} - \frac{1-s}{1+r}\mathbf{\Pi}\right]^{-1}$ monotonically transforms$\{y_i - z_i\}$ into $\{S_i\}$. It follows that processes $\frac{v_t}{u_t f(\theta_t)}$, S_t and $(y-z)_t$ have exactly the same ranks in their respective marginal distributions. Figure 3, panel (b) displays the scatterplot and the marginal histograms of $\left(\frac{S_{t-1}}{c}, \frac{S_t}{c}\right)$ and panel (c) shows the marginal and joint distributions of the ranks of $\frac{S_{t-1}}{c}$ and $\frac{S_t}{c}$.

The distribution of the cdfs of two variables X and Y is called a copula. We use a t-copula with parameters ρ and . Fitting this parametric copula using maximum likelihood, we obtain the following estimates: $\rho = 0.90$ and $= 10.65$.

Next we discretize the marginal distribution of $\frac{S_t}{c}$ using the midpoints of $N = 50$ equally spaced bins exactly covering the set of observed values of $\frac{S_t}{c}$. The respective probability p_i of each bin is computed using a smooth kernel estimator of the cdf. Let F be such a kernel cdf estimator. Let $[a_i, b_i]$ denote the limits of the ith bin with $\frac{S_i}{c} = \frac{a_i+b_i}{2}$. Then $p_i = F(b_i) - F(a_i)$. We also calculate a discrete approximation of the transition probability matrix across productivity states as

$$\pi_{ij} = \frac{C(F(b_i), F(b_j)) - C(F(a_i), F(b_j)) - C(F(b_i), F(a_j)) + C(F(a_i, F(a_j))}{p_i}$$

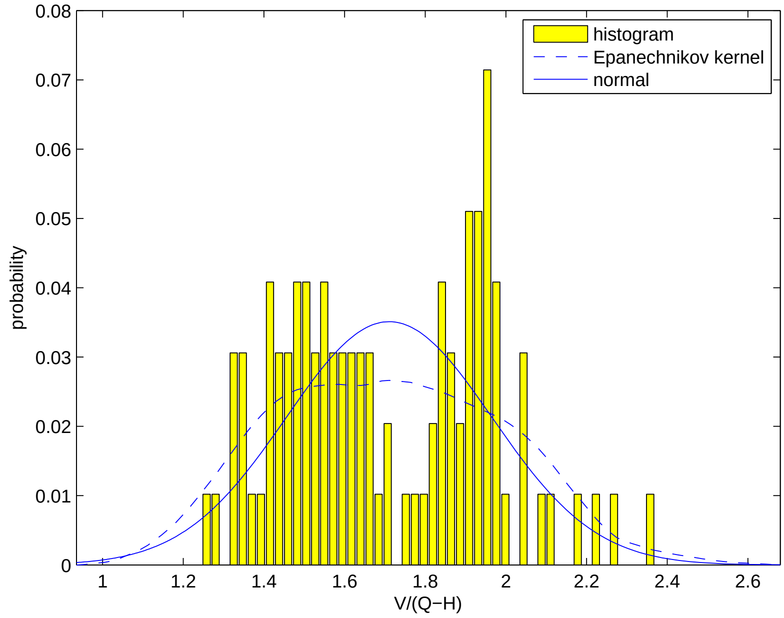
Finally, we estimate $\frac{\mathbf{y}-\mathbf{z}}{c} = \left(\frac{y_i - z_i}{c}\right)$ as

$$\frac{y_i - z_i}{c} = \frac{S_i}{c} - \frac{1-s}{1+r}\sum_j \pi_{ij}\frac{S_j}{c}. \tag{6}$$

Figure 4, panel (a) shows the support $\{S_i/c\}$ used for surplus states together with the associated marginal probabilities p_i in the form of a histogram. The normal density plot shows how different this distribution is from a normal density. Panel (b) shows

the discrete approximation of the marginal density of surplus values that we use in the estimation. Figure 5 displays the estimated surplus flow values $\frac{y_i - z_i}{c}$ as a function of $\frac{S_i}{c}$. It is approximately linear almost everywhere on the support

(a) Marginal Distribution of Vacancy-Hires Ratio



(b) Surplus States and Marginal Probabilities

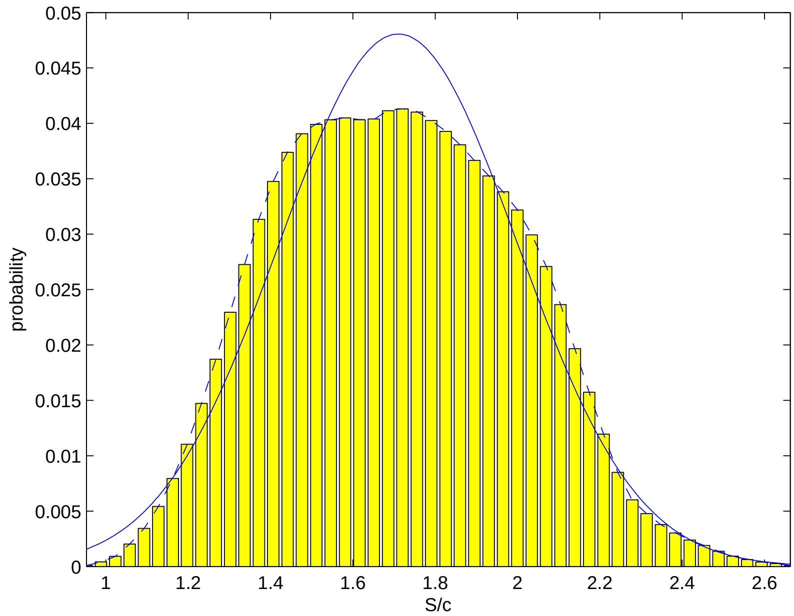
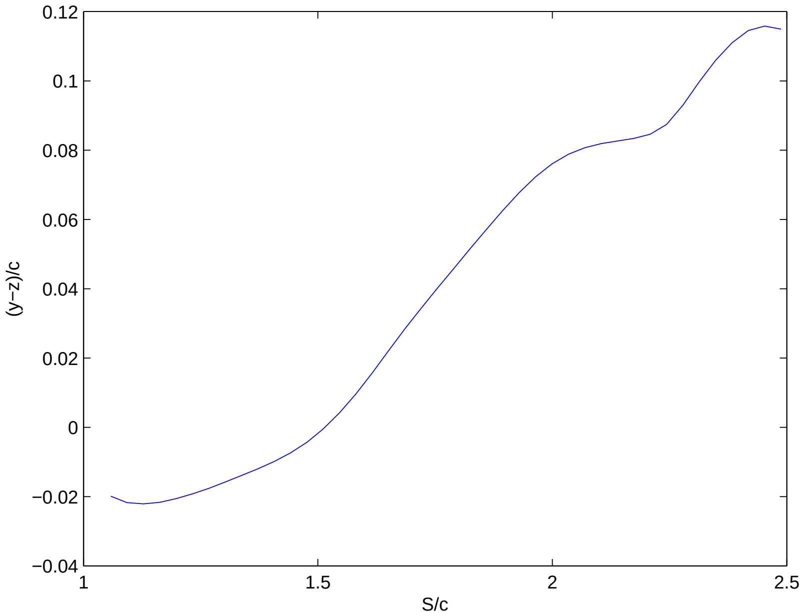


Figure 4: Marginal Distribution of Vacancy/Hires Ratio and State-Space Discretization



(Only states with a marginal probability greater than 0.1% are displayed)

Figure 5: Estimated Surplus Flows $y_i - z_i$ from Present Values S_i

Assume that $z_t = z_0$ is fixed. Then, we find that $z_0 = 1.313$ generates an elasticity of unemployment exit rate to productivity (a regression coefficient) of 7.56, which is the number found by Shimer. Increasing z_0 increases the elasticity by reducing the surplus flow as a share of z_0. Hence, we obtain the right elasticity with a surplus flow that lies indeed between -0.9% and 6.9% of the opportunity cost of working z_0. We also obtain an elasticity of unemployment to productivity of -4.35, which is also close to the -3.88 computable from Shimer (2005, Table 1).

Note that the dependence of unemployment to productivity seems to have changed over time. Using Hodrick-Prescott filtered series (with a smoothing parameter of 10^5 as in Shimer) of log unemployment rates and log labor productivity (lagged one quarter), we find an elasticity of -6.18 (std: 0.57) for the period 1947q1-1986q4 for an Rsquare of 43% and an elasticity of $+1.93$ (std: 1.39) for an Rsquare of 2% for the remaining period 1987q1-2008q4. Figure 8 shows how the fit has worsened after 1987.

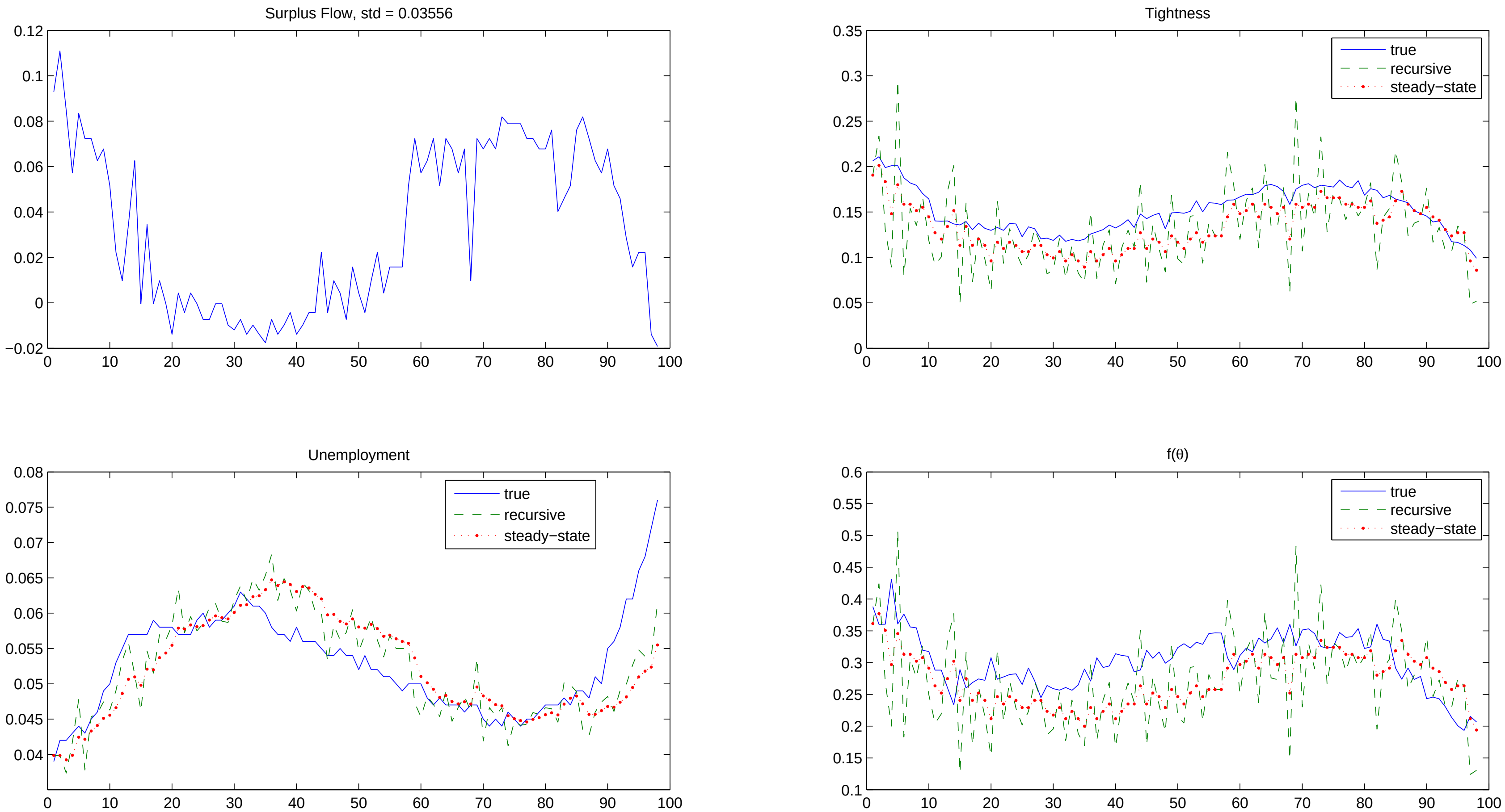


Figure 6: Model fit – Filtering Productivity Shocks

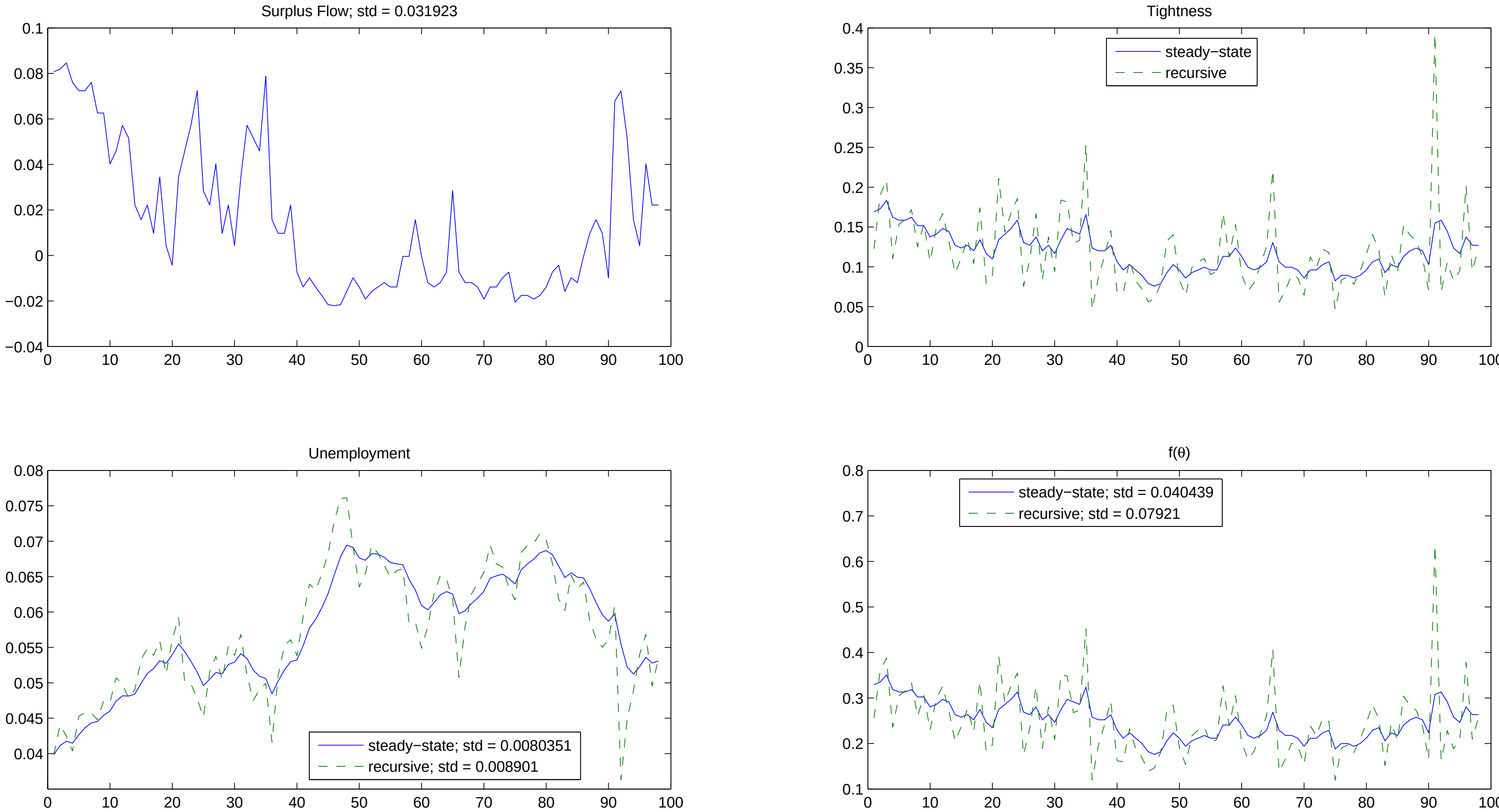
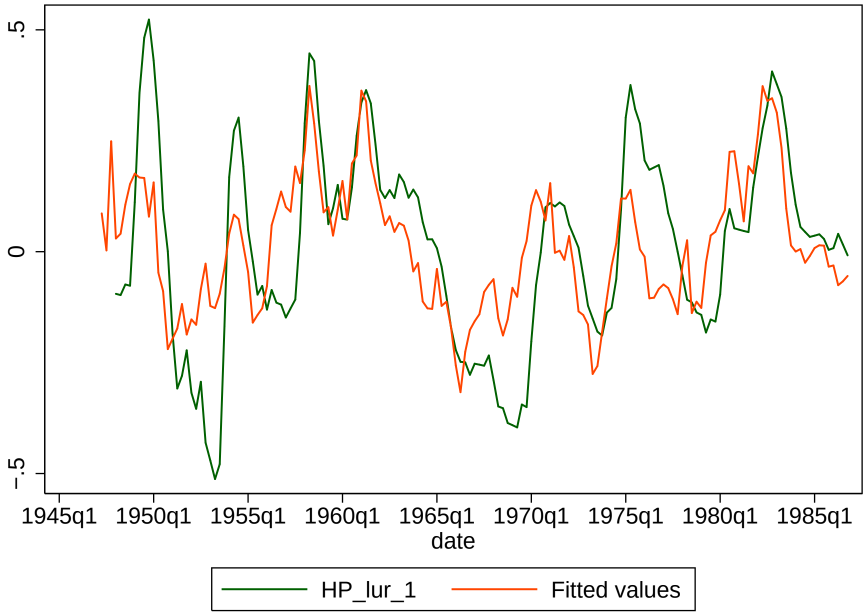
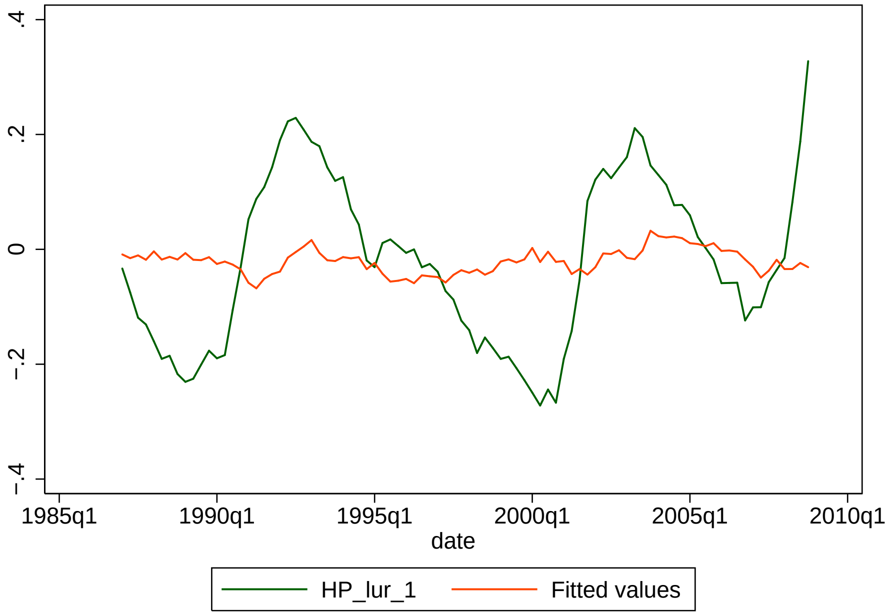


Figure 7: Dynamic Simulation

(a) Before 1987



(b) After 1987



(HP-filtered BLS series of logs with smoothing parameter 10^5)

Figure 8: Fitting the Rate of Unemployment by Labor Productivity

3 Wages

3.1 Present Values

Let $W_t(w)$ denote the present value of a wage contract w to the worker at time t. We write $W_t(w) \equiv W_i(w; u_t)$ if the productivity state at the beginning of period t is i and market tightness is θ_t. The Bellman equation for $W_t(w)$ is:

$$W_t(w) = w + \frac{1}{1+r} \sum_j \pi_{ij} \left[s U_j + (1-s) \Big(\lambda f(\theta_{t+1})(S_j + U_j) + [1 - \lambda f(\theta_{t+1})] W^*_{t+1}(w) \Big) \right].$$

In period $t+1$, the new productivity level j is first realized. Then, the separation shock is drawn. If the match is not destroyed, the employee draws an alternative offer with probability $\lambda f(\theta_{t+1})$. In this case, Bertrand competition between the incumbent employer and the poacher drives the wage contract to their common reservation value and the worker gets the whole surplus, S_j. If no outside offer has been drawn, the current contract w is continued if $0 \leq W_{t+1}(w) - U_j \leq S_j$. If, however, the productivity shock moves the current contract outside the bargaining set, i.e. $W_{t+1}(w) - U_j < 0$ or $W_{t+1}(w) - U_j > S_j$, then the contract must be renegotiated. We follow McLeod and Malcomson (1993) and Postel-Vinay and Turon (forth.) and assume that renegotiation takes the worker's surplus $W_{t+1}(w) - U_j$ to the closest point in the bargaining set $[0, S_j]$. That is, if $W_{t+1}(w) - U_j < 0$, the contract is readjusted to 0 and if $W_{t+1}(w) - U_j > S_j$, it is readjusted to S_j. So the continuation value is:

$$W^*_{t+1}(w) \equiv W^*_j(w; u_{t+1}) = \min\{\max\{W_j(w; u_{t+1}), U_j\}, S_j + U_j\}.$$

Using equation (2), we can derive the worker's surplus as:

$$W_t(w) - U_i = w - z_i + \frac{1-s}{1+r} \sum_j \pi_{ij} \left[\lambda f(\theta_{t+1}) S_j + [1 - \lambda f(\theta_{t+1})](W^*_{t+1}(w) - U_j) \right]$$

with

$$W^*_{t+1}(w) - {}_j = \min\{\max\{W_{t+1}(w) - {}_j, 0\}, S_j\}.$$

Define $\underline{w}_i(u)$ such that $W_i(\underline{w}_i; u) - {}_i = 0$ and $\overline{w}_i(u)$ such that $W_i(\overline{w}_i; u) - {}_i = S_i$. At any point in time, the support of the wage distribution lies inside the set $\Omega = \{\underline{w}_i(u), \overline{w}_i(u), i = 1, ..., N, u \in [0, 1]\}$. Let $g_t(w)$ denote the measure of workers employed at wage $w \in \Omega$ at the end of period $t - 1$.

3.2 Wage distribution

Let i be the productivity state of the economy in period $t - 1$. Conditional on the state of the economy not changing between $t - 1$ and t, the measure of wages is updated as follows. The measure $g_{t+1}(\underline{w}_i(u_t))$ of employees paid $\underline{w}_i(u_t)$ at the end of period t is equal to the measure of employees paid that wage in the preceding period who have not been laid off or poached plus the flow measure of previously unemployed workers who receive a job offer:

$$g_{t+1}(\underline{w}_i(u_t)) = (1 - s)[1 - f(\theta_t)]g_t(\underline{w}_i(\theta_t)) + f(\theta_t)u_t$$

where $\theta_t = q^{-1}\left(\frac{c}{S_i}\left(1 + (1 - s)\frac{1-u_t}{u_t}\right)\right)$. The measure of employees paid $\overline{w}_i(u_t)$ is equal to stock that is reconducted in absence of any shock plus the flow of employees who receive an outside offer:

$$g_{t+1}(\overline{w}_i(u_t)) = (1 - s)[1 - f(\theta_t)]g_t(\overline{w}_i(u_t)) + f(\theta_t)(1 - s)(1 - u_t),$$

and, for all $w \in \Omega\backslash\{\underline{w}_i(u_t), \overline{w}_i(u_t)\}$, only the employees who were already paid w and were not laid off or poached remain in the stock of workers paid w:

$$g_{t+1}(w) = (1 - s)[1 - f(\theta_t)]g_t(w).$$

Note that summing up these three equations yields the law of motion for the unemployment rate:

$$\begin{aligned} 1-u_{t+1} &= (1-s)[1- \ f(\theta_t)](1-u_t) + f(\theta_t)u_t + (1-s) \ f(\theta_t)(1-u_t) \\ &= (1-s)(1-u_t) + f(\theta_t)u_t. \end{aligned}$$

If, however, the state of the economy changes between $t-1$ and t, from i to $j \neq i$, then the renegotiation process has to be taken into consideration. Employees paid w such that $W_j(w;u_t) - \ _j < 0$ renegotiate their wages to $\underline{w}_j(u_t)$. Hence,

$$g_{t+1}(\underline{w}_j(u_t)) = (1-s)[1- \ f(\theta_t)] \sum_{w\in\Omega} \mathbf{1}\{W_j(w;u_t) - \ _j < 0\}\, g_t(w) + f(\theta_t)u_t,$$

where $\theta_t = q^{-1}\left(\frac{c}{S_j}\left(1+ \ (1-s)\frac{1-u_t}{u_t}\right)\right)$. Employees paid w such that $W_j(w;u_t) - \ _j \quad S_j$ are forced to accept wage $\overline{w}_j(u_t)$. Hence,

$$\begin{aligned} g_{t+1}(\overline{w}_j(u_t)) = (1-s)[1- \ f(\theta_t)] \sum_{w\in\Omega} \mathbf{1}\{W_j(w;u_t) - \ _j \quad S_j\}\, g_t(w) \\ + (1-s) \ f(\theta_j)(1-u_i). \end{aligned}$$

And for all $w \in \Omega\backslash\{\underline{w}_j(u_t), \overline{w}_j(u_t)\}$,

$$g_{t+1}(w) = (1-s)[1- \ f(\theta_t)]\mathbf{1}\{0 \leq W_j(w;u_t) - \ _j \leq S_j\}\, g_t(w).$$

3.3 An approximating equilibrium

Although the equilibrium process of (u_t, v_t, θ_t) can be easily calculated, wages depend on productivity and continuously updating unemployment. We therefore develop an approximating equilibrium where θ_t jumps to its steady-state value θ_i after a productivity shock.

Let $W_i(w)$ denote the present value of a wage w in state i. If θ_t jumps to θ_j immediately after a shock to productivity y_j,

$$W_i(w) - U_i = w - z_i + \frac{1-s}{1+r} \sum_j \pi_{ij} \Big[\lambda f(\theta_j) S_j + [1 - \lambda f(\theta_j)](W_j^*(w) - U_j) \Big]$$

with

$$W_j^*(w) - U_j = \min\{\max\{W_j(w) - U_j, 0\}, S_j\}.$$

Let $\underline{w}_i$ and $\overline{w}_i$ be such that $W_i(\underline{w}_i) - U_i = 0$ and $W_i(\overline{w}_i) - U_i = S_i$. Then, for all k,

$$\begin{aligned} W_k(\underline{w}_i) - U_k - (W_i(\underline{w}_i) - U_i) &= W_k(\underline{w}_i) - U_k \\ &= z_i - z_k + \frac{1-s}{1+r} \sum_j (\pi_{kj} - \pi_{ij}) \Big[\lambda f(\theta_j) S_j \\ &\qquad + [1 - \lambda f(\theta_j)](W_j^*(\underline{w}_i) - U_j) \Big] \end{aligned}$$

and

$$\begin{aligned} W_k(\overline{w}_i) - U_k - (W_i(\overline{w}_i) - U_i) &= W_k(\overline{w}_i) - U_k - S_i \\ &= z_i - z_k + \frac{1-s}{1+r} \sum_j (\pi_{kj} - \pi_{ij}) \Big[\lambda f(\theta_j) S_j \\ &\qquad + [1 - \lambda f(\theta_j)](W_j^*(\overline{w}_i) - U_j) \Big]. \end{aligned}$$

Having determined $W_k(\underline{w}_i) - U_k$ and $W_k(\overline{w}_i) - U_k$, wages then follow as

$$\underline{w}_i = z_i - \frac{1-s}{1+r} \sum_j \pi_{ij} \Big(\lambda f(\theta_j) S_j + [1 - \lambda f(\theta_j)](W_j^*(\underline{w}_i) - U_j) \Big)$$

and

$$\begin{aligned} \overline{w}_i &= S_i + z_i - \frac{1-s}{1+r} \sum_j \pi_{ij} \Big(\lambda f(\theta_j) S_j + [1 - \lambda f(\theta_j)](W_j^*(\overline{w}_i) - U_j) \Big) \\ &\Leftrightarrow \ \overline{w}_i = y_i - \frac{1-s}{1+r} \sum_j \pi_{ij} [1 - \lambda f(\theta_j)](W_j^*(\overline{w}_i) - U_j - S_j). \end{aligned}$$

We use a simple fixed point algorithm (of the form $x_n = T x_{n-1}$) to determine surpluses $W_k(\underline{w}_i) - U_k$ and $W_k(\overline{w}_i) - U_k$, and then wages $\underline{w}_i$ and $\overline{w}_i$.

3.3.1 Wage distribution.

The support of the wage distribution is the set $\Omega = \{\underline{w}_i, \overline{w}_i, i = 1, ..., N\}$. Let $g_t(w|i)$ denote the measure of workers employed at wage $w \in \Omega$ at time t and given that the state of the economy is i.

Conditional on the state of the economy not changing between t and $t+1$, the wage distribution is updated as:

$$\begin{aligned} g_{t+1}(\underline{w}_i|i) &= (1-s)[1-\lambda f(\theta_i)]g_t(\underline{w}_i|i) + f(\theta_i)u_t, \\ g_{t+1}(\overline{w}_i|i) &= (1-s)[1-\lambda f(\theta_i)]g_t(\overline{w}_i|i) + (1-s)\lambda f(\theta_i)(1-u_t), \end{aligned}$$

and, for all $w \in \Omega \backslash \{\underline{w}_i, \overline{w}_i\}$,

$$g_{t+1}(w|i) = (1-s)[1-\lambda f(\theta_i)]g_t(w|i).$$

Note that, summing over all $w \in \Omega$ yields the law of motion for unemployment:

$$u_{t+1} = u_t - f(\theta_i)u_t + s(1-u_t).$$

Tightness jumps but not unemployment.

If, however, the productivity state moves from i to $j \neq i$ at the beginning of period t, then,

$$g_{t+1}(\underline{w}_j|j) = (1-s)[1-\lambda f(\theta_j)] \sum_{w \in \Omega} \mathbf{1}\{W_j(w) - U_j \leq 0\}\, g_t(w|i) + f(\theta_j)u_t,$$

and

$$g_{t+1}(\overline{w}_j|j) = (1-s)[1-\lambda f(\theta_j)] \sum_{w\in\Omega} \mathbf{1}\{W_j(w) - U_j \geq S_j\}\, g_t(w|i) + (1-s)\lambda f(\theta_j)(1-u_t),$$

and, for all $w \in \Omega\backslash\{\underline{w}_j, \overline{w}_j\}$,

$$g_{t+1}(w|j) = (1-s)[1-\lambda f(\theta_j)]\mathbf{1}\{0 \leq W_j(w) - U_j < S_j\}\, g_t(w|i).$$

Unemployment is updated as:

$$u_{t+1} = u_t - f(\theta_j)u_t + s(1-u_t).$$

3.4 Simulations

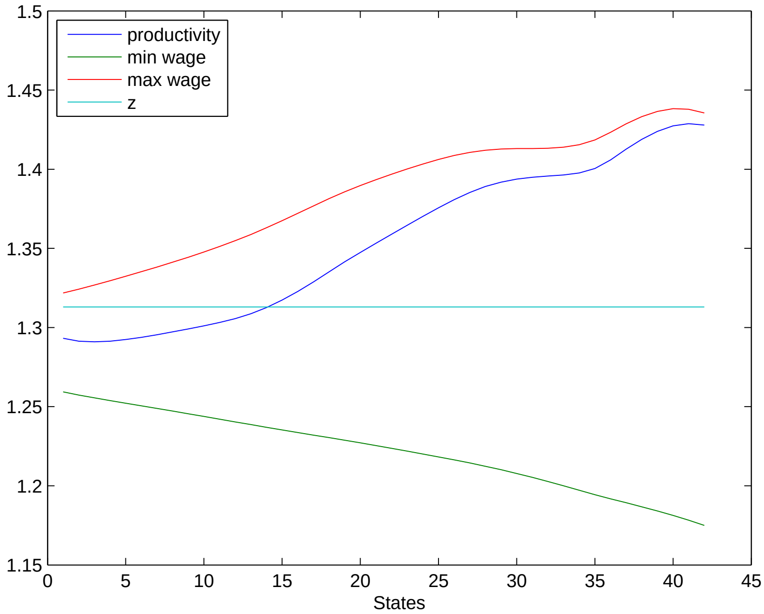
At the cost of renormalizing $c = 1-\alpha$, we may as well assume that the opportunity cost of employment is

$$z_i = z_0 + \alpha(y_i - z_0)$$

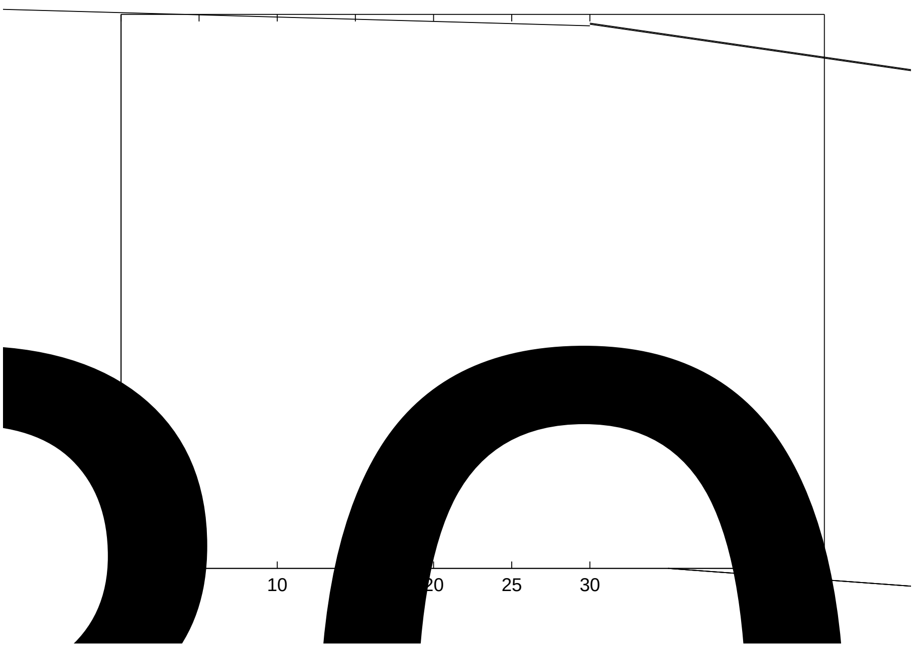
for the same calibrated value of 1.313 for z_0 and $\alpha \in [0, 1)$. Then, $y_i - z_i = (1-\alpha)(y_i - z_0)$ and everything holds with $\frac{1-\alpha}{c}$ in lieu of $\frac{1}{c}$ as unidentified scale factor. This looks as an innocuous change. However, with $\alpha \geq 0$, z_i is increasing with y_i. If $\alpha = 0$ then $z_i = z_0$ is constant and unemployed workers' reservation wages decrease with productivity as they accept a lower initial wage if future prospects improve. Figure 9 illustrates this point by showing how the wage bounds vary with productivity for two choices of α: 0 and 0.65.[2] The main result is that the cross-sectional wage variance always increases with productivity. The distribution is strongly skewed to the right, as can be seen by the relative proximity of the median (D5) with the 9th decile (D9).

[2]We burn 200 initial time periods to reach a stationary state.

(a) ✎ = 0


productivity
min wage
max wage
z
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
0
5
10
15
20
25
30
35
40
45
States

(b) ✎ = 0.65


10
25
30

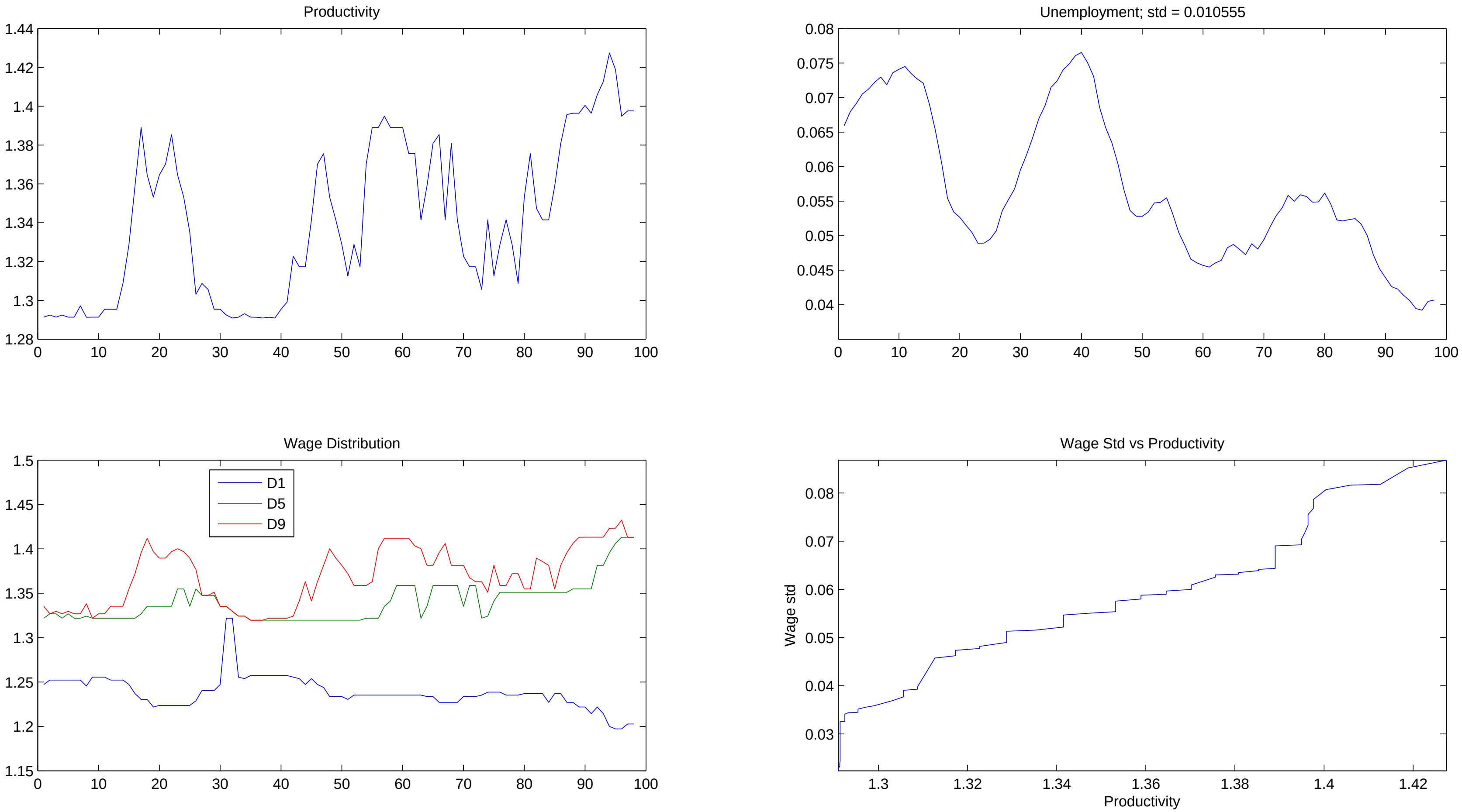


Figure 10: Wage Simulation - ϕ = 0

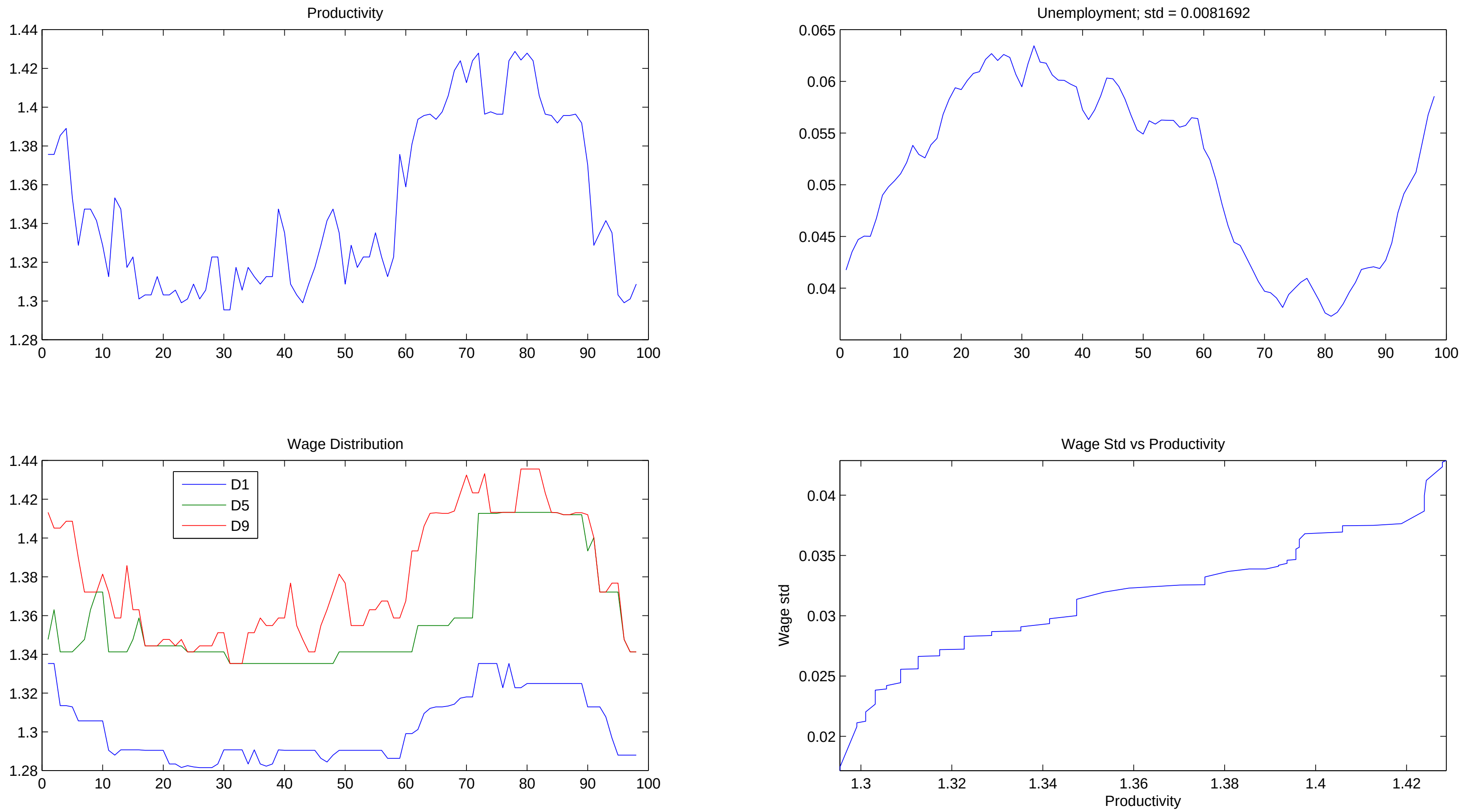


Figure 11: Wage Simulation - $= 0.65$

4 Worker heterogeneity

Suppose that there are M types of workers, and ℓ_m workers of each type (with $\sum_m \ell_m = 1$). Each type is characterized by a time-invariant characteristic x_m, $\ = 1, ..., M$, with $x_m < x_{m+1}$, such that the per-period output of a match (x_m, y_i) is $Q(x_m, y_i) \equiv y_i(\ )$. We assume that firms cannot direct their search to specific worker types. Let $S_i(\ )$ denote the corresponding surplus:

$$S_i(\ ) = y_i(\ ) - z_i(\ ) + \frac{1-s}{1+r} \sum_j \pi_{ij} S_j(\ )^+$$

with an obvious notation for $z_i(\ )$ and where $x^+ = \max(x, 0)$. After a productivity shock from i to j all matches yielding negative surplus are destroyed. Similarly, the value of uneployment is

$$_i(\ ) = z_i(\ ) + \frac{1}{1+r} \sum_j \pi_{ij} \ _j(\ ).$$

Market tightness is still defined as $\theta_t = \frac{v_t}{u_t + \kappa(1-s)(1-u_t)}$ where

$$u_t = \sum_{m=1}^{M} u_t(\ )\ell_m$$

is the aggregate unemployment rate, $u_t(\ )$ denoting the fraction of unemployed within group $\ $ of workers. We assume for simplicity the same $\ $ and s for each group of workers.

Let i be the state at the begining of period t. The free entry condition becomes:

$$cv_t = f(\theta_t) \sum_{m=1}^{M} u_t(\ )\ell_m S_i(\ )^+.$$

as only workers whose production generates a positive surplus can find a job. The law of motion of individual-specific unemployment rates is

$$u_{t+1}(\ ) = 1 - [(1-s)(1-u_t(\ )) + f(\theta_t)u_t(\ )]\mathbf{1}\{S_i(\ ) \quad 0\}.$$

Notice the disymmetry of match creation and match destruction. If $S_i(\quad) \leq 0$, all active matches are destroyed. But if $S_i(\quad)\quad 0$, only a fraction of unemployed workers fnd a job by the end of the period.

4.1 Steady-state

If the economy remains in state i for ever, the unemployment rate in group is

$$u_i(\quad) = \frac{s}{s+f_i}\mathbf{1}\{S_i(\quad)\quad 0\} + \mathbf{1}\{S_i(\quad) \leq 0\},$$

with $f_i \equiv f(\theta_i)$. The aggregate unemployment rate is:

$$u_i = \sum_{m=1}^{M} u_i(\quad)\ell_m = \frac{s}{s+f_i}L_i + 1 - L_i = 1 - \frac{f_i}{s+f_i}L_i,$$

where $L_i = \sum_{m=1}^{M} \ell_m \mathbf{1}\{S_i(\quad)\quad 0\}$ is the number of employable workers. Total search effort becomes

$$u_i + \quad (1-s)(1-u_i) = \frac{s + [1 - L_i + \quad (1-s)L_i]f_i}{s+f_i}$$

The free entry condition finally takes the following form:

$$\begin{aligned} c\theta_i &= f_i \sum_{m=1}^{M} \frac{u_i(\quad)\ell_m}{u_i + \quad (1-s)(1-u_i)} S_i(\quad)^+ \\ &= \frac{sf_i}{s + [1 - L_i + \quad (1-s)L_i]f_i} S_i \end{aligned}$$

with $S_i = \sum_{m=1}^{M} \ell_m S_i(\quad)^+$ being the aggregate surplus value. Therefore, the exit rate from unemployment is the following fixed point:

$$f_i = f\left(\frac{sf_i}{s + [1 - L_i + \quad (1-s)L_i]f_i}\frac{S_i}{c}\right).$$

4.2 Wages

Assuming as previously that θ_t jumps to its steady-state value θ_i after a productivity shock to i. Let $W_i(w, \ )$ denote the present value of a wage w in state i to a worker of type $\ $:.

$$W_i(w, \ ) - \ _i(\ ) = w - z_i(\ ) + \frac{1-s}{1+r} \sum_j \pi_{ij} \mathbf{1}\{S_j(\ ) \quad 0\} \Big[\ f(\theta_j) S_j(\ )$$

$$+[1 - \ f(\theta_j)](W_j^*(w, \ ) - \ _j(\ )) \Big]$$

with

$$W_j^*(w, \ ) - \ _j(\ ) = \min\{\max\{W_j(w, \ ) - \ _j(\ ), 0\}, S_j\}.$$

Let $\underline{w}_i(\ )$ and $\overline{w}_i(\ )$ be such that $W_i($

and

$$W_{k,i}(\ ) - S_i(\ ) = z_i(\ ) - z_k(\ ) + \frac{1-s}{1+r}\sum_j (\ _{kj} - \ _{ij})\mathbf{1}\{S_j(\ )\quad 0\}\times$$
$$\left[\ f(\ _j)S_j(\ ) + [1-\ f(\ _j)]W^*_{j,i}(\ )\right].$$

Having determined $\underline{W}_{k,i}(\ )$ and $\overline{W}_{k,i}(\ )$ for all k, i and , wages then follow as

$$\underline{w}_i(\ ) = z_i(\ ) - \frac{1-s}{1+r}\sum_j \ _{ij}\mathbf{1}\{S_j(\ )\quad 0\}\Big(\ f(\ _j)S_j(\ ) + [1-\ f(\ _j)]\underline{W}^*_{j,i}(\ )\Big)$$

and

$$\overline{w}_i(\ ) = S_i(\ ) + z_i(\ ) - \frac{1-s}{1+r}\sum_j \ _{ij}\mathbf{1}\{S_j(\ )\quad 0\}\Big(\ f(\ _j)S_j(\ ) + [1-\ f(\ _j)]\overline{W}^*_{j,m}(\ )\Big).$$

4.3 Wage distribution

The support of the wage distribution is the union of all sets $\Omega_m = \{\underline{w}_i(\ ), \overline{w}_i(\ ), \forall i\}$. Let $g_t(w|i,\ )$ denote the measure of workers employed at wage $w \in \Omega$ at time t and given that the state of the economy is i and the worker type is i.

Conditional on the state of the economy not changing between t and $t+1$, the wage distribution is updated as:

$$g_{t+1}(\underline{w}_i(\ )|i,\ ) = [(1-s)[1-\ f(\ _i)]g_t(\underline{w}_i(\ )|i) + f(\ _i)u_t(\ )]\mathbf{1}\{S_i(\ )\quad 0\},$$

$$g_{t+1}(\overline{w}_i(\ )|i,\ ) = \Big[(1-s)[1-\ f(\ _i)]g_t(\overline{w}_i(\ )|i,\ )$$
$$+ (1-s)\ f(\ _i)(1-u_t(\ ))\Big]\mathbf{1}\{S_j(\ )\quad 0\},$$

and, for all $w \in \Omega \backslash \{\underline{w}_i(\ ), \overline{w}_i(\ )\}$,

$$g_{t+1}(w|i,\ ) = (1-s)[1-\lambda f(\theta_i)] g_t(w|i,\ ) \mathbf{1}\{S_j(\ )\ \ 0\}.$$

If, however, the productivity state moves from i to $j \neq i$ at the beginning of period t, then,

$$g_{t+1}(\underline{w}_j(\ )|j,\ ) = \Big[(1-s)[1-\lambda f(\theta_j)] \sum_{w \in \Omega_m} \mathbf{1}\{\Delta_j(w,\ ) - \ _j(\ ) \le 0\}\, g_t(w|i,\ ) + f(\theta_j) u_t(\ )\ell_m \Big] \mathbf{1}\{S_j(\ )\ \ 0\},$$

where $\Delta_j(w,\ ) - \ _j(\ ) = \underline{\Delta}_{j,k}(\ )$ or $\overline{\Delta}_{j,k}(\ )$ depending on whether $w = \underline{w}_k(\ )$ or $\overline{w}_k(\ )$. Moreover,

$$g_{t+1}(\overline{w}_j(\ )|j) = \Big[(1-s)[1-\lambda f(\theta_j)] \sum_{w \in \Omega_m} \mathbf{1}\{\Delta_j(w,\ ) - \ _j(\ ) \ge S_j(\ )\}\, g_t(w|i,\ ) + (1-s)\lambda f(\theta_j)(1-u_t(\ ))\ell_m \Big] \mathbf{1}\{S_j(\ )\ \ 0\},$$

and, for all $w \in \Omega \backslash \{\underline{w}_j(\ ), \overline{w}_j(\ )\}$,

$$g_{t+1}(w|j) = (1-s)[1-\lambda f(\theta_j)] \mathbf{1}\{0 < \Delta_j(w,\ ) - \ _j(\ ) < S_j\}\, g_t(w|i,\ ) \mathbf{1}\{S_j(\ )\ \ 0\}.$$

4.4 Application

We specify a CES production function:

$$Q(x_m, y_i) = y_i(\ ) = \left(x_m^{\frac{\sigma-1}{\sigma}} + y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

Then,

$$\frac{\partial Q(x,y)}{\partial x \partial y} = \frac{1}{\ } x_m^{\frac{-1}{\sigma}} y_i^{\frac{-1}{\sigma}} \left(x_m^{\frac{\sigma-1}{\sigma}} + y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{2-\sigma}{\sigma-1}}$$

and

$$\frac{\partial \ln Q(x,y)}{\partial x \, \partial y} = -\frac{\quad - 1}{\qquad} x_m^{\frac{-1}{\sigma}} y_i^{\frac{-1}{\sigma}} \left(x_m^{\frac{\sigma-1}{\sigma}} + y_i^{\frac{\sigma-1}{\sigma}} \right)^{-2}.$$

Hence, $y_i(\quad + 1) - y_i(\quad)$ increases with i or y_i if $\quad 0$, and $\frac{y_i(m+1)}{y_i(m)}$ increases if $1/\quad 1$.

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